Ruteo de Vehículos

IN74O – MODELOS INDUSTRIALES

Agosto 2004

Ruteo de Vehículos en la Industria

•Asignación de vehículos a tareas o clientes.

•Ruteo de vehículos.

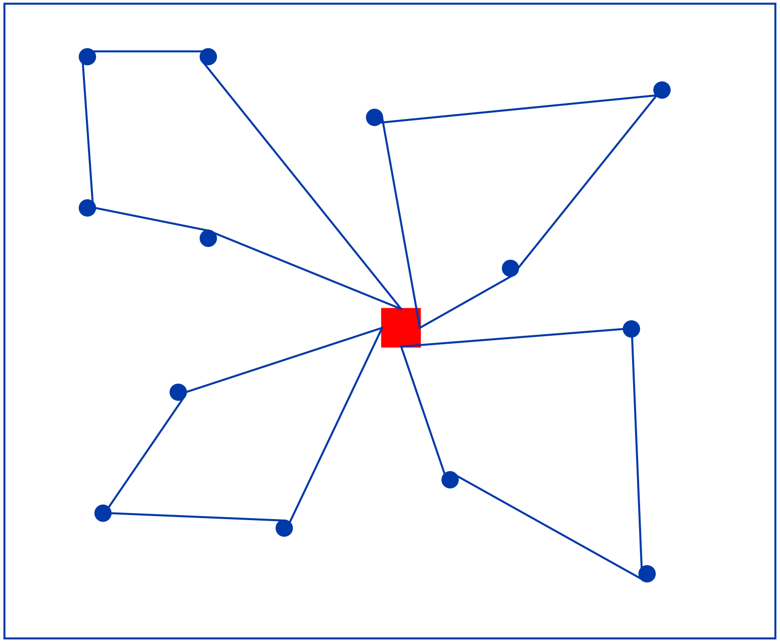
Trasporte de Pasajeros:
•Buses
•Taxis
•Tren

Productos:
•Embotelladora
•Supermercados.
•Minerales
•Bosques.
•Bencina

Servicios:
•Correo
•Basura
•Emergencia médica
•Servicio Técnico

Problema General de Ruteo de Vehículos

Diseñar rutas de vehículos para satisfacer requerimientos de clientes optimizando función objetiva dada.



● Clientes

■ Bodega

Objetivos

- Minimizar costos reales.
- Minimizar costos de contrato.
- Minimizar tiempo de llegada (emergencia).
- Maximizar la calidad del servicio (clientes atendidos a tiempo).
- Minimizar el riesgo de no cumplir la demanda.
- Maximizar beneficio neto.

Factores

- Número de depósitos:
  - Uno.
  - Múltiples (una o más plantas).
- Número de destinos:
  - Uno.
  - Múltiples (madera: puerto, aserraderos, planta de celulosa.).
- Demanda:
  - Determinística: entregas de una supertienda.
  - Semialeatoria: embotelladora.
  - Aleatoria:
    - En volumen: CCU.
    - En localización: fallas Chilectra.
    - En tiempo de viaje: Shell.

Factores

- Frecuencia de viajes:
  - Múltiples por día: Chuquicamata.
  - De duración de más de un día: Aserraderos Arauco.
- Tiempos de trabajo:
  - Con sobretiempo.
  - Sin sobretiempo.
- Tiempos y distancias de viaje:
  - Conocidos: minería.
  - Estimados: bosque.
  - Aleatorios: ciudad.

Factores

- Entregas periódicas, rutas que repiten clientes (por ejemplo, clientes que reciben dos veces por semana).
  - Petróleo.
  - Bebidas.
- Ruteo con inventario:
  - Bombas de bencina: los clientes deben ser atendidos antes que se acabe la bencina.
- Problemas de congestión.
- Ligazón cliente-vehículo.
- Transbordos: camión-tren.
- Costos de ida y vuelta distintos entre clientes.

Factores

- Camiones con compartimentos para distintos productos: Bancura.
- Restricciones en la duración de la ruta (por ejemplo, terminar antes de 8 horas).
- Ventanas de tiempo:
  - Supermercado permite descargar entre 6-8 y 21-23 hrs.
  - Documentos bancarios.
  - Prohibición de entrar al centro en determinadas horas.
- Relaciones de precedencia fija entre algunos clientes.
- Tamaño de flota variable.
- Penalidad por no cumplir con algún cliente (subcontrato).

Métodos de Solución

Manuales:
- Experiencia del operador.
- Uso de mapas, afiches, cartas Gantt, pizarras, etc.
- Solución por lógica

Heurísticos:
- Utilización de computadores.
- Aproximados a la solución óptima.

Optimización:
- Modelos matemáticos.
- Aproximaciones con heurísticas.

Caso Real

Constraint Programming and Column Generation Methods to Solve the Dynamic Vehicle Routing Problem for Repair Services

Motivation

- Real Problem: Dynamic dispatch of technicians of Xerox Chile to repair failures of their machines (color and black-and-white digital printers, digital presses, multifunction devices, and digital copiers) along the day.
- Xerox strategic objective: Client satisfaction, so technical service becomes relevant.
- Clients have different priorities, which define different goal response times for clients at different priority levels .

Motivation

- The goal response time is defined as the maximum allowable time for a technician to reach the client measured from the time of the service request

- Interesting feature of this specific problem is that usually there are some technicians specialized in certain type of machines.

- Service times depend on the specific failure of the visited machine: final service (repair) time not always matches the description of the failure provided by the client at the time of the request. Once the repairman reaches a machine, he can estimate the repair time with much more certainty.

Motivation

- The objective function for assigning technicians and jobs is to minimize two components:
  - **the sum of the differences between goal response times of clients and the effective service time provided by Xerox**
  - **the sum of travel times.**

- The way in which Xerox assigns their jobs is determined by the currently relative position of clients in a queue kept by the dispatcher sorted by priority.

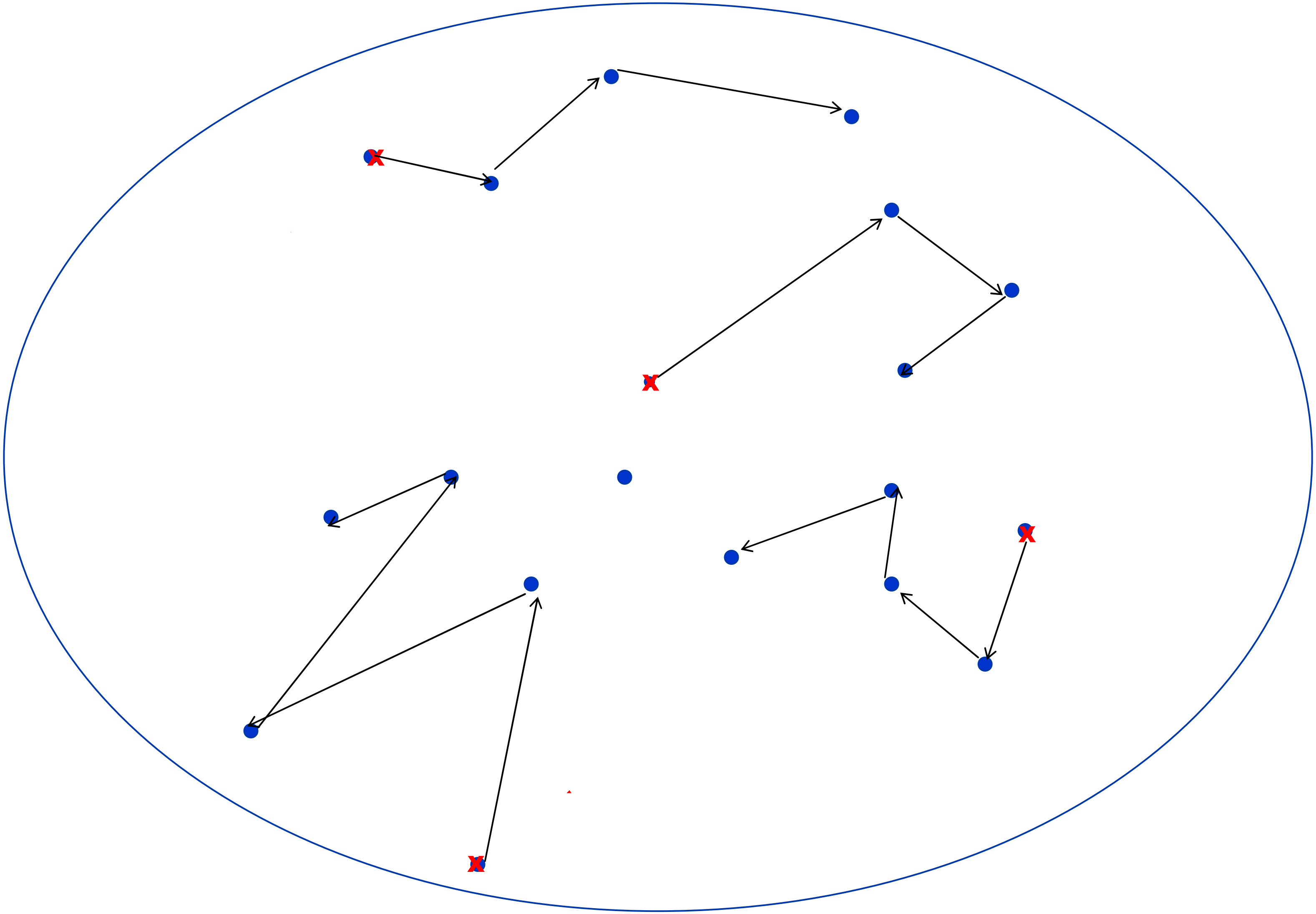
Problem

- 400 call daily.
- 100 technicians.

$\left.\right\}$ **4 jobs for technician in average**

- Information:
  - Average time service for type of machine : $\bar{t} \sim 1.5hrs$
  - Average travel time between zones (comunas). : $\bar{s} \sim 25 \min$

Problem:
Great variability in response time



Approaches

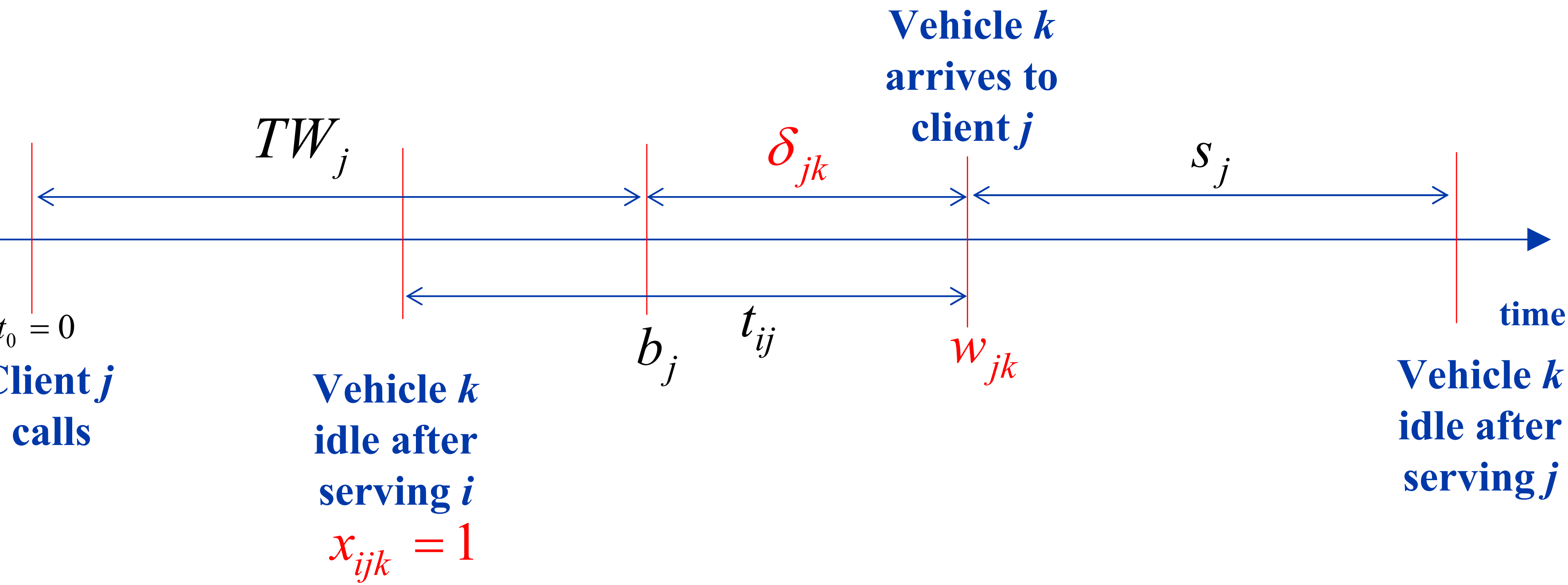
- Strategic, based on queuing theory.
  - Dynamic Travelling Salesman Problem (Psaraftis,1988)
  - Dynamic Travelling Repairman Problem (Bertsimas et. al., 1991)
- Algorithmic approach:
  - Analytical Models
  - Heuristics and Metaheurstics
    - Tabu Seach (Gendreau et. al., 1999), Ant Colony System adapted to solve DVRP (Montemanni, 2002).

Proposed formulation and modeling

- For this problem, only the upper bound of the soft time window (related to the goal response time of each client) is considered, since the objective is to serve each requirement as soon as possible.

- Service times are quantitatively longer and less accurate than travel times, therefore the former will play a more relevant role than the latter into the proposed decision rules.

Model: VRPTW



Variables

$$x_{ijk} : \begin{cases} 1 \text{ if vehicle } k \text{ goes from client } i \text{ to } j \\ 0 \text{ otherwise} \end{cases}$$

w_{ik} : time in which vehicle k starts service of client i

δ_{ik} : time window violation of vehicle k to service client i

FO: Minimize TW violation and travel time

$$\min_{x,\delta} \underbrace{\sum_{k\in K}\sum_{i\in I}\delta_{ki}}_{\substack{\text{Violaciones}\\ \text{de las ventanas}\\ \text{de tiempo}}} + \overbrace{\beta}^{\substack{\text{Penalización}\\ \text{tiempos}\\ \text{de viaje}}} \underbrace{\sum_{k\in K}\sum_{i,j\in I} t_{ij}x_{ijk}}_{\substack{\text{Tiempos}\\ \text{de viaje}}}$$

Todos las máquinas deben ser atendidas por un y solo un técnico.

$$\sum_{k\in K}\sum_{j\in I} x_{ijk} = 1$$

$$\forall i \in \{I \backslash m_{I+1}\}$$

De todas las máquinas, en cola de espera, entra y sale el mismo técnico

$$\sum_{\mathrm{i}} x_{ijk} - \sum_{\mathrm{i}} x_{jik} = 0$$

$$\forall j \in \{m_{K+1}, \ldots, m_I\}, \forall k \in K$$

El inicio del servicio de cada máquina debe ser anterior al final del día

$$w_{ik} \le L \sum_{j \in I} x_{jik}$$
$$\forall i \in I, \forall k \in K$$

Concordancia entre tiempos:
Si técnico *k* visita máquina *i* y luego a máquina *j*, entonces el tiempo de inicio de servicio de *j* debe ser mayor que el iempo de inicio de servicio de *i* más el tiempo de reparación de *i* más el iempo de viaje entre *i* y *j*

$$w_{ik} + s_i + t_{ij} - w_{jk} \le (1 - x_{ijk}) * M$$
$$\forall i, j \in I, \forall k \in K$$

Se incurre violación de ventana de tiempo si el tiempo de inicio de servicio es mayor que la cota superior de la ventana de tiempo.

$$w_{ik} - \delta_{ik} \le b_i$$
$$\forall i \in I, \forall k \in K$$

Naturaleza de las variables

$$x_{ijk} \in \{0,1\}$$
$$w_{i,k}, \delta_{i,k} \ge 0 \quad \forall i, j \in I, \forall k \in K$$

Características del Modelo

- El VRPTW es NP-hard.

- Programación y resolución resolución con CPLEX 7.5 permitió resolver instancias con 20 clientes y 5 técnicos en 3.5 hrs. (Pentium IV 2.2 Ghz, 256 RAM).

- Instancias de tamaño real no es posible resolver con este modelo en tiempo razonable.

Alternativas de Solución:
•Heurísticas •Metaheurísticas (Tabu, etc). •Formulación Alternativa.

→ **Column Generation**

Dantzig Wolfe Decomposition
Column Generation

Decompose Original Problem into:

- Master Problem – Choose routes
  OF and consistency (coverage) constraint
- Sub Problem – Generate Routes
  Modified OF, route constraints and time

$$\min_{x,\delta} \sum_{k \in K} \sum_{i \in I} \delta_{ki} + \sum_{k \in K} \sum_{i,j \in I} t_{ij} x_{ijk}$$

$$\sum_{k \in K} \sum_{j \in I} x_{ijk} = 1 \qquad \forall i \in \{ I \backslash m_{I+1} \}$$

Master Problem: Set Partitioning Model

$$\sum_{\mathrm{i}} x_{ijk} - \sum_{\mathrm{i}} x_{jik} = 0$$

$$\forall j \in \{ m_{K+1}, \ldots, m_I \}, \forall k \in K$$

$$w_{ik} + s_i + t_{ij} - w_{jk} \le (1 - x_{ijk}) * M \qquad \forall i, j \in I, \forall k \in K$$

$$x_{ijk} \le c_{jk} \qquad \forall i, j \in I, \forall k \in K$$

$$w_{ik} \le L \sum_{j \in I} x_{ijk} \qquad \forall i \in I, \forall k \in K$$

$$\delta_{ik} \ge [w_{ik} - b_i] \qquad \forall i \in I, \forall k \in K$$

$$x_{ijk} \in \{0, 1\}$$

$$w_{i,k}, \delta_{i,k} \ge 0 \quad \forall i, j \in I, \forall k \in K$$

COLUMN GENERATOR Sub Problem

Master Problem
Set Partitioning
Model

Dual
ultipliers

Columns

COLUMN
GENERATOR
Sub Problem

$$P: \quad \min_{x} \sum_{r\in R} c^r x^r$$

$$\sum_{r\in R} a_i^r x^r = 1 \quad \forall i \in I \longrightarrow \alpha_i$$

$$x^r \in \{0,1\}, \quad r \in R$$

$$x^r = \begin{cases} 1 \text{ si ruta r es seguida} \\ 0 \text{ si no} \end{cases}$$

$$c^r = \text{costo de la ruta r}$$

$$a_i^r = \begin{cases} 1 \text{ si máquina } i \text{ pertenece a ruta} \\ 0 \text{ si no} \end{cases}$$

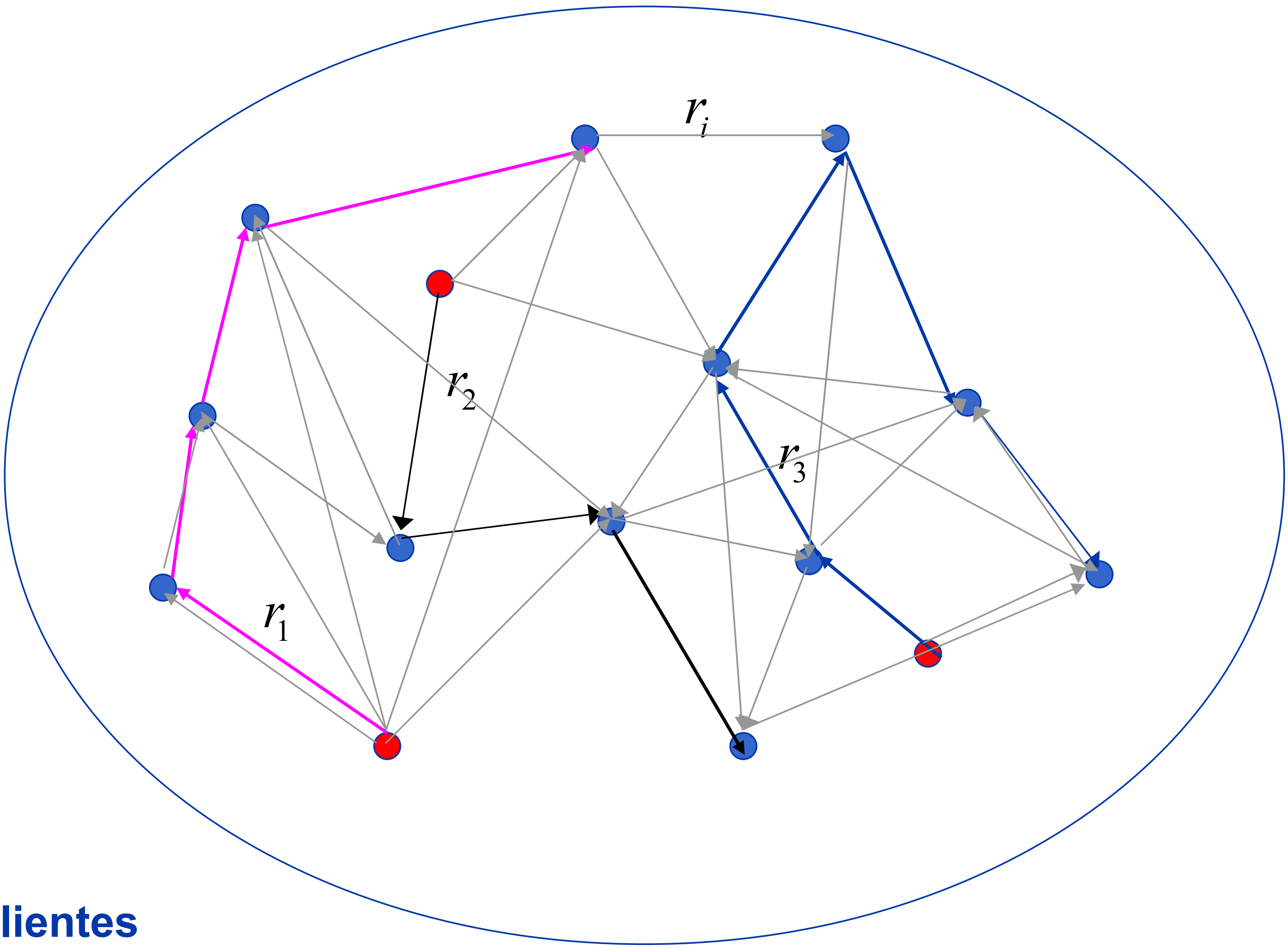
$$R = \text{Cjto. de Rutas factibles}$$

Reduced Cost Column

Suma de los multiplicadores duales de las máquinas que pertenecen a esta columna

$$\overline{c_k} = \sum_{i\in P^k} \delta_{ki} + \beta \sum_{i,j\in P^k} t_{ij} - \overbrace{\sum_{i\in P^k} \alpha_i}$$

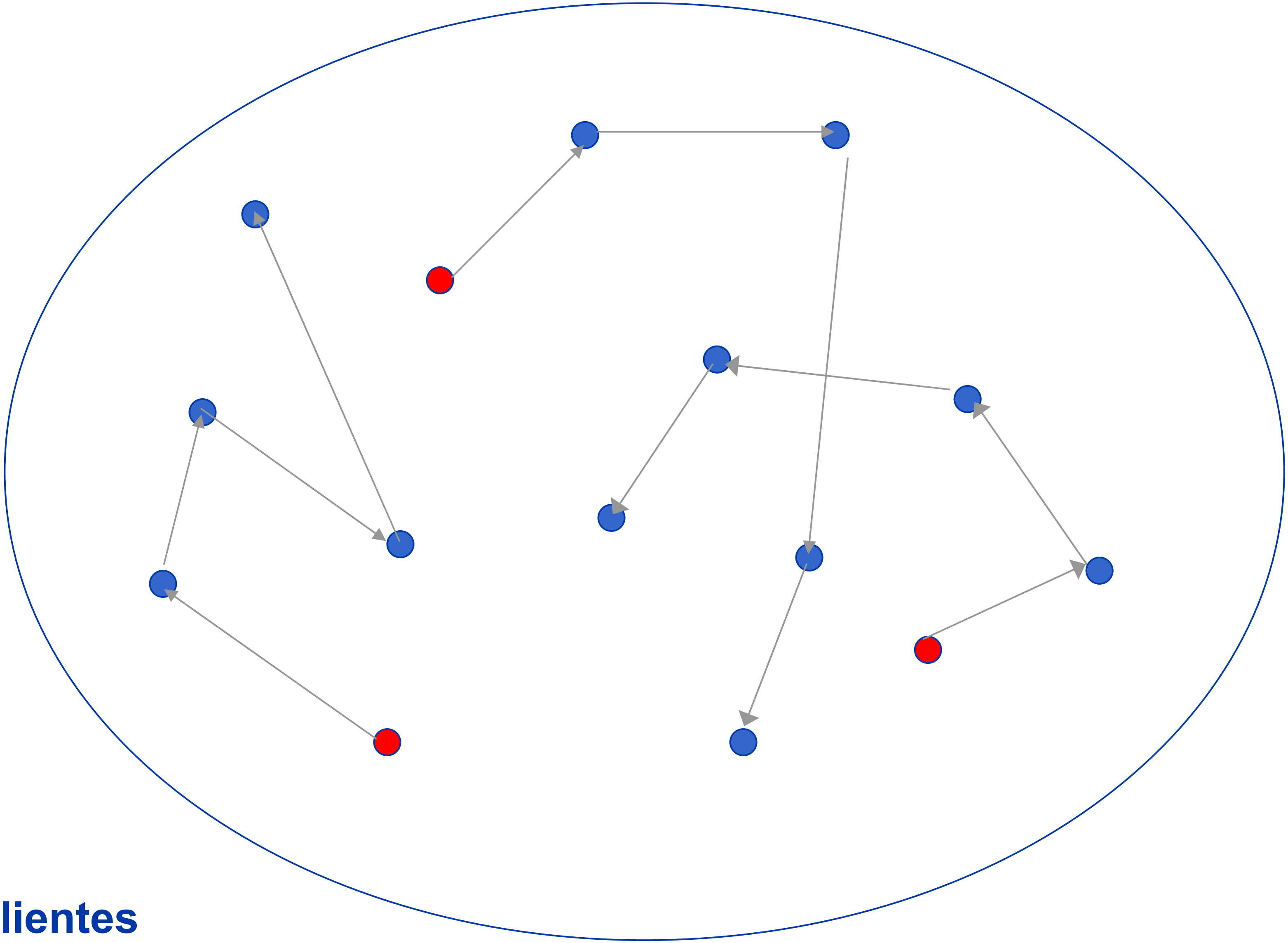
2. Column Generation cial para formar solución factible



Clientes

Técnicos

3. Selección de las mejores columnas



Clientes

Técnicos

Description of the General Process

1. Generate a pool of initial routes: For all tecnician generate a route with diferent machines, whit greedy heuristic.

2. Solve a linear relaxation of the master problem with the generated pool of routes.

3. From the dual variables associated to the constraints of each machine in the master problem, generate new routes with the minimum reduced cost, adapted from the sub-problem. If there exists a route with negative reduced cost *cr** , go to 4, otherwise go to 5.

4. Using the sub-problem generate all possible columns with reduced cost less than $cr^*\gamma$ and length *L*, where γ is a control parameter that satisfies $0 < \gamma < 1$, thus $cr \in [cr^*, cr^*\gamma] < 0$
   This helps control the columns generated at each iteration. If *cr** is a large negative number, it may be convenient to generate many columns in the range $[cr^*, cr^*\gamma]$ where will be large negative numbers. Go to 2.

5. Solve IP associated with the master problem including all the columns generated in the previous steps, obtaining the final routes to be followed by each technician.

Generate feasible routes

	r_1	r_2	r_3	$\cdots$	r_{R_1}
c_1	1	1	1	$\cdots$	0
c_2	1	0	1	$\cdots$	1
c_3	0	1	0	$\cdots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
c_i	1	1	1	$\cdots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
c_m	0	0	1	$\cdots$	1

MP
LR SPP

Generate feasible paths

	r_1	r_2	r_3	$\cdots$	r_{R_1}	Σ
c_1	1	1	1	$\cdots$	0	1
c_2	1	0	1	$\cdots$	1	1
c_3	0	1	0	$\cdots$	1	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
c_i	1	1	1	$\cdots$	1	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
c_m	0	0	1	$\cdots$	1	1

Are the choosen columns the optimal configuration?

MP
LR SPP

$\xrightarrow{\alpha_i}$

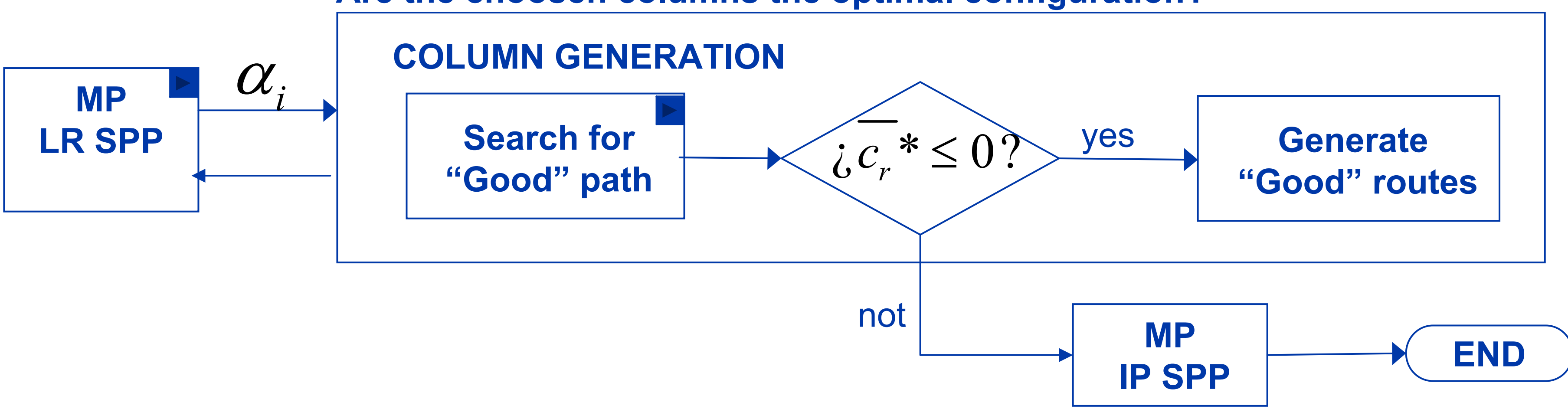
COLUMN GENERATION

Search for “Good” route

Generate feasible paths

	r_1	r_2	r_3	$\cdots$	r_{R_1}	$\sum$	r_{R_1+1}	r_{R_1+2}	$\cdots$	r_{R_2}	$\sum$	r_{R_2+1}	r_{R_2+2}	$\cdots$	r_{R_3}	$\sum$
c_1	1	1	1	$\cdots$	0	1	1	1	$\cdots$	0	1	0	0	$\cdots$	1	1
c_2	1	0	1	$\cdots$	1	1	0	1	$\cdots$	1	1	0	0	$\cdots$	0	1
c_3	0	1	0	$\cdots$	1	1	0	0	$\cdots$	0	1	1	0	$\cdots$	0	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
c_i	1	1	1	$\cdots$	1	1	1	0	$\cdots$	1	1	1	1	$\cdots$	0	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
c_m	0	0	1	$\cdots$	1	1	0	0	$\cdots$	0	1	0	1	$\cdots$	1	1

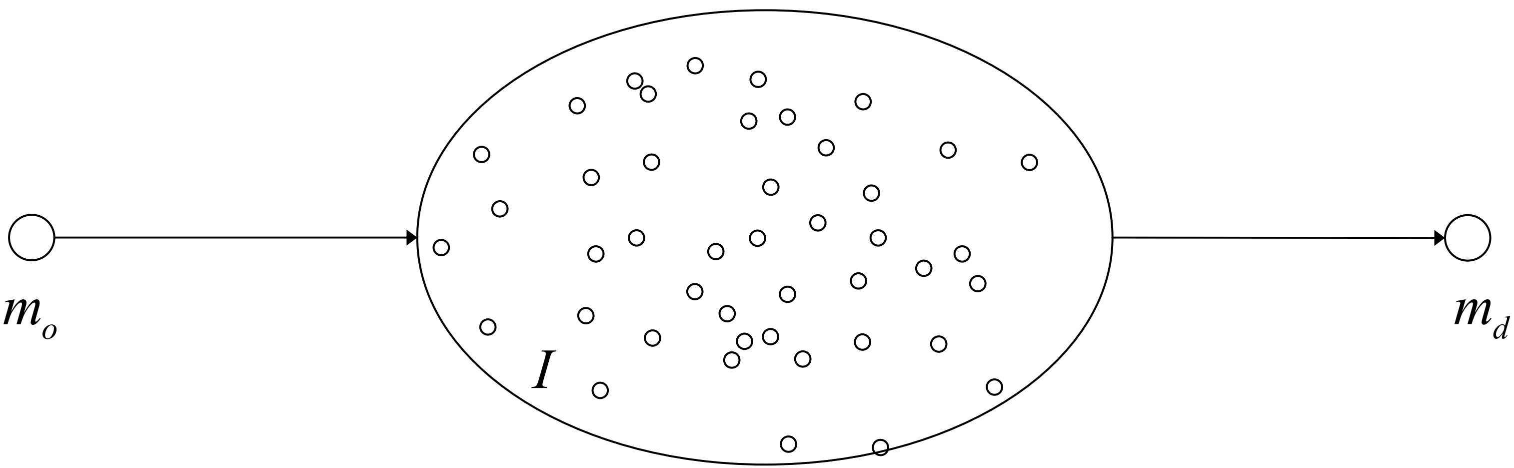
Are the choosen columns the optimal configuration?



Sub Problem: Shortest Path Problem on G=(N,A)

Encontrar el camino más corto entre *mo* y *md*

$$Costo\ de\ arco_{ij} = tiempo\ de\ viaje + violación\ ventana\ de\ tiempo\ nodo\ j - variable\ dual\ de\ nodo\ j$$



$$\text{Costo Path } m_o \text{ a } m_d = F_{m_o - m_d} = \sum_{i \in Path} \delta_i + \beta \sum_{(i,j) \in Path} t_{ij} - \sum_{i \in Path} \alpha_i$$

Sub Problem: Shortest Path Problem on G=(N,A)

Problema NP-Hard
Métodos de Solución

- **Programación Dinámica**
- **Constraint Programming**

$F(S, i, w)$: costo mínimo de la ruta desde m_o hasta $i \in M \setminus m_o$, visitando todos los nodos del conjunto $S \subseteq M \setminus m_o$ solamente una vez y sirviendo al nodo i en w o después.

$$F(\phi, m_o, 0) = 0$$

$$F(S, j, w) = \min_{i} \Big\{ F(S - \{j\}, i, w') + (t_{ij} + \delta j - \alpha_j) \mid i \in S - \{j\},$$

$$w' \leq w - (t_{ij} + s_i), w' \leq L, \delta_j = \text{máx}\{0, w - b_j\} \Big\},$$

$$\forall S \subseteq M, j \in S \quad \text{y} \quad w \leq L.$$

Sub Problem: Shortest Path Problem on G=(N,A)

- Problema NP-Hard.
- Largo pequeño de las rutas permite resolverlo en tiempo razonable

Metodología Usada: Constraint Programming
Rápida resolución dada estructura del problema (pocas máquinas por técnico).

Constraint Programming

- A computer programming methodology
- Solves
  - Constraint satisfaction problems
  - Combinatorial optimization problems
- Methodology
  - Represent a model of a problem in a computer programming language.
  - Describe a search strategy for solving the problem

Constraint satisfaction problems:	**Combinatorial Optimization Problems**
Find a Feasible Solution. Subject to Constraints. Over a set of values of decision variables	•Minimize (or maximize) an Obj Function •Subject to Constraints. •Over a set of values of decision variables

Constraint Programing Model

SETS

M :	clients
$MT \subseteq M$:	clients with technician
$MQ \subseteq M$:	clients in queue

PARAMETERS

s_i :	reparation time of client i
b_i :	due time of client i
F :	period end of evaluation
tv_{ij} :	travel time between clients i and j
L :	lenght of the path to generate
c_i :	dual variable of client i given by the LR of SSP
$tvmax$:	maximum time of trip allowed in the path to generate

VARIABLES

$MaqSeq[l] \in M$:	secuence of clients in path to generate, $l = 0..L$
$w[l] \in [0..F]$:	time to begin service of client l in position l in the path, $l = 0..L$
$d[l] \in [0..F]$:	time windows violation, $l = 1..L$
$t[l] \in [0..tvmax]$:	travel time to clients in position l - 1 to l

OBJECTIVE FUNCTION

$$Min \ \ ReduceCost = \left[\left(B * \sum_l d[l] + \sum_l t[l] \right) - \sum_l c_{MaqSeq[l]} \right]$$

SUBJECT TO

1. Time to begin the service

$$w[l] = w[l-1] + s_{MaqSeq[l-1]} + t[l] \qquad , \forall l = 1..L$$

2. Violation of time windows

$$d[l] \geq w[l] - b_{MaqSeq[l-1]} \qquad , \forall l = 0..L$$

3. Travel time between clients in the path

$$t[l] = tv_{MaqSeq[l-1], MaqSeq[l]} \qquad , \forall l = 1..L$$

4. Path begin time (depends of the state of service of the first client)

$$w[0] = b_{MaqSeq[0]}$$

5. All the clients in the path must be different

$$MaqSeq[m] <> MaqSeq[n] \qquad , \forall m < n = 0..L$$

$$alldiferent(MaqSeq)$$

6. First client in the path must have technician

$$MaqSeq[0] \in MT$$

7. Other clients in the path be not have technician

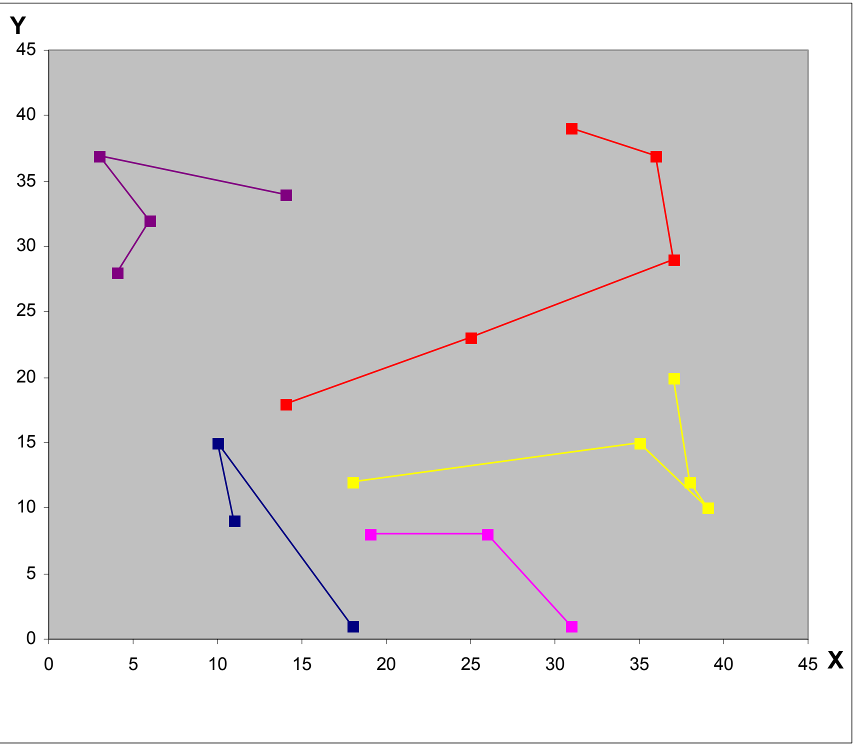
$$MaqSeq[l] \in MO \qquad , \forall l = 1..L$$

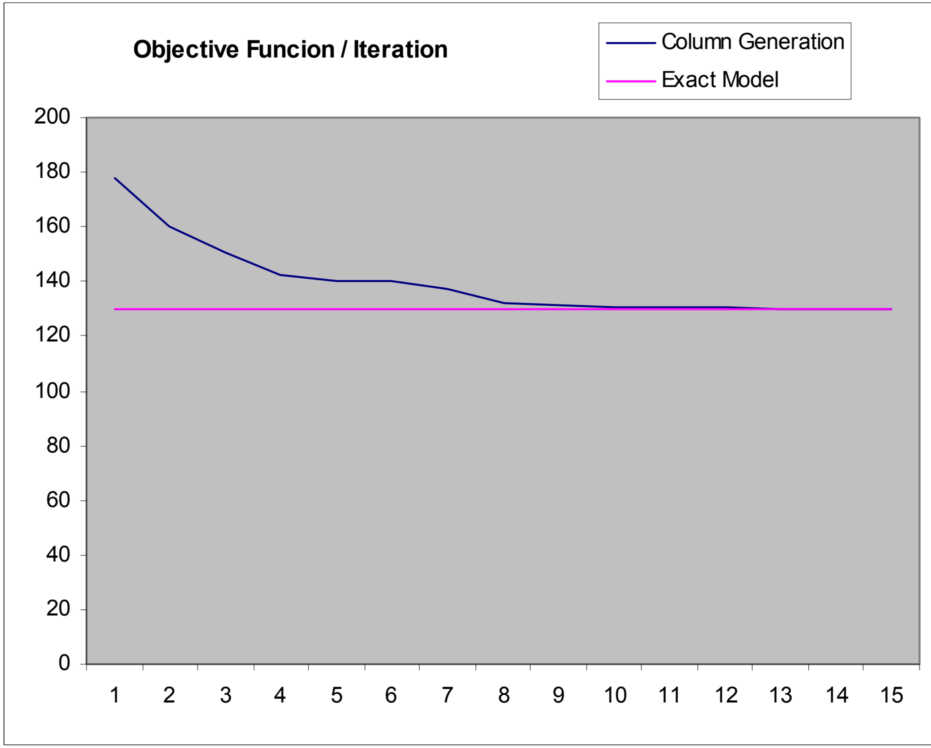
Exact Model v/s Column Generation

Instance: 5 technicians, 20 clients

Exact Solution in 3.5 hrs.

Same solution with Column Generation in 15 iterations, 3.7 sec.





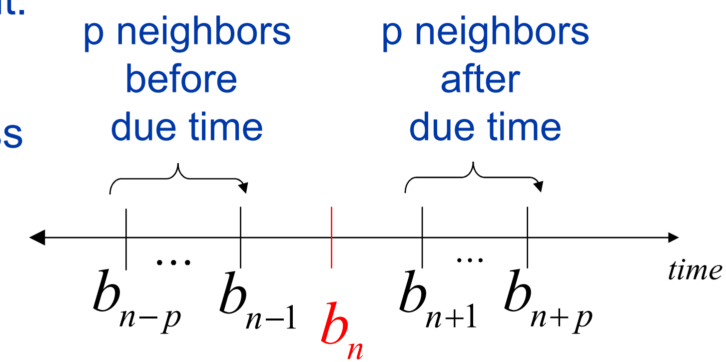
Real Instance

- Model coded in OPL Studio 3.5, and solved by using CPLEX 7.5 and SOLVER 5.2
- Real Operation: Normal working day, 36 clients (machines), 9 technicians.
- Optimization total time 500 seg
- The following maps compare observed routing versus optimized routing.
- Illogical patterns are due to TW and priority constraints

	REAL	CG
TOTAL TRAVEL TIME (min)	918	656
TOTAL Violation TW (min)	3965	1530
Z*	**8848**	**3716**

Dynamic Insertion

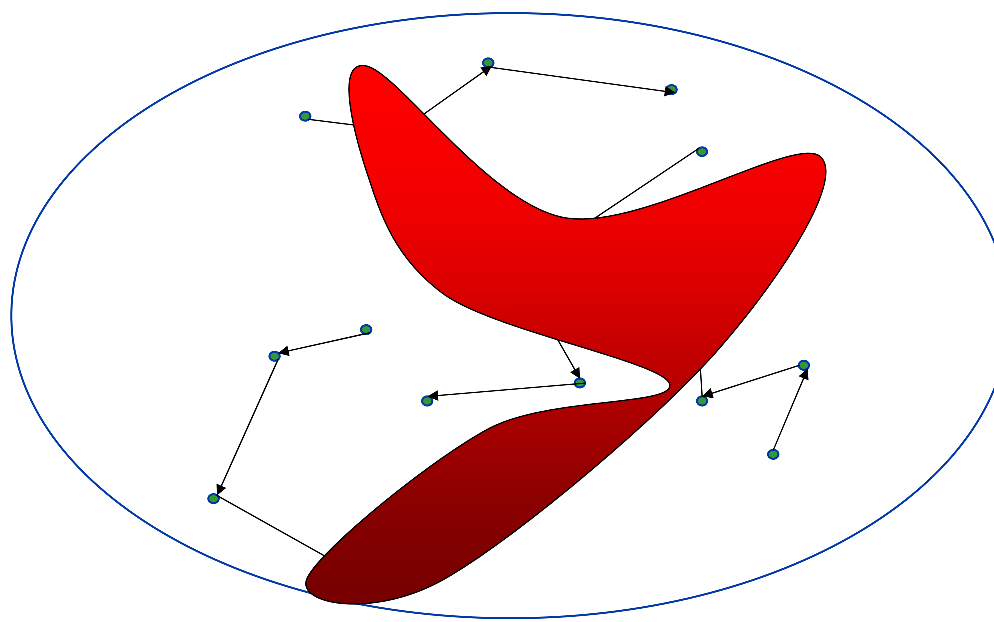
1. Generate a Neighborhood around new client.
2. Insert it in the path with minimum cost.
3. After N new clients run the complete process



Neighborhood

•Due Time(p): p neighbors before and after due time

•Distance(q): q neighbors by distance proximity




Real instance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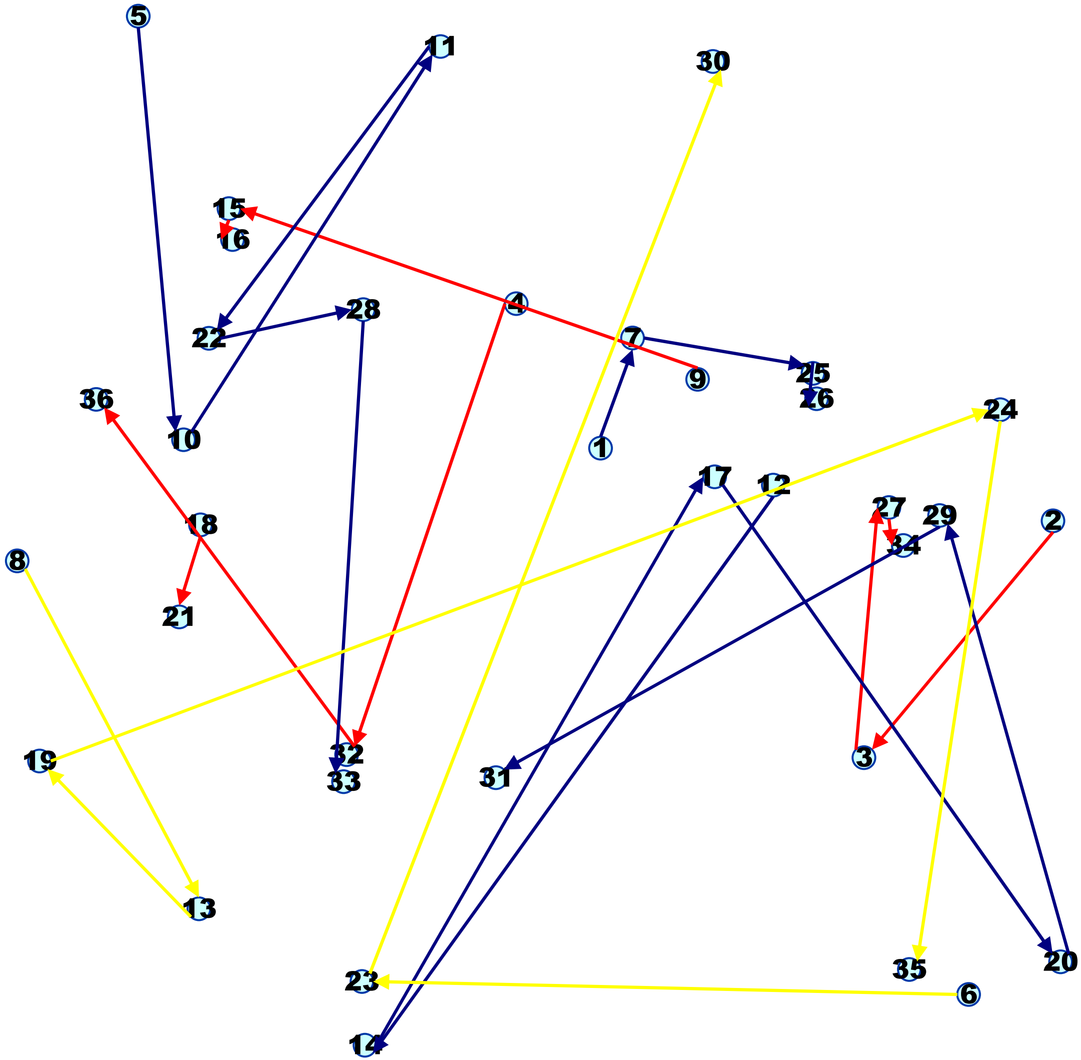
Real Path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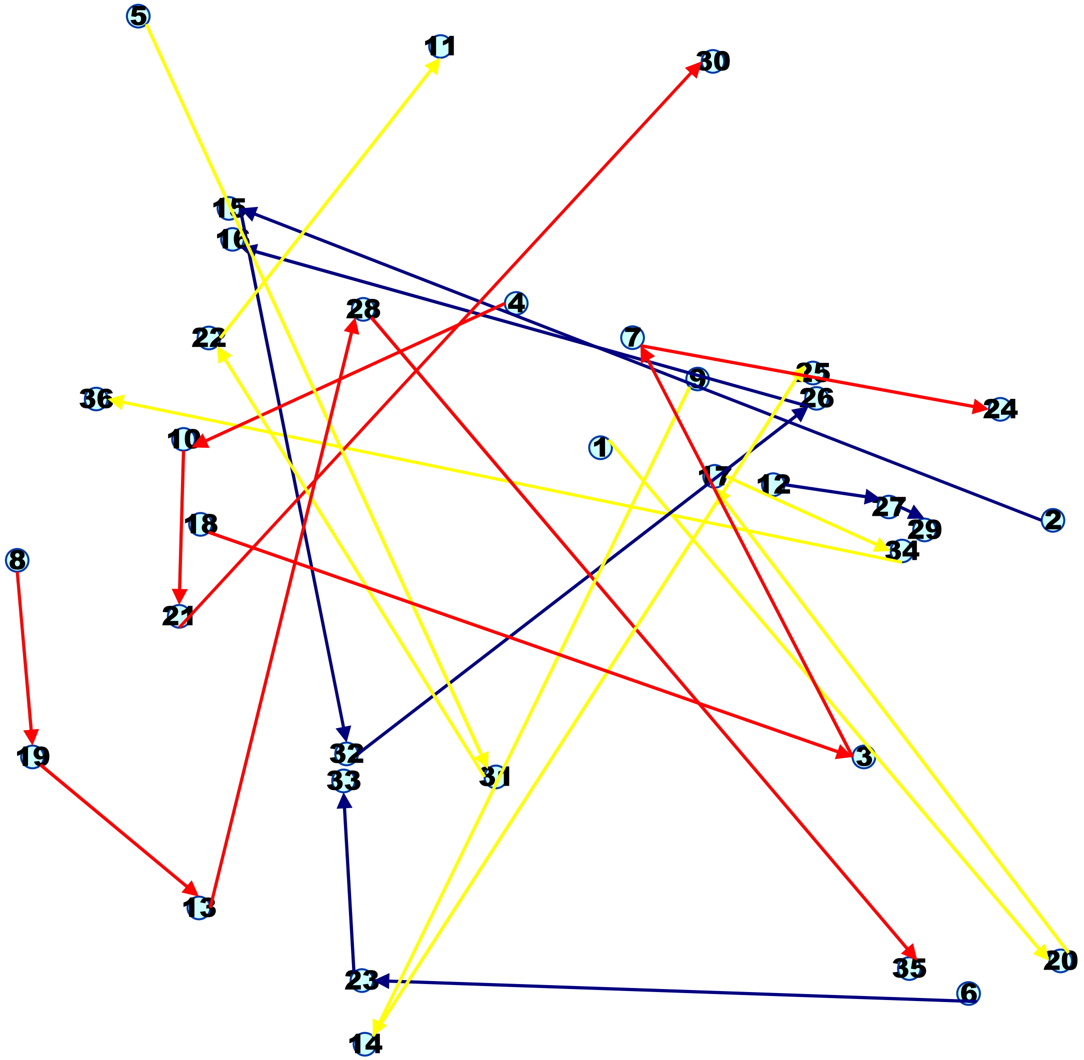
OPT Path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OPT PATHS

Path	Maq	w	d	s	trm	tv
1	m1	755	0	60	755	0
	m20	838	73	30	765	23
	m17	892	0	80	929	24
	m34	993	0	90	1059	21
	m36	1109	0	145	1890	26
2	m2	1002	0	120	1002	0
	m15	1152	0	120	1716	30
	m32	1290	0	70	2032	18
	m26	1371	0	180	1652	11
	m16	1581	0	120	1718	30
3	m4	1022	0	100	1022	0
	m10	1152	307	85	845	30
	m21	1262	315	60	947	25
	m30	1354	0	150	1893	32
4	m5	1024	0	45	1024	0
	m31	1083	0	90	1780	14
	m22	1190	0	120	1832	17
	m11	1325	0	40	1779	15
5	m6	1980	0	60	1980	0
	m23	2068	0	20	2250	28
	m33	2113	0	60	2214	25
6	m8	943	0	135	943	0
	m19	1101	116	120	985	23
	m13	1244	245	60	999	23
	m28	1352	0	30	1701	48
	m35	1425	0	90	2010	43
7	m9	1068	0	60	1068	0
	m14	1143	0	90	1834	15
	m25	1244	0	90	1651	11
8	m12	1662	0	75	1662	0
	m27	1761	0	30	2040	24
	m29	1812	0	60	1955	21
9	m18	970	0	50	970	0
	m3	1045	30	180	1015	25
	m7	1249	444	195	805	24
	m24	1474	0	105	1758	30
	TOTAL		**1530**			**656**
		Z*	**3716**			

REAL PATHS

Path	Maq	w	d	s	trm	tv
1	m2	660	0	120	1002	19
	m3	810	0	180	1015	30
	m27	1680	0	30	2040	30
	m34	1800	741	90	1059	0
2	m4	1670	648	100	1022	20
	m36	2085	53	70	2032	22
	m32	1820	0	145	1890	26
3	m18	870	0	50	970	0
	m21	930	0	60	947	0
4	m9	1020	0	60	1068	30
	m15	1710	0	120	1716	60
	m16	1860	142	120	1718	17
5	m20	1665	3	75	1662	15
	m17	1770	0	90	1834	20
	m29	910	0	80	929	20
	m12	765	0	30	765	0
	m14	1020	0	60	1955	20
	m31	1980	200	90	1780	40
6	m10	1665	641	45	1024	15
	m5	720	0	85	845	30
	m28	1980	201	40	1779	60
	m22	1800	0	120	1832	15
	m11	1740	39	30	1701	30
	m33	2130	0	60	2214	42
7	m7	900	145	60	755	41
	m1	600	0	195	805	14
	m25	990	0	90	1651	18
	m26	1650	0	180	1652	0
8	m6	990	0	60	1980	20
	m23	990	0	20	2250	30
	m30	1680	0	150	1893	30
9	m8	705	0	135	943	10
	m13	930	0	60	999	20
	m19	1800	815	120	985	150
	m24	1995	237	105	1758	15
	m35	2110	100	90	2010	10
	TOTAL		**3965**			**918**
		Z*	**8848**			

Ruteo de Vehículos

IN74O – MODELOS INDUSTRIALES

Agosto 2004

$$\min z = cx$$
$$Ax = b$$
$$x \geq 0$$

n variables, m restricciones
n>>m,
No se conocen todas las columnas de A

$$\text{sea } B \text{ base factible}$$
$$x = (x_B, x_n)$$
$$x_N = 0, x_B = B^{-1}b \geq 0$$

$$\text{Buscar "mejor" columna}$$
$$\min \overline{c_j} = \min c_j - c_B B^{-1} a_j = \overline{c_j}^*$$

$$\text{sea } \alpha = c_B B^{-1} = \text{ vector de multiplicadores duales}$$

¿Óptimo? costo reducido de todas las columnas debe ser ≥ 0

$$\overline{c_j} = c_j - c_B B^{-1} a_j$$

$$¿\ \overline{c_r}^* \leq 0\ ?$$

Encontrar nueva Ruta con menor $\overline{c_r}*$

$$\left[\min_r\left(c_r - \sum_{i\in Maq} \pi_i y_{ir}\right)\right] = \overline{c_r}*$$

con

c_r = costo de la ruta r (sobreventanas y tiempo de viaje)

y_{ir} = 1 si maquina i pertenece a ruta r, 0 si no