vigas hiperestáticas

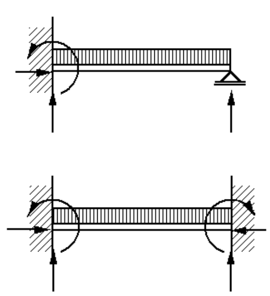
ESTRUCTURAS 2

2

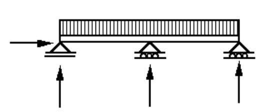
profesora: Verónica Veas ayudante: Preeti Bellani

Vigas Hiperestáticas

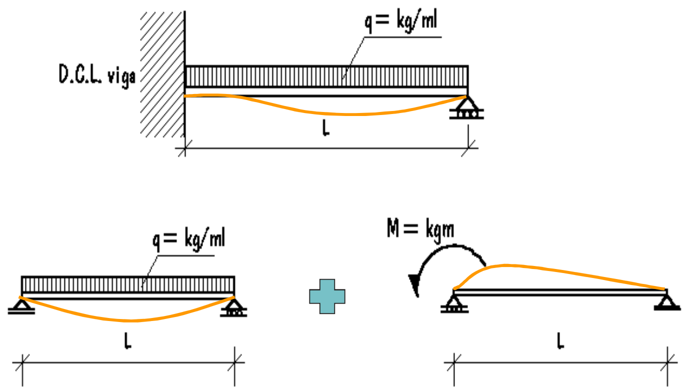
a



b

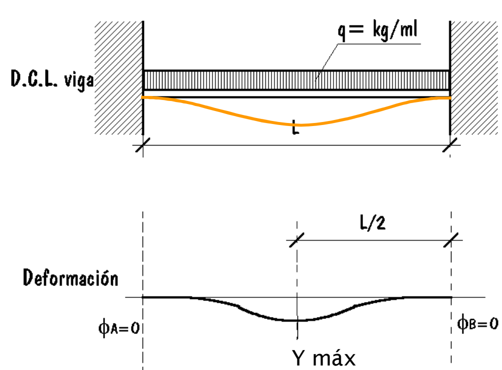


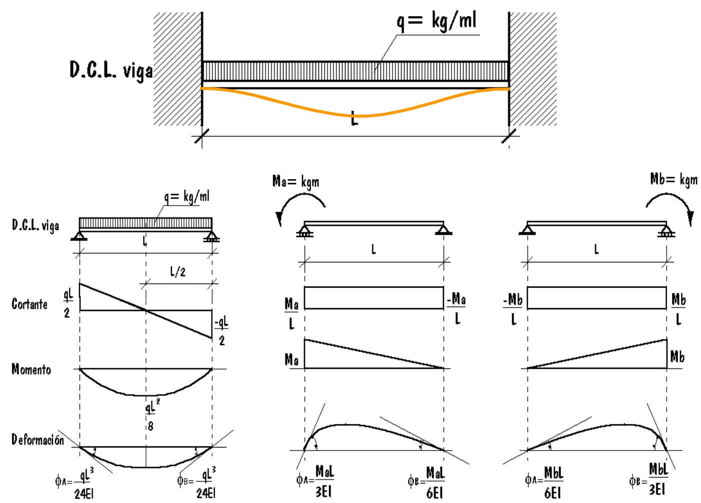
Concepto de vigas hiperestáticas por empotramiento

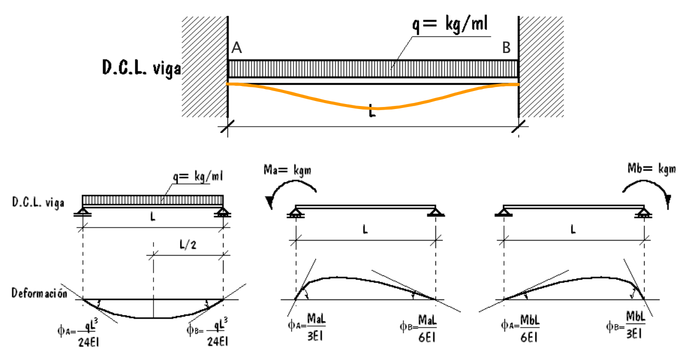


$\emptyset_{\text{empotramiento}} = 0$

Ejemplo Viga bi-empotrada con carga uniformemente repartida







$\varnothing_A = 0$

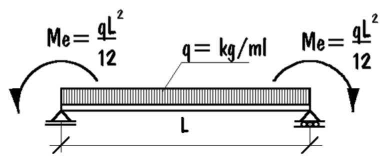
$$0 = -\frac{qL^3}{24EI} + \frac{M_A L}{3EI} + \frac{M_B L}{6EI}$$

$$\frac{2M_A L + M_B L}{6EI} = \frac{qL^3}{24EI}$$

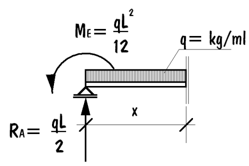
$M_A = M_B = M_E$

$$\frac{3M_E L}{6EI} = \frac{qL^3}{24EI}$$

$$\boxed{M_E = \frac{qL^2}{12}}$$



$$Ra = Rb = \frac{qL}{2}$$



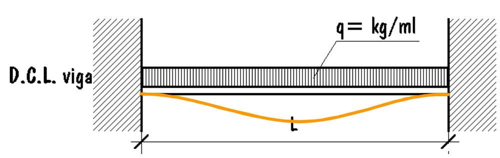
$$Mx = \frac{qLx}{2} - \frac{qx^2}{2} - M_E$$

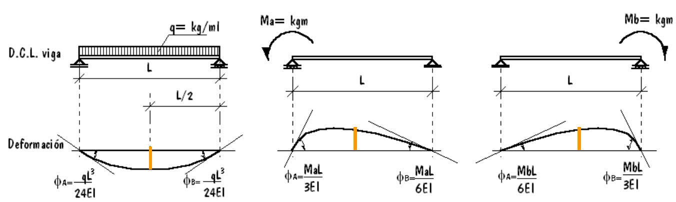
El momento máximo en una viga simétrica se encuentra en X=L/2

$$M_{(L/2)} = \frac{qL}{2}\frac{L}{2} - \frac{q}{2}\left(\frac{L}{2}\right)^2 - M_E$$

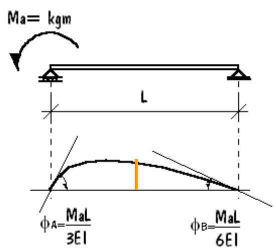
$$M_{(L/2)} = \frac{qL^2}{4} - \frac{qL^2}{8} - \frac{qL^2}{12}$$

$$M_{MAX} = \frac{qL^2}{24}$$





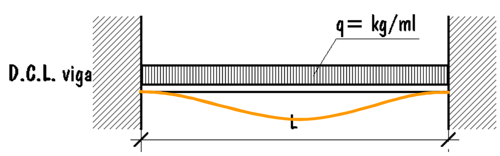
$$Y_{L/2} = \frac{5qL^4}{384EI} \qquad Y_{L/2} = \frac{ML^2}{16EI} \qquad Y_{L/2} = \frac{ML^2}{16EI}$$

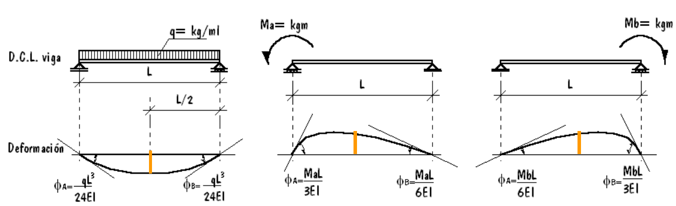


$$Y_{L/2} = \frac{ML^2}{16EI}$$

$$Y_{L/2} = \frac{qL^2}{12} \frac{L^2}{16EI}$$

$$Y_{L/2} = \frac{qL^4}{192EI}$$

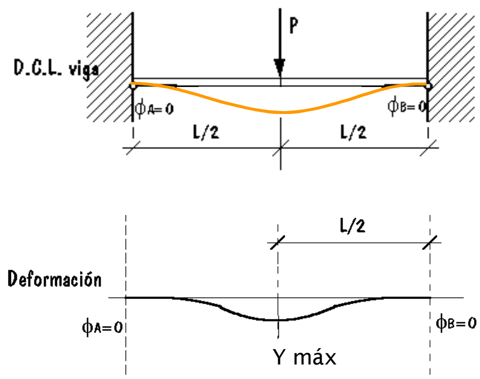


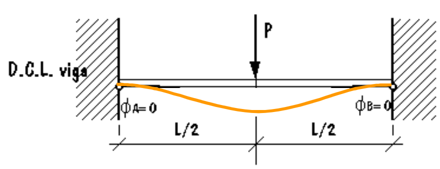


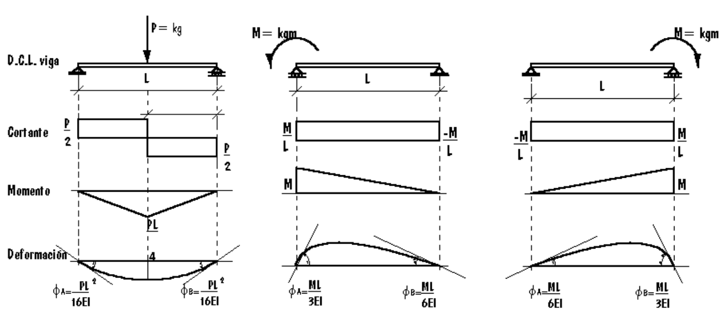
$$Y_{MAX} = \frac{5qL^4}{384EI} - \frac{qL^4}{192EI} - \frac{qL^4}{192EI}$$

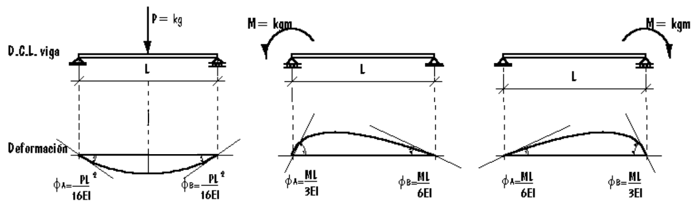
$$\boxed{Y_{MAX} = \frac{qL^4}{384EI}}$$

Ejemplo Viga bi-empotrada con carga puntual al centro









$$\varnothing_A = 0$$

$$M_A = M_B = M_E$$

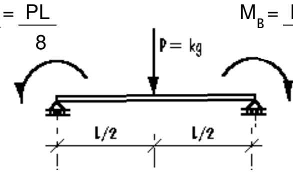
$$0 = \frac{-PL^2}{16EI} + \frac{M_A L}{3EI} + \frac{M_B L}{6EI}$$

$$\frac{3M_E L}{6EI} = \frac{PL^2}{16EI}$$

$$\frac{2M_A L + M_B L}{6EI} = \frac{PL^2}{16EI}$$

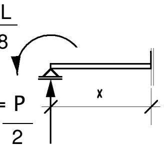
$$M_E = \frac{PL}{8}$$

$$M_A = \frac{PL}{8} \qquad M_B = \frac{PL}{8}$$



$$Ra = Rb = \frac{P}{2}$$

$$M_A = \frac{PL}{8}$$



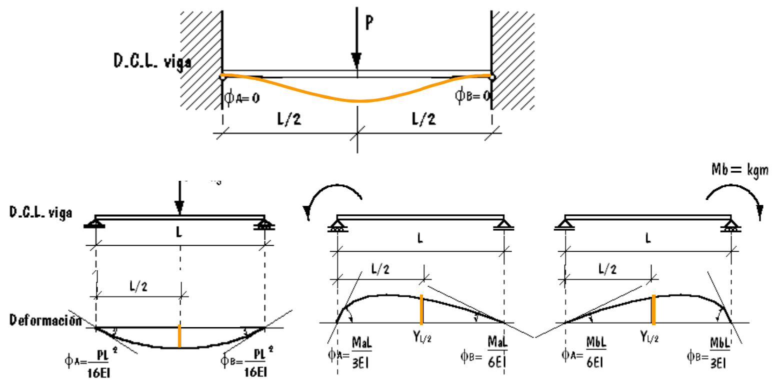
$$R_A = \frac{P}{2}$$

$$Mx = \frac{-PL}{8} + \frac{PX}{2}$$

El momento máximo en una viga simétrica se encuentra en X=L/2

$$M_{(L/2)} = \frac{-PL}{8} + \frac{P}{2}\,\frac{L}{2}$$

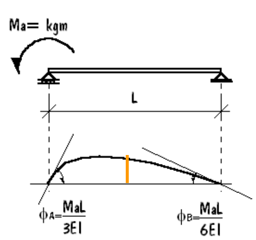
$$M_{MAX} = \frac{PL}{8}$$



$$Y_{L/2} = \frac{PL^3}{48EI}$$

$$Y_{L/2} = \frac{ML^2}{16EI}$$

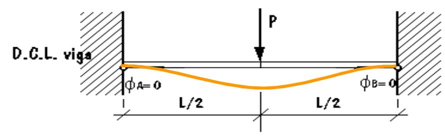
$$Y_{L/2} = \frac{ML^2}{16EI}$$

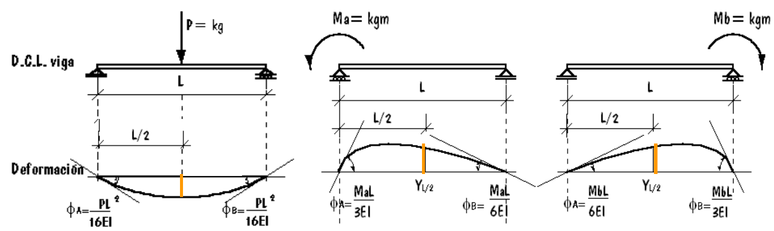


$$Y_{L/2} = \frac{ML^2}{16EI}$$

$$Y_{L/2} = \frac{PL}{8}\,\frac{L^2}{16EI}$$

$$Y_{L/2} = \frac{PL^3}{128EI}$$

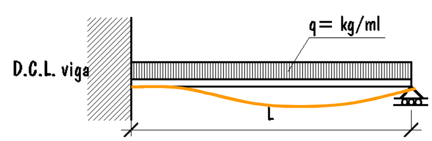


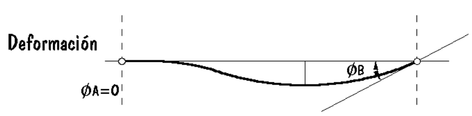


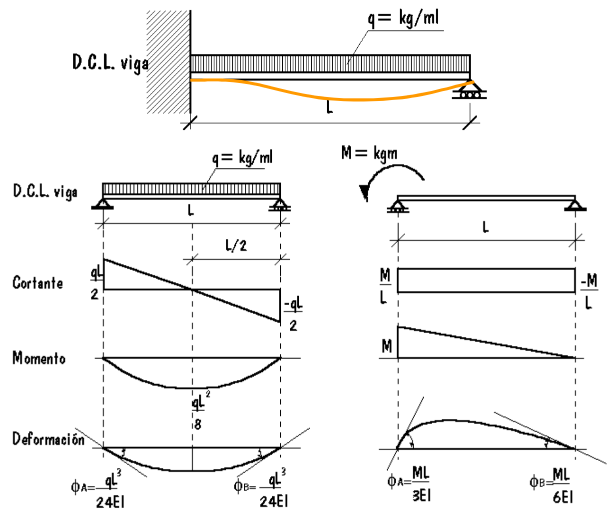
$$Y_{MAX} = \frac{PL^3}{48EI} - \frac{PL^3}{128EI} - \frac{PL^3}{128EI}$$

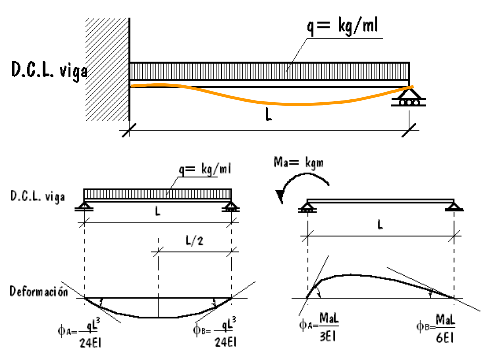
$$Y_{MAX} = \frac{PL^3}{192EI}$$

Ejemplo Viga empotrada-apoyada con carga uniformemente repartida







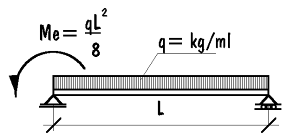


$\varnothing_A = 0$

$$0 = \frac{-qL^3}{24EI} + \frac{MeL}{3EI}$$

$$\frac{MeL}{3EI} = \frac{qL^3}{24\,EI}$$

$$\boxed{Me = \frac{qL^2}{8}}$$

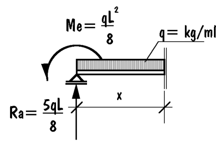


$$Ra = \frac{qL}{2} + \frac{Me}{L} = \frac{qL}{2} + \frac{qL}{8} = \frac{5qL}{8}$$

$$Rb = \frac{qL}{2} - \frac{Me}{L} = \frac{qL}{2} - \frac{qL}{8} = \frac{3qL}{8}$$

$$Mx = \frac{5qLx}{8} - \frac{qx^2}{2} - Me$$

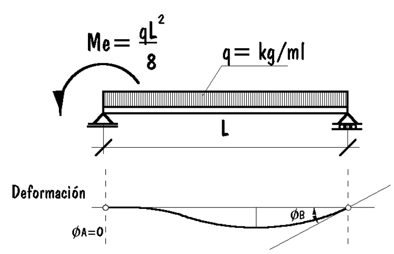
$$M_{MAX} = \frac{25qL^2}{64} - \frac{25qL^2}{128} - \frac{qL^2}{8}$$



$$V_x = 0$$

$$M_{MAX} = \frac{9qL^2}{128}$$

$$\frac{5qL}{8} - qx = 0 \qquad X = \frac{5L}{8}$$



$$EI\frac{d^2y}{dx^2} = \frac{5qLx}{8} - \frac{qL^2}{8} - \frac{qx^2}{2}$$

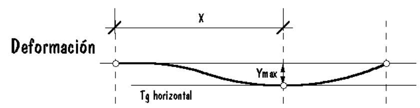
$$EI\frac{dy}{dx} = \frac{5qLx^2}{16} - \frac{qL^2x}{8} - \frac{qx^3}{6} + C_1$$

$$EI.y = \frac{5qLx^3}{48} - \frac{qL^2x^2}{16} - \frac{qx^4}{24} + C_1x + C_2$$

Condiciones de apoyo

Si X=0 **C1=0**

Si X=0 ó X=L **C2=0**



Flecha máx en dy/dx=0

$$\frac{5qLx^2}{16} - \frac{qL^2 x}{8} - \frac{qx^3}{6} = 0$$

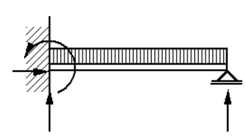
X = 0,58L

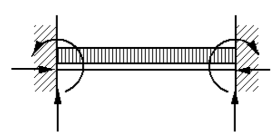
$$Y = \frac{5qL}{48\,EI}(0{,}58L)^3 - \frac{qL^2}{16EI}(0{,}58L)^2 - \frac{q}{24\,EI}(0{,}58L)^4$$

$$Y_{MAX} = \frac{qL^4}{185EI} = 0{,}005\,\frac{qL^4}{EI}$$

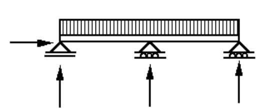
Vigas Hiperestáticas

a



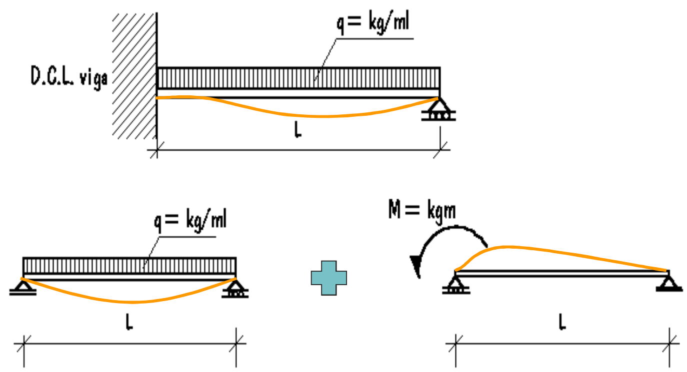


b



Concepto de vigas hiperestáticas por empotramiento

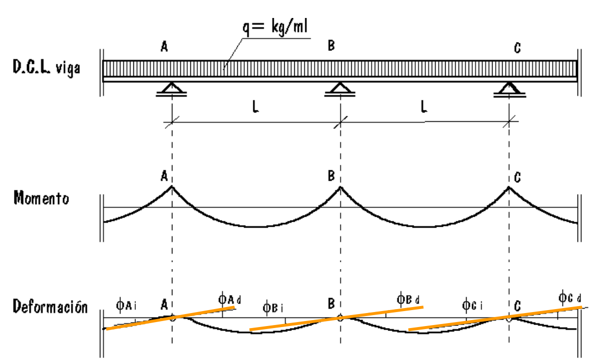
a



$Ø_{\text{empotramiento}} = 0$

Concepto de vigas hiperestáticas por continuidad

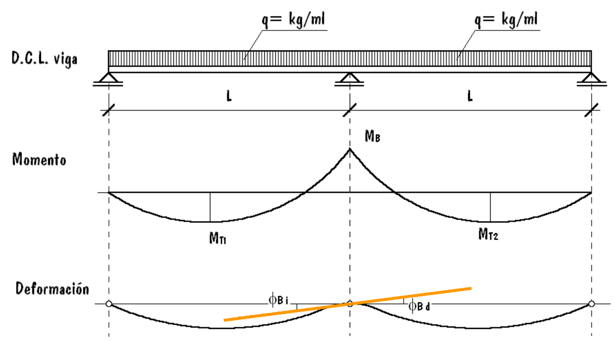
b



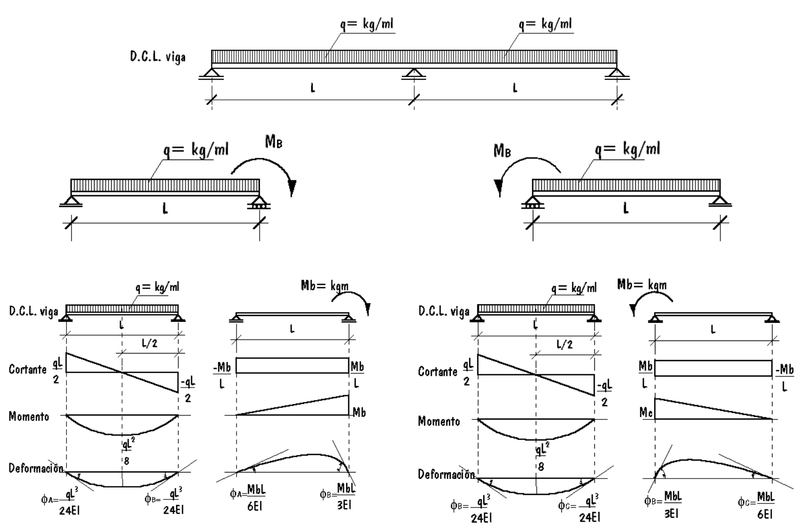
$\phi B_{\text{izquierdo}} = - \phi B_{\text{derecho}}$

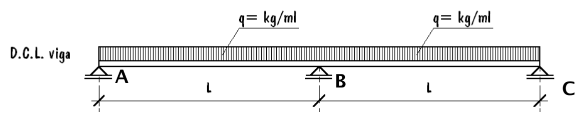
por ángulos opuestos por el vértice

Ejemplo Viga de dos tramos con carga uniformemente repartida



$$\phi B_{izquierdo} = - \phi B_{derecho}$$



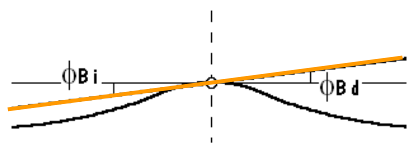


Se igualan los valores de ángulos a ambos lados del apoyo B para determinar el momento de continuidad entre ambos tramos.

$$\Sigma\phi_{B\ \text{Izquierdo}} = -\Sigma\phi_{B\ \text{derecho}}$$

$$-\frac{qL^3}{24EI}+\frac{M_BL}{3EI} = -\left(-\frac{qL^3}{24EI}+\frac{M_BL}{3EI}\right)$$

$$\frac{2M_BL}{3EI}=\frac{qL^3}{12EI} \quad \ldots * EI/L$$



$$\boxed{M_B = \frac{qL^2}{8EI}}$$

$$R_B = \frac{qL}{2}+\frac{M_B}{L}=\frac{5qL}{8}$$

$$R_C = \frac{qL}{2}-\frac{M_B}{L}=\frac{3qL}{8}$$

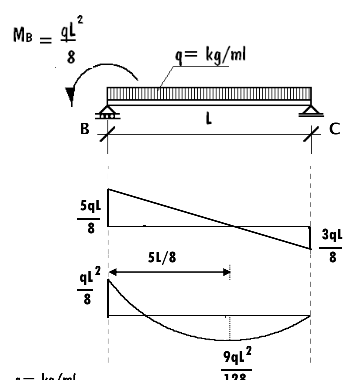
$$Mx=\frac{5qLx}{8}-\frac{qx^2}{2}-\frac{qL^2}{8}$$

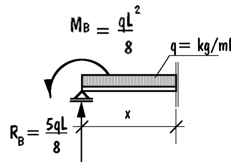
$$V_x = 0$$

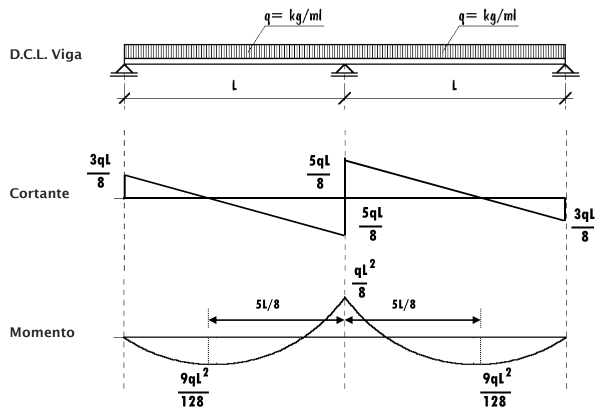
$$\frac{5qL}{8}-qx=0 \qquad X=\frac{5L}{8}$$

$$M_{MAX} = \frac{25qL^2}{64}-\frac{25qL^2}{128}-\frac{qL^2}{8}$$

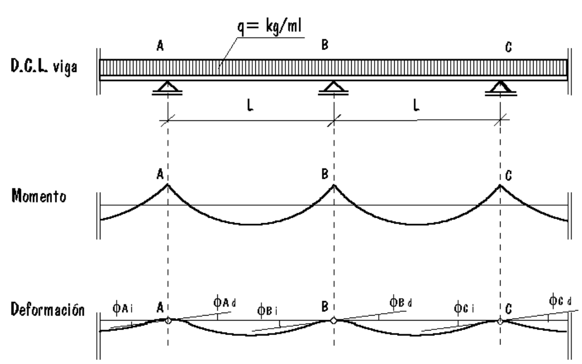
$$\boxed{M_{MAX} = \frac{9qL^2}{128}}$$

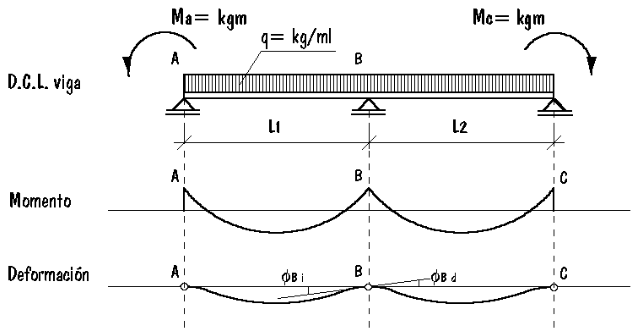




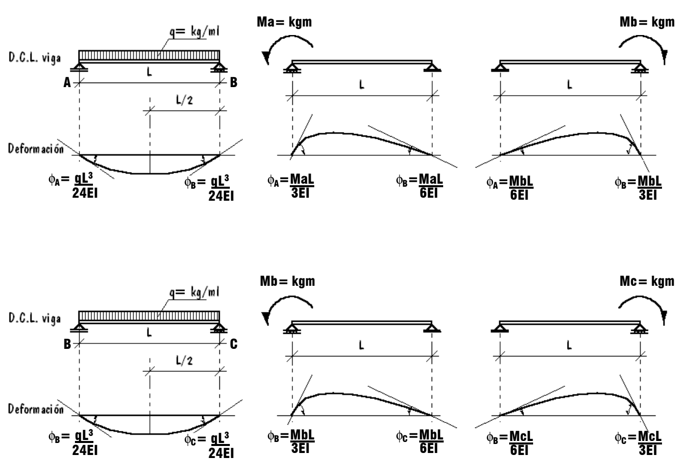


Teorema de los tres momentos o Clapeyron





$\Sigma\phi B$ izquierdo= $-\Sigma\phi B$ derecho



$\Sigma\phi B$ izquierdo= $-\Sigma\phi B$ derecho

$$-\frac{qL_1^3}{24EI} + \frac{MaL_1}{6EI} + \frac{MbL_1}{3EI} = -\left[-\frac{qL_2^3}{24EI} + \frac{MbL_2}{3EI} + \frac{McL_2}{6EI}\right]$$

$$\frac{MaL_1}{6EI} + \frac{MbL_1}{3EI} + \frac{MbL_2}{3EI} + \frac{McL_2}{6EI} = \frac{qL_1^3}{24EI} + \frac{qL_2^3}{24EI}$$

Reemplazando L/EI por λ (módulo de flexibilidad)

$$\frac{Ma\lambda_1}{6} + \frac{Mb\lambda_1}{3} + \frac{Mb\lambda_2}{3} + \frac{Mc\lambda_2}{6} = \frac{qL_1^2\lambda_1}{24} + \frac{qL_2^2\lambda_2}{24} \quad /*6$$

Al amplificar la expresión 6 veces se obtiene

$$Ma\lambda_1 + 2Mb\lambda_1 + 2Mb\lambda_2 + Mc\lambda_2 = 6\left[\frac{qL_1^2\lambda_1}{24} + \frac{qL_2^2\lambda_2}{24}\right]$$

$$Ma\lambda_1 + 2Mb(\lambda_1+\lambda_2) + Mc\lambda_2 = 6\left[\frac{qL_1^2\lambda_1}{24} + \frac{qL_2^2\lambda_2}{24}\right]$$

Si EI = constante y λ = L/EI λ = L

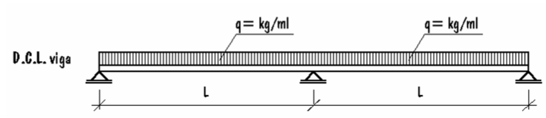
Reemplazando λ = L en la ecuación se tiene:

$$MaL_1 + 2Mb(L_1 + L_2) + McL_2 = 6\left[\frac{qL_1^3}{24} + \frac{qL_2^3}{24}\right]$$

Reemplazando $\frac{qL_1^3}{24}$ por Tc_1 y $\frac{qL_2^3}{24}$ por Tc_2

$$MaL_1 + 2Mb(L_1+L_2) + McL_2 = 6(Tc_1 + Tc_2)$$

Ejemplo Viga de dos tramos con carga uniformemente repartida



$$M_aL_1 + 2M_b(L_1+L_2) + M_cL_2 = 6(T_{c1} + T_{c2})$$

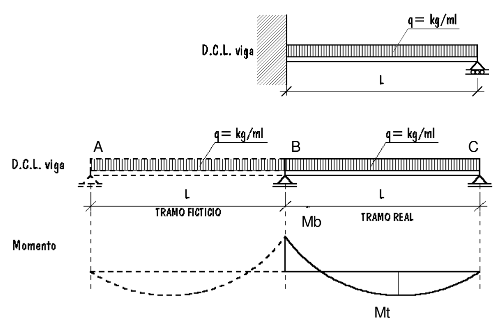
$$0L_1 + 2M_b(L_1+L_2) + 0L_2 = 6\left[\frac{qL_1^3}{24} + \frac{qL_2^3}{24}\right]$$

$$2M_b(L_1+L_2) = \frac{qL_1^3}{4} + \frac{qL_2^3}{4}$$ **Si** $L_1=L_2$

$$2M_b\,2L = \frac{qL^3}{2}$$

$$\boxed{M_b = \frac{qL^2}{8}}$$

Ejemplo Viga empotrada en un extremo y apoyada en el otro con carga uniformemente repartida



$$M_aL_1 + 2M_b(L_1+L_2) + M_cL_2 = 6(T_{c1} + T_{c2})$$

$$00 + 2M_b(0+L) + 0L = 6\left[0 + \frac{qL^3}{24}\right]$$

$$2M_b\,L = \frac{qL^3}{4}$$

$$\boxed{M_b = \frac{qL^2}{8}}$$