ESTRUCTURAS II

vigas hiperestáticas
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2

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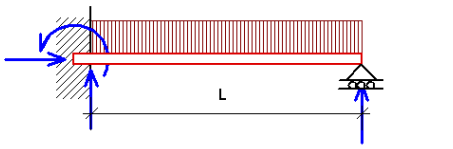
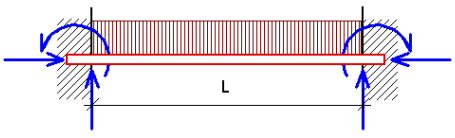
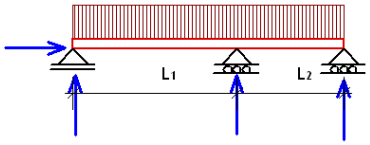
Vigas Hiperestáticas

a



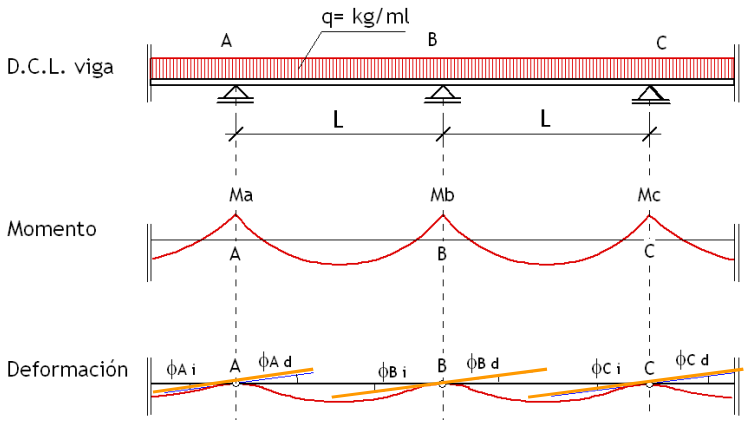


b



Concepto de vigas hiperestáticas por continuidad

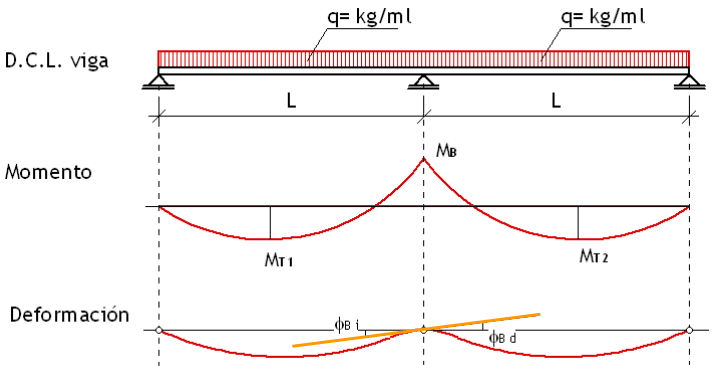
b



$\phi B_{izquierdo} = - \phi B_{derecho}$

por ángulos opuestos por el vértice

Ejemplo Viga de dos tramos con carga uniformemente repartida



$\phi B_{izquierdo} = - \phi B_{derecho}$

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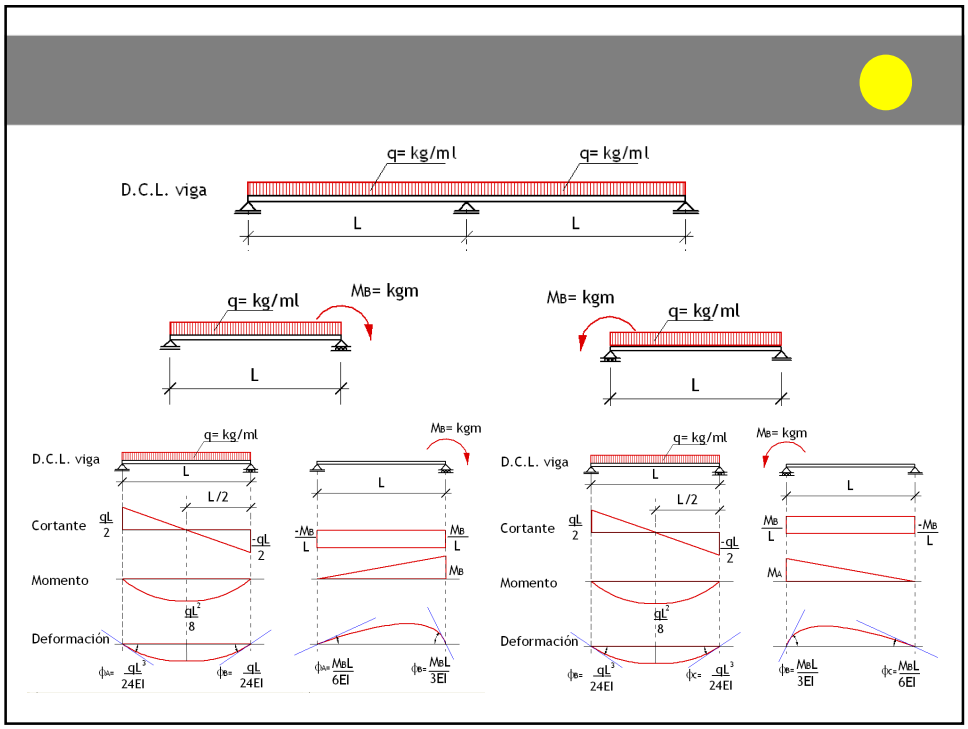
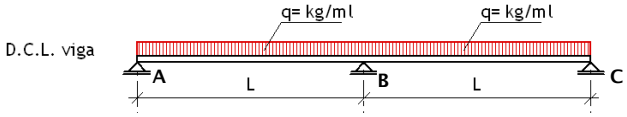




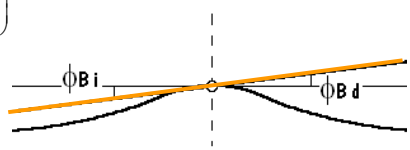
Se igualan los valores de ángulos a ambos lados del apoyo B para determinar el momento de continuidad entre ambos tramos.

$$\Sigma \phi_{B\ Izquierdo} = -\Sigma \phi_{B\ derecho}$$

$$-\frac{qL^3}{24EI} + \frac{M_B L}{3EI} = -\left(-\frac{qL^3}{24EI} + \frac{M_B L}{3EI}\right)$$

$$\frac{2M_B L}{3EI} = \frac{qL^3}{12EI}$$

$$\boxed{M_B = \frac{qL^2}{8}}$$



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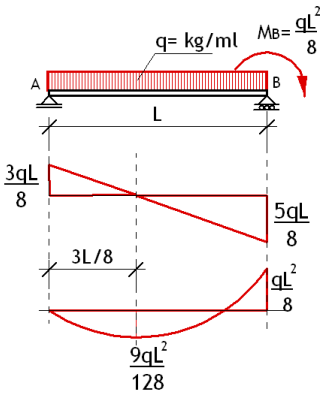
$$R_A = \frac{qL}{2} - \frac{Mb}{L} = \frac{qL}{2} - \frac{qL}{8} = \frac{3qL}{8}$$

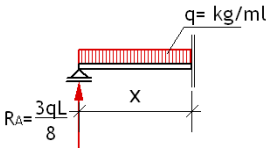
$$R_{Bizq} = \left(\frac{qL}{2} + \frac{Mb}{L}\right) = \left(\frac{qL}{2} + \frac{qL}{8}\right) = \frac{5qL}{8}$$



$$V_x = 0$$

$$0 = \frac{3qL}{8} - qx$$

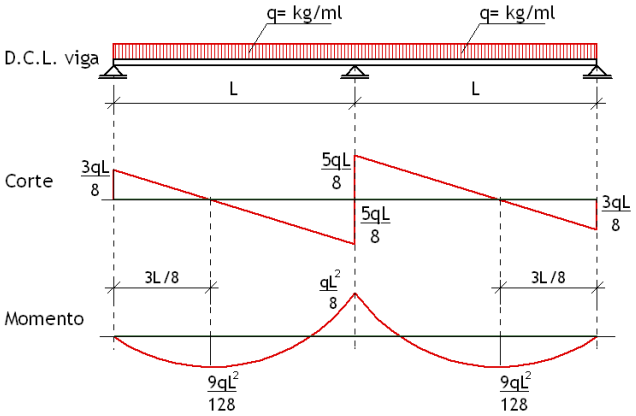
$$x = \frac{3L}{8}$$



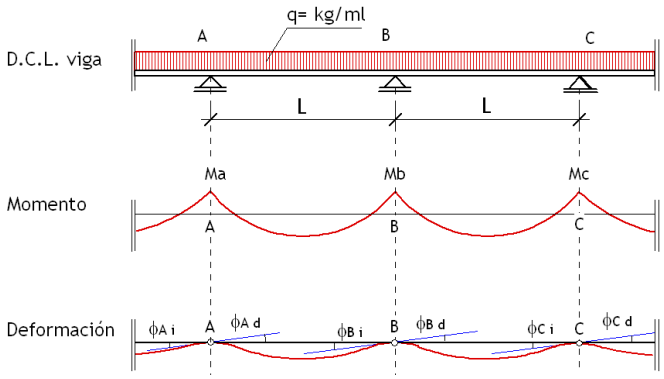
$$Mx = \frac{3qLx}{8} - \frac{qx^2}{2}$$

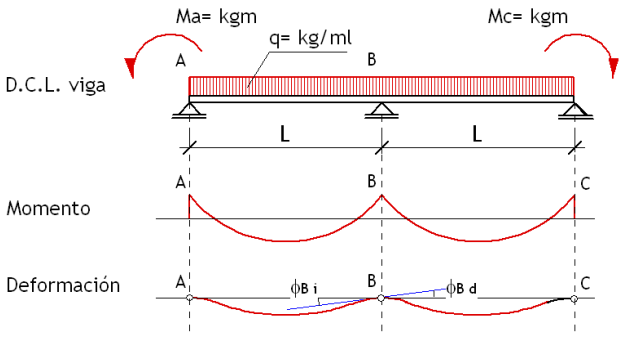
$$M_{MAX} = \frac{9qL^2}{64} - \frac{9qL^2}{128}$$

$$M_{MAX} = \frac{9qL^2}{128}$$



Teorema de los tres momentos o Clapeyron





ΣϕB izquierdo= -ΣϕB derecho

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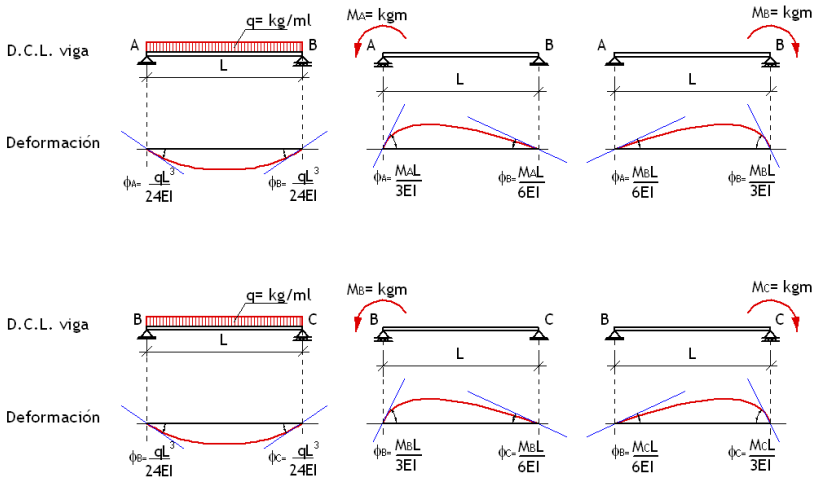


$$\Sigma\phi B \text{ izquierdo} = -\Sigma\phi B \text{ derecho}$$

$$-\frac{qL_1^3}{24EI}+\frac{M_AL_1}{6EI}+\frac{M_BL_1}{3EI}=-\left(-\frac{qL_2^3}{24EI}+\frac{M_BL_2}{3EI}+\frac{M_CL_2}{6EI}\right)$$

$$\frac{M_AL_1}{6EI}+\frac{M_BL_1}{3EI}+\frac{M_BL_2}{3EI}+\frac{M_CL_2}{6EI}=\frac{qL_1^3}{24EI}+\frac{qL_2^3}{24EI}$$

Reemplazando L/EI por λ (módulo de flexibilidad)

$$\frac{M_A\lambda_1}{6}+\frac{M_B\lambda_1}{3}+\frac{M_B\lambda_2}{3}+\frac{M_C\lambda_2}{6}=\frac{qL_1^2\lambda_1}{24}+\frac{qL_2^2\lambda_2}{24} \qquad /*6$$

Al amplificar la expresión 6 veces se obtiene

$$M_A\lambda_1+2M_B\lambda_1+2M_B\lambda_2+M_C\lambda_2=6*\left[\frac{qL_1^2\lambda_1}{24}+\frac{qL_2^2\lambda_2}{24}\right]$$

$$M_A\lambda_1+2M_B\left(\lambda_1+\lambda_2\right)+M_CC\lambda_2=6*\left[\frac{qL_1^2\lambda_1}{24}+\frac{qL_2^2\lambda_2}{24}\right]$$

Si EI = constante y $\lambda = L/EI$ $\lambda = L$

Reemplazando $\lambda = L$ en la ecuación se tiene:

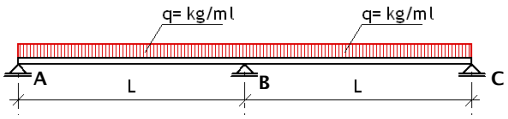
$$M_A L_1 + 2M_B \left(L_1 + L_2 \right) + M_C L_2 = 6 * \left[\frac{qL_1^3}{24} + \frac{qL_2^3}{24} \right]$$

Reemplazando $\frac{qL_1^3}{24}$ por T_{C1} y $\frac{qL_2^3}{24}$ por T_{C2}

$$M_A L_1 + 2M_B \left(L_1 + L_2 \right) + M_C L_2 = 6 * \left[T_{c_1} + T_{c_2} \right]$$

Ejemplo Viga de dos tramos con carga uniformemente repartida

D.C.L. viga



$$M_A L_1 + 2M_B \left(L_1 + L_2 \right) + M_C L_2 = 6 * \left[T_{c_1} + T_{c_2} \right]$$

$$0 * L_1 + 2M_B \left(L_1 + L_2 \right) + 0 * L_2 = 6 * \left[\frac{qL_1^3}{24} + \frac{qL_2^3}{24} \right]$$

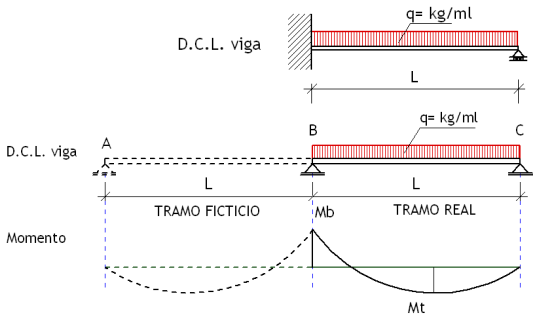
$$2M_B \left(L_1 + L_2 \right) = \frac{qL_1^3}{4} + \frac{qL_2^3}{4}$$

Si $L_1 = L_2$

$$2M_B \; 2L = \frac{qL^3}{2}$$

$$M_B = \frac{qL^2}{8}$$

Ejemplo Viga empotrada en un extremo y apoyada en el otro con carga uniformemente repartida



$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6 * \left[T_{c_1} + T_{c_2}\right]$$

$$0 * L_0 + 2Mb * \left(L_0 + L_1\right) + 0 * L_1 = 6 * \left[0 + \frac{qL_1^3}{24}\right]$$

$$2M_B\, L = \frac{qL^3}{4}$$

$$M_B = \frac{qL^2}{8}$$