$\underline{P_1|}$ • $P(x)$ de grado 3

(a) $P(x) = ax^3 + bx^2 + cx + d$

• $P(x)$ mónico $\longrightarrow a = 1$

$P(x) = x^3 + bx^2 + cx + d$

• $(x-2)$ factor $\longrightarrow P(2) = 0$

$8 + 4b + 2c + d = 0 \quad (1)$

• $(x+1)$ factor $\longrightarrow P(-1) = 0$

$-1 + b - c + d = 0 \quad (2)$

• $P(x) : (x-1)$ tiene resto 2 $\rightarrow P(1) = 2$

$1 + b + c + d = 2 \quad (3)$

Tenemos:

$(1) \quad 4b + 2c + d = -8$

$(2) \quad b - c + d = 1$

$(3) \quad b + c + d = 1$

$\longrightarrow (3)-(2) \quad 2c = 0 \Rightarrow \boxed{c = 0}$

$(1) \quad 4b + d = -8$

$(2) \quad b + d = 1$

$\longrightarrow (1)-(2) \quad 3b = -9 \Rightarrow \boxed{b = -3}$

$(2) \quad -3 + d = 1 \longrightarrow \boxed{d = 4}$

$$\boxed{P(x) = x^3 - 3x^2 + 4}$$

(b) ¿Es -2 raíz?

Reemplazamos: $P(-2) = -8 - 12 + 4 = -16 \neq 0$

luego, -2 no es raíz de $P(x)$.

(c) raíces racionales

$P(x) = x^3 - 3x^2 + 4$

Sabemos que $(x-2)$ divide a $P(x)$. Así

$$
\begin{array}{l}
(x^3-3x^2+4):(x-2) = x^2 - x - 2 \\
-(x^3-2x^2) \\
\hline
0 - x^2 + 4 \\
\quad -(-x^2+2x) \\
\hline
\quad 0 \quad -2x+4 \\
\qquad -(-2x+4) \\
\hline
\qquad\quad 0
\end{array}
$$

$\Rightarrow P(x) = (x-2)(x^2-x-2)$

$= (x-2)(x-2)(x+1)$

Raíces racionales:

$x = 2, \quad x = -1$

$\underline{P_2}$ | $G(x) = 0{,}01(x^3 - 5x^2 - 44x - 60)$

factoricemos $G(x)$!

Raíces racionales, $\frac{p}{q}$ donde $p \mid -60$ y $q \mid 1$

$\{\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12, \pm 15, \pm 20, \pm 30, \pm 60\}$

Con $x = -2$:

$G(-2) = -8 - 20 + 88 - 60 = -88 + 88 = 0$ ✓

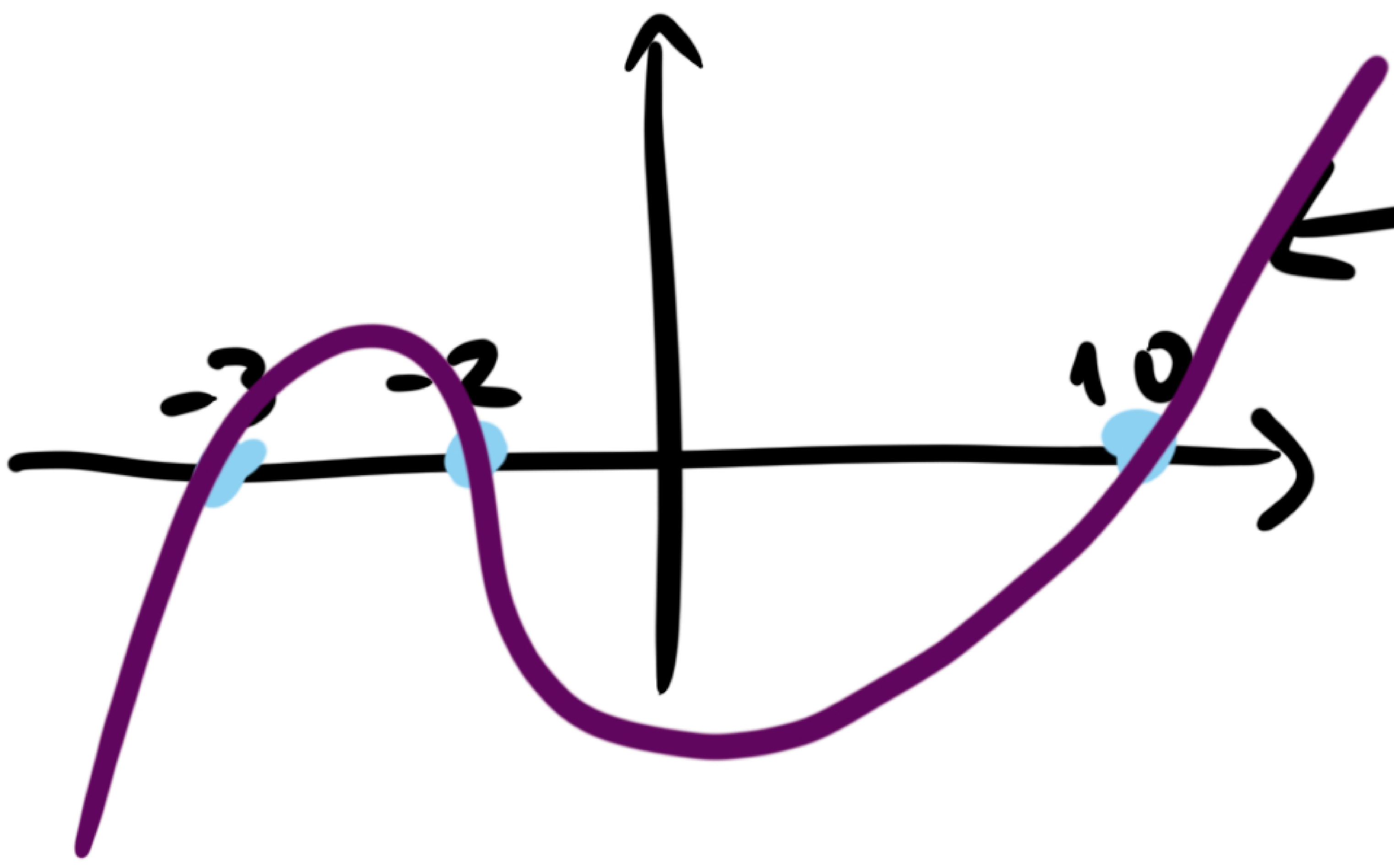
$\Rightarrow -2$ es raíz. $\therefore G(x) = q(x)\cdot(x+2)$

dividiendo:

$$(x^3 - 5x^2 - 44x - 60) : (x+2) = x^2 - 7x - 30$$
$$-(x^3 + 2x^2)$$
$$0 - 7x^2 - 44x$$
$$-(-7x^2 - 14x)$$
$$0 - 30x - 60$$
$$-(-30x - 60)$$
$$0 \,//$$

$$G(x) = (x+2)(x^2 - 7x - 30)\cdot 0{,}01$$
$$= (x+2)(x-10)(x+3)\cdot 0{,}01$$

	$-\infty$ a -3	-3 a -2	-2 a 10	10 a ∞
$x+3$	$-$	0 $+$	$+$	$+$
$x+2$	$-$	$-$	0 $+$	$+$
$x-10$	$-$	$-$	$-$	0 $+$
	$(-)$	$(+)$	$(-)$	$(+)$



ya que x positivo por el problema el primer valor x con $G(x) \geq 0$ es $\underline{x = 10}$ /

P3 $P(x) = Kx^3 + x^2 - K^2 + 11$

(a) determine K para que $(x+1)$ factor de $P(x)$

Basta con que $x = -1$ sea raíz de $P(x)$!

$$P(-1) = -K + 1 - K^2 + 11 \overset{!}{=} 0 \qquad | \cdot (-1)$$

$$\Leftrightarrow K^2 + K - 12 = 0$$

$$\Leftrightarrow (K+4)(K-3) = 0 \Rightarrow \underline{K = -4} \text{ o } \underline{K = 3}$$

$\underline{\text{con } k=-4}$ | $P(x) = -4x^3 + x^2 - 5$ $(x+1)$ factor!

$$(-4x^3+x^2-5):(x+1) = -4x^2 + 5x - 5$$

$$\begin{array}{l} -(-4x^3-4x^2) \\ \hline 0 + 5x^2 - 5 \\ \quad -(5x^2+5x) \\ \hline \quad 0 \quad -5x-5 \\ \qquad -(-5x-5) \\ \hline \qquad\quad 0 \end{array}$$

$\Rightarrow P(x) = (x+1)(-4x^2+5x-5)$

$\Delta = 25 - 4(-4)(-5)$
$= 25 - 80 < 0$
$\Rightarrow$ no tiene raíces reales

$\Rightarrow$ $\underline{\text{Solo } -1}$

$\underline{\text{con } k=3}$ | $P(x) = 3x^3 + x^2 + 2$ $(x+1)$ factor

$$(3x^3+x^2+2):(x+1) = 3x^2 - 2x + 2$$

$$\begin{array}{l} -(3x^3+3x^2) \\ \hline 0 - 2x^2 + 2 \\ \quad -(-2x^2-2x) \\ \hline \quad 0 + 2x + 2 \\ \qquad -(2x+2) \\ \hline \qquad\quad 0 \end{array}$$

$\Rightarrow P(x) = (x+1)(3x^2-2x+2)$

$\Delta = 4 - 4 \cdot 3 \cdot 2$
$= 4 - 24 = -20 < 0$
$\Rightarrow$ no tiene raíces reales

$\Rightarrow$ $\underline{\text{Solo } -1}$

P4 $P(x) = 9x^3 - 6x^2 - 5x + 2$

(a) factorice

Raíces racionales: $\left\{\pm 1, \pm \frac{1}{3}, \pm \frac{1}{9}, \pm 2, \pm \frac{2}{3}, \pm \frac{2}{9}\right\}$

Con $x=1$: $P(1) = 9 - 6 - 5 + 2 = 11 - 11 = 0$

$\Rightarrow$ $x=1$ es raíz $\rightarrow$ $(x-1)$ factor.

$$
\begin{array}{l}
(9x^3 - 6x^2 - 5x + 2) : (x-1) = 9x^2 + 3x - 2 \\
\underline{-(9x^3 - 9x^2)} \\
\quad 0 + 3x^2 - 5x \\
\quad\quad \underline{-(3x^2 - 3x)} \\
\quad\quad\quad 0 \quad -2x + 2 \\
\quad\quad\quad\quad \underline{-(-2x + 2)} \\
\quad\quad\quad\quad\quad 0
\end{array}
$$

$\Rightarrow P(x) = (x-1)(9x^2 + 3x - 2)$

Raíces:

$\Delta = 9 - 4 \cdot 9 \cdot (-2)$

$= 9 + 72 = 81$

$\Rightarrow x = \dfrac{-3 \pm 9}{18}$

$x_1 = 6/18 = 1/3$

$x_2 = \dfrac{-12}{18} = \dfrac{-2}{3}$

$9x^2 + 3x - 2$

$= 9\left(x - \frac{1}{3}\right)\left(x + \frac{2}{3}\right)$

$= (3x - 1)(3x + 2)$

$\Rightarrow P(x) = (x-1)(3x-1)(3x+2)$

$P(x) = (x-1)(3x-1)(3x+2)$

b) $P(x) > (3x+2)$

$\Leftrightarrow (x-1)(3x-1)(3x+2) > (3x+2)$

$\Leftrightarrow (x-1)(3x-1)(3x+2) - (3x+2) > 0$

$\Leftrightarrow \left[(x-1)(3x-1) - 1\right](3x+2) > 0$

$\Leftrightarrow (3x^2-4x+1-1)(3x+2) > 0$

$\Leftrightarrow (3x^2-4x)(3x+2) > 0$

$\Leftrightarrow x(3x-4)(3x+2) > 0$

P.C: $x=0$, $x=4/3$, $x=-2/3$

	$-\infty$ a $-2/3$	$-2/3$ a 0	0 a $4/3$	$4/3$ a ∞
x	$-$	$-$	$+$	$+$
$3x-4$	$-$	$-$	$-$	$+$
$3x+2$	$-$	$+$	$+$	$+$
	$(-)$	$(+)$	$(-)$	$(+)$

$S = \left]\frac{-2}{3}, 0\right[\cup \left]\frac{4}{3}, \infty\right[$

c) S es acotado inferior pero no superiormente.

$CS(S) = \emptyset$ $\quad \inf(S) = \frac{-2}{3}$ $\quad \min(S) \nexists$

$CI(S) = \left]-\infty, \frac{-2}{3}\right]$ $\quad \sup(S) \nexists$ $\quad \max(S) \nexists$