Universidad de Chile
Programa Académico de Bachillerato
Álgebra y Geometría, segundo semestre 2020

Ejercicio resuelto 9

1. Encuentre la matriz representante, en bases canónicas, de las siguientes transformaciones lineales:

   (a) $T : M_{23}(\mathbb{R}) \to M_{32}(\mathbb{R})$ tal que $T(A) = A^T$.

   (b) $T : \mathbb{R}^2 \to \mathbb{R}^3$ tal que $T\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ y $T\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$.

Solución. **(a)** Sean $C = \{v_1, v_2, \ldots, v_6\}$ la base canónica de $M_{23}(\mathbb{R})$ y $C' = \{w_1, w_2, \ldots, w_6\}$ la base canónica de $M_{32}(\mathbb{R})$. Entonces

$$T\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = w_1 ,$$

$$\Rightarrow [Tv_1]_{C'} = (1,0,0,0,0,0)$$

$$T\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix} = w_5 ,$$

$$\Rightarrow [Tv_3]_{C'} = (0,0,0,0,1,0)$$

$$T\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} = w_4 ,$$

$$\Rightarrow [Tv_5]_{C'} = (0,0,0,1,0,0)$$

$$T\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} = w_3 ,$$

$$\Rightarrow [Tv_2]_{C'} = (0,0,1,0,0,0)$$

$$T\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = w_2 ,$$

$$\Rightarrow [Tv_4]_{C'} = (0,1,0,0,0,0)$$

$$T\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} = w_6$$

$$\Rightarrow [Tv_6]_{C'} = (0,0,0,0,0,1) ,$$

$$\Rightarrow [T]_C^{C'} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Solución. (b) Sean

$$C = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \text{ y } C' = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

las bases canónicas de $\mathbb{R}^2$ y $\mathbb{R}^3$ respectivamente. Tenemos que

$$\begin{aligned} \begin{pmatrix} 1 \\ 0 \end{pmatrix} &= \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} \\ \Rightarrow T\begin{pmatrix} 1 \\ 0 \end{pmatrix} &= T\begin{pmatrix} 2 \\ -1 \end{pmatrix} + T\begin{pmatrix} -1 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]_{C'} \end{aligned}$$

$$\begin{aligned} \begin{pmatrix} 0 \\ 1 \end{pmatrix} &= \begin{pmatrix} 2 \\ -1 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 1 \end{pmatrix} \\ \Rightarrow T\begin{pmatrix} 0 \\ 1 \end{pmatrix} &= T\begin{pmatrix} 2 \\ -1 \end{pmatrix} + 2T\begin{pmatrix} -1 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix} = \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]_{C'} \end{aligned}$$

$$\Rightarrow [T]_C^{C'} = \begin{pmatrix} 3 & 5 \\ 2 & 3 \\ 1 & 2 \end{pmatrix}.$$