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ON THE INTERNAL STABILITY OF GRANULAR SOILS

R. Jonathan Fannin, P.Eng., Professor of Civil Engineering, UBC, Vancouver, British Columbia, Canada
Ricardo Moffat, Graduate Research Assistant, UBC, Vancouver, British Columbia, Canada

ABSTRACT:

The onset of internal instability in potentially unstable soils is governed by geometric and hydrodynamic constraints. Interpretations of laboratory studies on reconstituted specimens have led, in the last 25 years, to empirical criteria that define a threshold to the onset of instability. The development of those empirical criteria is reviewed. New laboratory data are then presented, and compared with selected data reported in the literature. They broadly confirm the relevance of the empirical design criteria which, it is noted, address only geometric constraints to internal stability. In practice, concern exists for the risk posed by seepage flow through potentially unstable soil: a need remains to better address hydrodynamic influences, and resulting total loss of soil.

RÉSUMÉ:

Le commencement de l'instabilité interne des sols potentiellement instables est gouverné par des contraintes géométriques et hydrodynamiques. L'interprétation d'études menées en laboratoire sur des specimens reconstitués a mené, durant les 25 dernières années, à des critères empiriques définissant le seuil du commencement de l'instabilité. Le développement de ces critères empiriques est passé en revue dans cet article. De nouvelles données expérimentales sont présentées et compares avec des données sélectionnées provenant de la littérature. D'une manière générale, ces données confirment la pertinence des critères de conception empiriques qui, il est noté, considèrent seulement les contraintes géométriques de stabilité interne. En pratique, il existe des inquietudes concernant le risque posé par l'écoulement à travers les sols potentiellement instables et il reste donc un besoin de prendre en compte les influences hydrodynamiques et les pertes de sol totales résultantes.

1. INTRODUCTION

The term "filtration", as used with reference to civil engineering works, describes the restriction of particle migration from a soil (the "base" soil) into or through an adjacent medium (the "filter" material) as a consequence of groundwater seepage. The filtration process itself is predicated on the development, over time, of a stable interface between the base soil and the filter material. In construction practice, there is a considerable body of experience with the use of granular soils as a filter material, and a growing body of experience with the use of geotextiles as a filter material. Irrespective of the filter material, granular or geotextile, the principal design criteria against which performance is assessed are a criterion for retention of the base soil and a criterion for relative permeability at the interface of base soil and filter. Additionally, a granular filter should not become unduly segregated during construction, as a result of the method of placement, and should not be susceptible to any significant loss of the finer fraction of the grading curve during the service life, as a consequence of seepage flow. The design criteria are empirical, and are typically derived from experimental studies on laboratory test specimens.

In this paper we review a series of experimental studies that have been instrumental in guiding the development of empirical design criteria for granular filters. Specific consideration is given to the scope of the experimental work, its direct relevance to field conditions, and the implications for confidence in assessing the potential for internal instability of a soil.

2. CHARACTERISTICS OF GRANULAR FILTERS

Granular filters are primarily specified with reference to the range and shape of the particle size distribution curve, with additional consideration given to the mineralogy of the granular material and the thickness to which it is placed. A schematic illustration of the causal relations between characteristics of a granular filter, and functional requirements against which performance is assessed, is given in Figure 1. Three of those functional requirements are:

- base soil retention
- permeability
- internal stability

The characteristic size of the finer fraction (for example, D_{15}) influences the size distribution of the pores, and hence the capacity for retention of the base soil, and the permeability of the filter itself. The quantity and size of the smallest particles also exert an influence on permeability. Lastly, the gradation of the filter determines the potential for any internal instability and hence internal migration of fines.

In addition, there are two functional requirements against which the ease of construction is assessed, namely:

- segregation potential
- placement and durability

The quantity and size of the largest particles exert an influence on segregation potential, as does the shape of the gradation curve. Mineralogy of the granular material, and thickness to which it is placed, act to control the durability and construction method respectively.

The causal relations illustrated in Figure 1 are commonly described by a series of design criteria that should be satisfied by the granular filter. The design criteria are empirical, in that they have been established from interpretation of experimental observations, with occasional consideration of theoretical analysis and practical constraints. A brief review of the origins of these empirical design criteria is given below, with an emphasis placed on the scope of the laboratory test programme and the range of test variables that was examined. A summary of the selected filtration studies is given in Table 1.

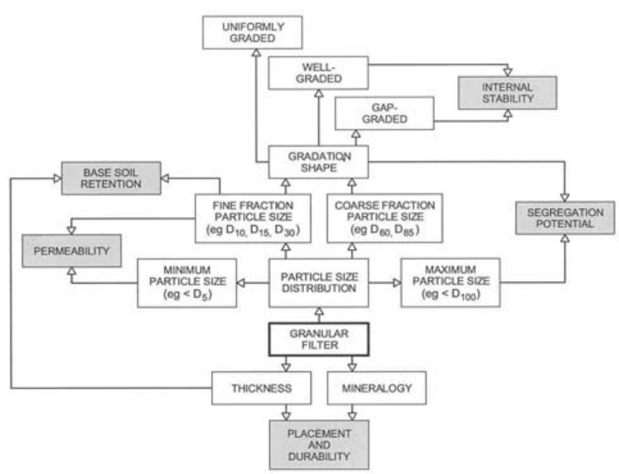

Figure 1. Functional requirements of granular filters

Table 1. A summary of selected laboratory filtration studies

Author	Specimen Size	Surcharge	Water Quality	Hydraulic gradient	Flow direction	Vibration	Duration
Bertram (1940)	h = 6 cm d = 5 or 10 cm	no surcharge	distilled and de-aired	6-8; 18-20; 10-12	downward or upward	no	2-4 h;
Karpoff (1955)	h = 20 cm d = 20 cm	no surcharge	not mentioned	1 - 24	downward	no	not mentioned
Sherard et al. (1984)	h = 16.5 cm d = 10 cm.	no surcharge	distilled (fine soil) tap water (coarse soil)	4 kg/cm^2 supply pressure	downward	optional shaking	5-10 min
Lafleur (1984)	h = 15 cm d = 15 cm	back pressure	not mentioned	8	downward	no	50 - 880 h
Kenney and Lau (1985)	h =20-50 cm d =24.5-58 cm	10 kPa	re-circulated water	"large seepage velocity"	downward	manual tapping	30 - 100 h
Skempton and Brogan (1994)	h =15.5 cm d = 13.9 cm	no surcharge	not mentioned	0 - 1	upward	no	1.5 h
Honjo et al. (1996)	h =10 cm d = 15 and 30 cm	0.9 kPa	tap water	36-52 kPa supply pressure, or 2.5 - 19 gradient	downward	manual tapping	2 h
Tomlinson and Vaid (2000)	h =4 cm d = 10 cm	50 - 400 kPa	tap water	25	downward	no	3 1/2 h
UBC (current study)	h = 10 cm d = 10 cm	25 kPa	distilled and de-aired	0.1-15	downward	automatic tapping	11 h

2.1 A review of selected filtration studies

Bertram (1940) reported the results of tests, conducted under the supervision of Casagrande, which established key issues for consideration in laboratory studies of permeability and, correspondingly, on studies of filtration phenomena. Permeation of the soil specimen with de-aired water was found essential in all but the shortest of tests, since it eliminates the opportunity for any dissolved air in a water supply to come out of solution in the specimen during testing. The presence of air yields a reduction in the degree of saturation and hence in permeability. Use of distilled water was also recommended, to prevent any tendency for suspended solids to accumulate within the specimen during testing, which again would yield a reduction in permeability. Constant-head tests (see Figure 2) were performed on specimens that were 50 mm in diameter, with a length of 60 mm (base soil) and 60 mm (filter soil). The base soils were uniformly graded fine sands and the filter materials were uniformly graded coarse sands. A few tests were performed on well-graded specimens of fine to medium gravel that were 100 mm in diameter. No surcharge pressure was applied to the specimen. Unidirectional flow was imposed in a downward, and for selected critical cases in an upward direction, using de-aired and distilled water, over a test duration of approximately 2 h (at i = 18 to 20) or 4 h (at i = 6 to 8).

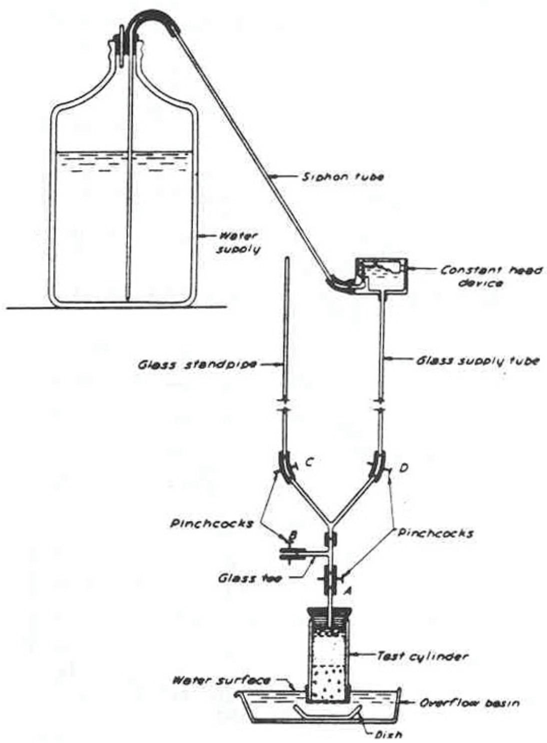

Figure 2. Filtration test assembly (after Bertram, 1940)

Any incompatibility of the base soil and filter, as a result of seepage flow, was found to initiate relatively quickly:

"It was observed that the movement of the base layer and rearrangement of the grains occurred only at the very beginning of a test and that visible movement ceased within three to five minutes."

Interpretation of the results showed, for uniformly graded sands:

"The minimum critical ratio of the 15 per cent size of the filter to the 85 per cent size of the base at the limit of stability is approximately 6."

The observations of Bertram (1940) provide the basis of filtration design criteria for soil retention and permeability advocated by Terzaghi and Peck (1948), based on earlier recommendations of Terzaghi (1939), and illustrated in Figure 3, where:

"Experiments have shown that a material satisfies the essential requirements for a filter if its 15 per cent size D_{15} is at least four times as large as that of the coarsest layer of soil in contact with the filter and not more than four times as large as the 85 per cent size D_{85} of the finest adjoining layer of soil."

Terzaghi et al. (1996) illustrate graphically the relation between this criterion for soil retention and the laboratory results of Bertram (1940). Inspection shows the design criterion of 4 or 5 provides a margin of safety against inadequate retention of the base soil, which occurs at a ratio between 8 and 6. Tomlinson and Vaid (2000) show this ratio to be sensitive to the hydraulic gradient, for ratios between 12 and 8. Piping occurs almost spontaneously at a ratio greater than 12.

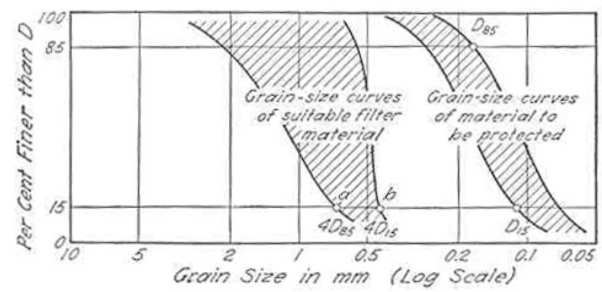

Figure 3. Design criteria (after Terzaghi and Peck, 1948)

Karpoff (1955) conducted a limited series of tests, intended to further guide filtration design criteria. The base soils used were a well-graded silt, a uniformly graded fine sand and medium sand, and a well-graded gravelly sand. The filter materials were uniform gradations of medium or coarse sand, or fine to medium gravel. Constant-head tests were performed on specimens that were 200 mm in diameter, with a length of 200 mm (base soil) and 200 mm (filter soil). A few tests were performed on a uniform coarse gravel filter material. No surcharge pressure was applied to the specimen. Unidirectional flow was imposed in a downward direction, using a water supply for which no details were provided, over a duration that was not reported (at values of *i* in the range 1 to 24).

The following three observations of Karpoff (1955) provide the basis of filtration design criteria, later adopted in several guidelines, for segregation potential and additional qualifiers for base soil retention, where:

1. *"The filter material should pass the 75 mm (3") screen for minimizing particle segregation and bridging during placement. Also filters must not have more than 5 per cent minus 0.075 mm (No. 200) particles to prevent excessive movement of fines in the filter and into drainage pipes causing clogging."*
2. *"The gradation curves of the filter and the base material should be approximately parallel in the range of the finer sizes, because the stability and proper function of protective filters depend upon skewness of the gradation curve of the filter toward the fines, giving a support to the fines in the base."*
3. *"In designing of filters for base materials containing particles larger than 4.75 mm (No. 4) size the base material should be analyzed on the basis of the gradation of material smaller than No. 4 size."*

Lafleur (1984) sought to independently validate the latter recommendation of Karpoff (1955) from tests on base soils that were well-graded gravelly silt-sands, selected as representative of tills used extensively in dam construction in the James Bay project. The filter materials were uniform gravels, and well-graded gravel-sands. Constant-head tests (see Figure 4) were performed on specimens that were 150 mm in diameter, with a length of 150 mm (base soil) and 200 mm (filter soil). An effective cell pressure of 100 kPa was applied, over a back pressure of approximately 800 kPa that was applied to ensure saturation of the soils. Unidirectional flow was imposed in a downward direction, over a test duration varying between 50 and 880 h, at values of i up to 8. Regarding the influence of time on particle migration as a result of seepage flow, most of the movement was found to take place within 50 h, with a potential for slow continued movement thereafter.

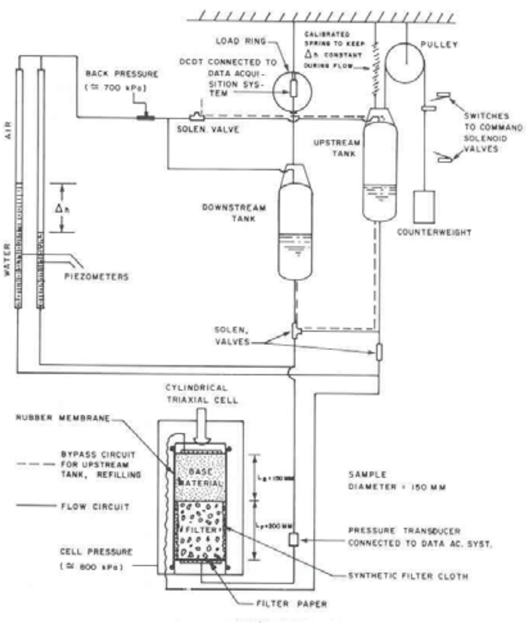

Figure 4. Permeameter test assembly (after Lafleur, 1984)

The observations of Lafleur (1984) confirm the recommendation of Karpoff (1955) to use the gradation smaller than the 4.75 mm (No.4) size for gravelly base soils for purposes of filtration design, in which case the criteria of Bertram (1940) were confirmed as applicable. The behaviour was attributed to the distribution of coarser particles in the soil matrix causing them to exert no significant influence on fines migration.

Sherard et al. (1984a) conducted an extensive series of tests, intended to validate independently the recommendation of Bertram (1940) for soil retention. The base soil was a uniform fine, medium or coarse sand. The filter material was a uniform gradation coarse sand, uniform gravel, or well-graded sandy gravel. The test specimens were 100 mm in diameter, see Figure 5, with a length between 50 and 100 mm (base soil) and 125 to 175 mm (filter soil). No surcharge load was applied to the specimen. Unidirectional flow was imposed in a downward direction, using the building water supply, at the supply pressure and therefore without control of the hydraulic gradient. The test duration was 5 to 10 min, after which the specimen was placed on a shake-table for 60 s if little or no base soil had migrated through the filter layer.

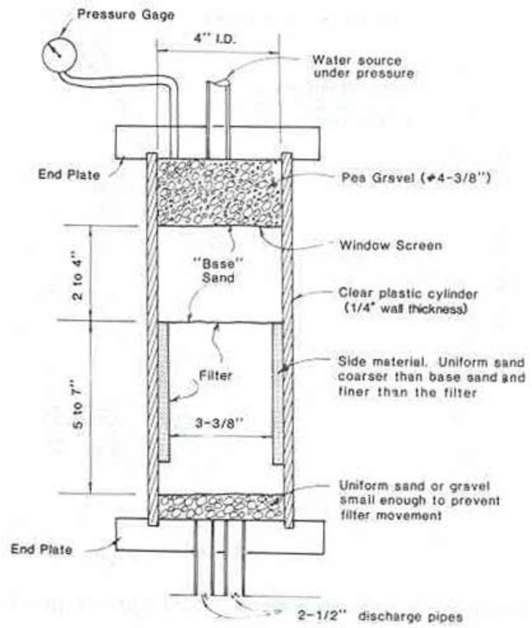

Figure 5. Soil specimen (after Sherard et al., 1984a)

From interpretation of the results, Sherard et al. (1984a) concluded the design criterion of Bertram (1940) is conservative, but not unduly so, for filters with a D_{15} greater than 1.0 mm. Alternative recommendations were made for finer filters suitable for base soils comprising fine-grained silts and clays (Sherard et al., 1984b). Importantly, the authors noted that:

"The preceding conclusions apply to base soils that are internally stable. For these stable base soils, the filter prevents entry of the coarse particles that accumulate in a thin skin on the filter face and prevent entry of the finer base particles. For certain gap-graded and unstable, coarse, broadly graded base soils, usually graded from clay sizes to gravels with D_{85} larger than 2 mm, the base soil fines may be able to enter the filter voids even if the coarser particles cannot. Filter criteria for these internally unstable base soils need to be applied differently, using procedures outside the scope of this study."

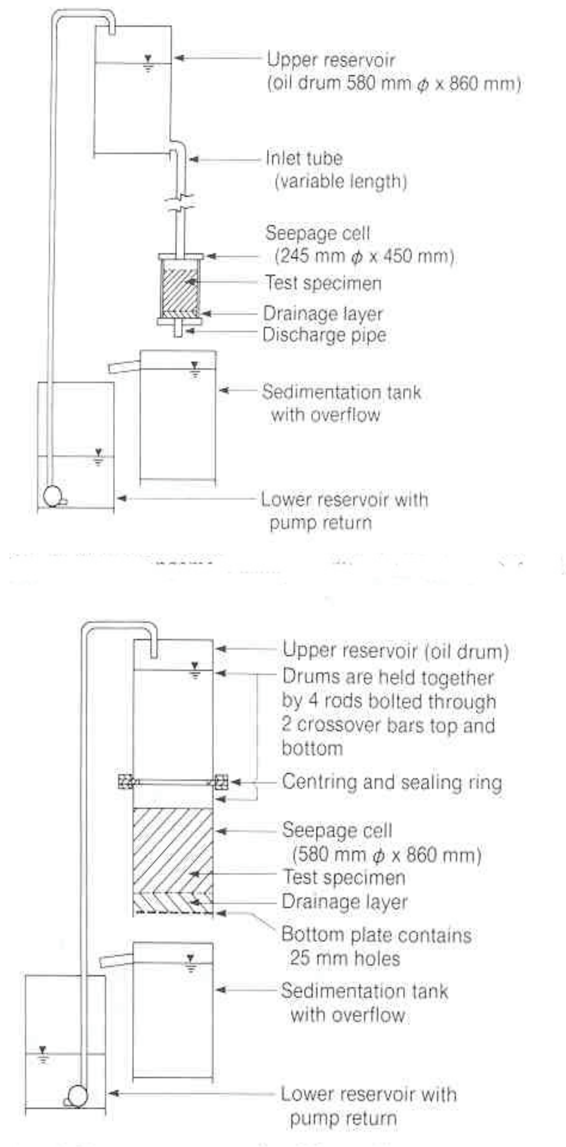

Figure 6. Test arrangement (after Kenney and Lau, 1985)

Recognising the internal stability of a granular material results from an ability to prevent the loss of its own small particles due to disturbing influences such as seepage and vibration, Kenny and Lau (1985) conducted a series of tests to define a threshold between stable and potentially unstable gradations. Constant-head tests (see Figure 6) were performed on specimens of approximate diameter 245 mm or 580 mm. The smaller specimen has a length of 450 mm (base soil), and the larger one a length of 860 mm (base soil).

The base soils were well-graded sandy gravels and the filter materials a uniform medium or coarse gravel, or uniform distribution of coarse gravel and cobbles. A low surcharge pressure of 10 kPa was applied. Unidirectional flow was imposed in a downward direction, using a water supply for which no details were provided, for a test of duration between 30 and 100 h (at an unspecified hydraulic gradient). Mild vibration was applied to the specimen throughout the test, and found to have a profound influence on the response of some the soils.

Interpretation of the results yielded a threshold criterion between stable and potentially unstable gradations. The criterion is based on a method of describing the shape of the grading curve and, therefore, is insensitive to grain size of the soil (see Figure 7).

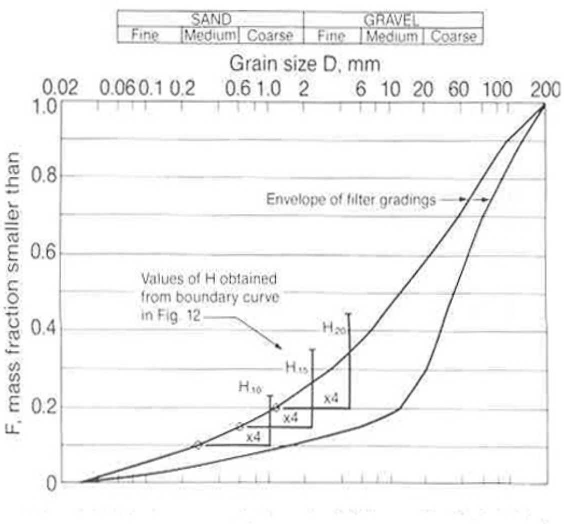

Figure 7. Stability criterion (after Kenney and Lau, 1985)

As illustrated, a discrete envelope of points (H) is established for selected intervals on the grading curve (F). If the grading curve lies below this envelope of points, over a designated portion of its finer end, then the gradation is deemed potentially unstable. The postulated boundary between stable and potentially unstable grading curves was defined as H/F = 1.3. The criterion is empirical, being defined from interpretation of the laboratory results (Kenney and Lau, 1985):

"Filter materials that exhibit unstable gradings in these tests show their potential for instability, but whether or not they would behave as unstable materials in practice would depend on the conditions of particle transport to which they were subjected. It can also be stated that any exaggeration of the hydrodynamic conditions in the tests provides an additional margin of safety to help offset the effects of particle segregation, which inevitably occurs during construction".

The study of Kenney and Lau (1985) generated significant discussion. Ripley (1986) underscored the likelihood of segregation occurring during construction as the range of particle sizes becomes wider. Comments by Milligan (1986), and additional work by Sherard and Dunnigan (1986), led Kenney and Lau (1986) to perform additional tests and redefine the postulated boundary between stable and potentially unstable grading curves as H/F = 1.

Lafleur et al. (1989) used the equipment and test methodology described by Lafleur (1984) to examine the response of base soils exhibiting a broad gradation that were stable according to the criterion of Kenney and Lau (1985, 1986). The base soils were very well graded silty gravel-sands, with a trace to some clay, and gap-graded silty gravels. The filter materials were uniform gravels, and well-graded gravel-sands. The mass of soil particles that migrated from the base soil into the filter material was used to define incompatibility, and establish the "indicative base size" for purposes of soil retention. For gap graded soils, it corresponded to the grading curve below the gap since the coarser particles above the gap do not contribute to the self-filtration process. For very well graded soils, it corresponded to the midpoint of the grading curve (D_{50}).

Skempton and Brogan (1994) report findings from piping tests on well graded and gap graded sandy gravels that broadly confirm the Kenney and Lau (1985, 1986) criterion for internal stability. The test specimens were 139 mm in diameter, see Figure 8, with a length of approximately 155 mm (base soil).

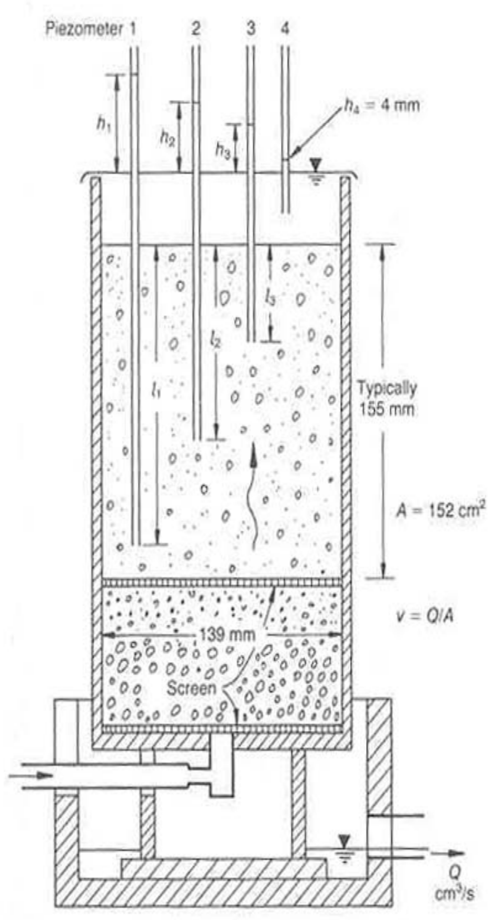

Figure 8. Test specimen (after Skempton and Brogan, 1994)

After saturation of the specimen, flow was imposed in an upward direction, and increased in small steps until piping occurred. The test duration was about 1.5 h. They found:

"There is an abrupt transition from unstable to stable states at about the limits defined by Kenney and Lau (1985, 1986) in terms of a stability index derived from the shape of the grain size distribution curves, and also at (or just below) the limit proposed by Kezdi (1979) based on the filter ratio of the sand and gravel components".

The limit proposed by Kezdi (1979) involves dividing the soil into a fine and coarse component, using a selected fines content on the grading curve (see Figure 8). If the two components satisfy the filtration rule of Terzaghi (1939), where $D'_{15}/d'_{85}<4$, then the composite gradation will be self-filtering and therefore internally stable.

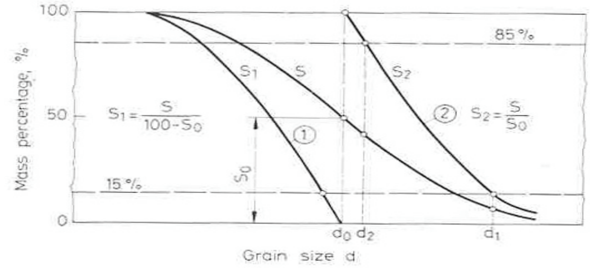

Figure 9. Stability criterion (after Kezdi, 1979)

Honjo et al. (1996) further examined aspects of internal stability, through self-filtration, in widely and gap graded soils. The widely graded soils were gravelly sands. The gap graded soils were predominantly sands in which the medium sand fraction was absent. The test specimens were either 150 mm or 300 mm in diameter, see Figure 10, with a length of 100 mm (base soil) that was supported on a metal screen filter. A light surcharge of 0.9 kPa was applied to the specimen. Unidirectional flow of distilled water was imposed in a downward direction, either with head control (at a hydraulic gradient of 2.5 to 14) or alternatively by pumping. Vibration was applied by lightly tapping the permeameter with a rubber hammer. The test duration was 2 h.

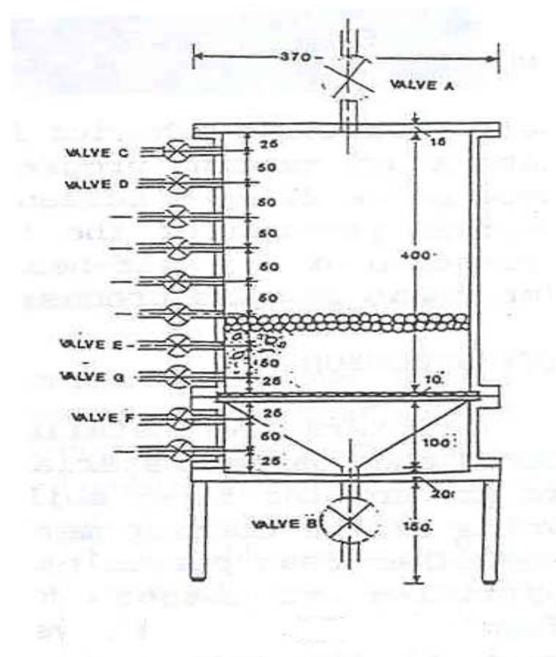

Figure 10. Permeameter assembly (after Honjo et al., 1996)

Interpretation of the results suggested the ratio D_{95}/D_{85} is a good indicator for internal instability in a well graded base soil, with the onset of particle migration and a commensurate loss of soil occurring at a threshold value between 15 and 20. For gap graded soils, the potential for internal stability is described with reference to a gap ratio, defined by the ratio of maximum to minimum grain size for the gap portion of the grain size distribution curve. Instability appears to be sensitive to the fines content of the soil, and commences at a threshold gap ratio of 3 for soils with 40 % or more fines and a larger threshold value of 4 for soils with 30% or less fines.

3. A COMPARISON OF SELECTED DATA ON INTERNAL INSTABILITY

Moffat (2002) used a permeameter described by Hameiri and Fannin (2001) to test selected gradations of Kenney and Lau (1985) and Honjo et al. (1996), with the objective of contrasting the response against the simple method of assessing internal stability proposed by Kezdi (1979). The test specimens were 100 mm in diameter, see Figure 11, with a length of 100 mm (base soil) that was supported on a metal screen filter. A light surcharge of 25 kPa was applied to the specimen. In a multistage test, unidirectional flow of de-aired distilled water was in a downward direction, to impose a hydraulic gradient in the range 0.1 to 15. Vibration was applied in some stages, by controlled striking of the base of the permeameter with the rod of a miniature air hammer, at a frequency of 7 Hz. The test duration was approximately 11 h.

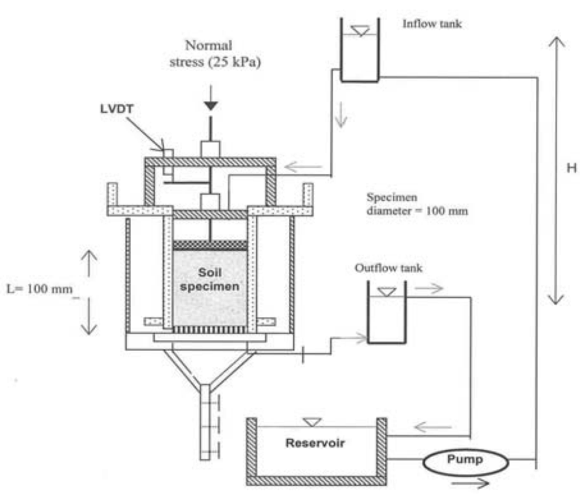

Figure 11. UBC permeameter assembly

A comparison of the three series of laboratory data is made, with reference to the per cent loss of soil from the specimen during a test and the grain size ratio D'_{15}/d'_{85} (see Figure 12). Inspection shows the total loss of soil to increase with the ratio of grain size. It is less than 10% for values of D'_{15}/d'_{85} less than 4. At values of grain size ratio greater than 4, there is a wide range in the percentage loss of soil. Hence it would appear the value of D'_{15}/d'_{85} of 4 defines a threshold between stable and potentially unstable, which is consistent with the criterion of Kezdi (1979). General agreement is also inferred with the criterion of Kenney and Lau (1985), since soils that were classified as internally stable in that study exhibited no loss of soil.

Results from the study of Moffat (2002) also yield general agreement with results from the studies of Kenney and Lau (1985, 1986) and Honjo et al. (1996), given the similar magnitudes of total loss (see Table 2). One combination of results is for a stable grading, Soil K, which yields a small difference in measured total loss of soil through particle migration. The other combinations of results describe potentially unstable gradings, for which the difference in total loss is more significant. The difference is attributed to the percentage of grading curve that is potentially mobile, together with the variation in seepage flow, energy from vibration, geometry of

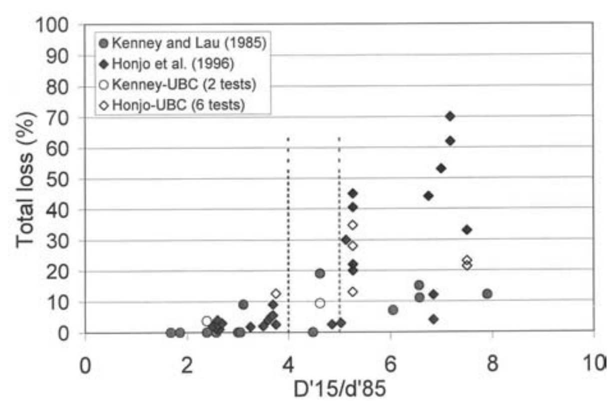

Figure 12. Synthesis of selected data

the supporting metal filter screen and test duration between each of the three studies. In other words we speculate that, for potentially unstable soils, factors limiting total loss of soil are the quantity of potentially mobile grains and the opening size of the exit boundary, while factors controlling rate of loss of soil are the severity of hydrodynamic conditions and time.

Table 2. Loss of soil through particle migration

Soil code	Total loss (%)	Comments
Kenney and Lau (1985)		
K	0	manual tapping (D_{15}=20mm)
D	19	manual tapping (D_{15}=20mm)
Honjo et al. (1996)		
G1-D	2.5	manual tapping; 0.6 mm screen
G3-C	20	manual tapping; 0.83 mm screen
G4-C	33	manual tapping; 1.2 mm screen
UBC tests		
K	3.7	automatic tapping (60 min); 2.76 mm screen
D	9.3	automatic tapping (60 min); 6.5 mm screen
G1-D	12.5	automatic tapping (60 min); 1.15 mm screen
G3-C	13	automatic tapping (60 min); 0.86 mm screen
G3-C	28	automatic tapping (60 min); 0.45 mm screen
G3-C	34.8	automatic tapping (60 min); 0.86 mm screen
G4-C	21.4	no vibration; 1.15 mm screen
G4-C	23.1	no vibration; 1.15 mm screen

4. SUMMARY REMARKS

(1) In potentially unstable soils, Kovacs (1981) notes the onset of internal instability is governed both by a geometric constraint and a hydrodynamic constraint. In geotechnical engineering practice, concern exists for the impact of seepage flow through a potentially unstable soil. This requires a threshold to the onset of instability be defined, and the extent of instability be quantified.

(2) Interpretations of laboratory permeameter studies have led to empirical criteria in the literature (Kezdi, 1979; Kenney and Lau, 1985 and 1986) that appear to define a threshold to the onset of instability. Those empirical criteria are based on geometric constraints alone, and do not explicitly account for hydrodynamic constraints. New laboratory data lend further confidence to the use of empirical criteria to identify potentially unstable soil gradations.

(3) A synthesis of the data from this and selected previous studies indicates the extent of the instability is wide ranging in terms of total loss of soil. While there is a basis for reasonable confidence in identifying soils that are potentially unstable, the role of hydrodynamic influences and the implications of any instability cannot be described with confidence.

5. ACKNOWLEDGEMENTS

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