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Horizontal Mergers: An Equilibrium Analysis

By JOSEPH FARRELL AND CARL SHAPIRO*

We analyze horizontal mergers in Cournot oligopoly. We find general conditions under which such mergers raise price, and show that any merger not creating synergies raises price. We develop a procedure for analyzing the effect of a merger on rivals and consumers and thus provide sufficient conditions for profitable mergers to raise welfare. We show that traditional merger analysis can be misleading in its use of the Herfindahl Index. Our analysis stresses the output responses of large firms not participating in the merger. (JEL 022,612)

Mergers between large firms in the same industry have long been a public policy concern. In the United States, Section 7 of the Clayton Act (as amended by the Celler-Kefauver Act) prohibits mergers that “substantially decrease... competition or tend... to create a monopoly.” Under the Hart-Scott-Rodino Act, large firms must report any proposed substantial merger to the Department of Justice and the Federal Trade Commission, which evaluate the merger’s likely effect on competition and can choose to permit or to oppose it.

In evaluating proposed mergers, federal antitrust officials generally apply rules summarized in the Department of Justice’s Merger Guidelines (1984).¹ An important part of merger analysis under these guide-

lines involves estimating the effect of a proposed merger on market concentration. In particular, the analyst is instructed to pay careful attention to the initial *level* of concentration in the industry and the predicted *change* in concentration due to the merger. Roughly speaking, the guidelines permit mergers that will not increase concentration by very much or that will leave it low even after the merger. This reflects a view that anticompetitive harm is an increasing function of concentration, which is measured using the Herfindahl-Hirschman index, H , defined as the sum of the squares of the firms’ market shares.

The Merger Guidelines, while surely more sophisticated than what they replaced, are not based on explicit analysis of how a merger will affect equilibrium output and welfare. This theoretical shortcoming leads to two basic problems with the guidelines’ use of concentration measures.

The first such problem is the curious rule that the guidelines use to estimate the effect of a merger on H . This rule takes the initial market shares of the merging firms, s_1 and s_2 , and assumes that the new entity’s market share will be $s_1 + s_2$, so that the merger will increase H by $(s_1 + s_2)^2 - (s_1^2 + s_2^2) = 2s_1s_2$. But if indeed all firms maintain their pre-merger outputs, then the merger will affect neither consumers nor nonparticipant firms, so it will be socially desirable if and only if it is privately profitable. If, as is more likely, outputs change in response to the merger, then the $2s_1s_2$ formula is wrong. Equilibrium analysis, such as we provide below, is neces-

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¹For a discussion of these guidelines, see the symposium in the *Journal of Economic Perspectives*, Fall 1987.

sary to compute the change in H in a logically consistent manner.

The second problem is deeper and more serious. Implicitly, the guidelines assume a reliable (inverse) relationship between market concentration and market performance. In particular, the entire approach presumes that a *structural* change, such as a merger, that increases the equilibrium value of H also systematically reduces equilibrium welfare, W , defined as the sum of consumer and producer surplus or equivalently the difference between gross consumer benefits and production costs.

Is there in fact such a reliable relationship between changes in market concentration and changes in economic welfare? In some very special circumstances, there is. For example, if n equally efficient firms with constant unit cost compete as Cournot oligopolists, then W is increasing in n and $H = 1/n$, so there is a rigid (inverse) relationship between H and W as n varies.² But if the competing firms are not equally efficient, or if there are economies of scale, there is no reason to expect that concentration and welfare will move in opposite directions in response to a merger.³

The dangers of identifying changes in H with changes in W are suggested by the fact that, starting at a Cournot equilibrium, welfare rises with a small change in firms' outputs if and only if

$$(1) \quad \frac{dX}{X} + \frac{1}{2} \frac{dH}{H} > 0,$$

where X is aggregate output. (See the Appendix for a derivation of this condition.) Of course, increases in output ($dX > 0$) tend to raise welfare, since price exceeds marginal cost in equilibrium. But if a firm with a large market share increases its output, then H ,

X , and W will all rise. And condition (1) shows that for any given (percentage) change in X , welfare is more likely to rise if H increases; it is even possible to have $dX < 0$, $dH > 0$, and yet $dW > 0$.⁴

How can increases in the concentration be associated with increases in welfare? In Cournot equilibrium, larger firms have lower marginal costs, so welfare is enhanced if a fixed total output X is shifted toward them and away from smaller, less efficient firms. But such shifts will increase concentration.

This observation is not a theoretical curiosity. Critics of U.S. antitrust policy have long argued that large firms may be large because they are efficient. If so, then economic welfare may be enhanced if these efficient firms acquire more of the industry's productive capital and thus increase their market share.⁵ One means to do this is by buying the assets of smaller, less efficient rivals. Any useful theory of oligopoly and horizontal mergers should account for the role that mergers may play in this process.

Given the complex relationship between concentration, output, and welfare, a careful analysis of the welfare effects of mergers is badly needed. The few existing theoretical analyses of the effects of horizontal mergers have used very special models, and their insights of course reemerge below.⁶ But our goal is more ambitious: we use Cournot oligopoly theory, with quite general cost and demand functions, to study the output and welfare effects of mergers. At a theoretical level, we develop some techniques for analyzing welfare changes in Cournot markets;

⁴For example, consider a Cournot duopoly in which firm 1 has low marginal cost and hence a large market share, and firm 2 has high marginal cost and is small. Closing down firm 2 will scarcely reduce welfare (since its marginal cost is close to the price). And if firm 1 expands at the same time, the net welfare effect will be favorable, since firm 1's marginal cost is distinctly less than the price. Such an output shift would lower X , raise H and yet raise W .

⁵For an influential statement of this view, see for instance Harold Demsetz (1973, 1974).

⁶Steven Salant, Sheldon Switzer, and Robert Reynolds (1983), Raymond Deneckere and Carl Davidson (1985), and Martin Perry and Robert Porter (1985) are the most prominent papers.

²In this model, aggregate output, X , is also a sufficient statistic for welfare, as it is more generally if total production costs depend only on aggregate output, and not on the distribution of output across firms.

³For further discussion of the relationship between changes in H and changes in W , see Farrell and Shapiro (1990).

these techniques apply not only to mergers but more generally. At a policy level, we find surprisingly general sufficient conditions for when a merger should be approved or prohibited.

Our policy analysis is in two parts. First, we study a merger's effect on price. This inquiry will be most telling for those antitrust practitioners who see consumer welfare as the sole objective of antitrust policy. We provide a necessary and sufficient condition for a merger to raise price (Proposition 1) and show in general that mergers in Cournot oligopoly raise price if they generate no synergies between the merging firms (Propositions 2 and 3). We also show that firms with large market shares must achieve impressive synergies or scale economies if their merger is to reduce price.

Second, we study a merger's effect on welfare, W . Here we emphasize the *external* effects on consumers and on nonparticipant firms. In this, our work is squarely in the tradition of mainstream economic analysis, whose presumption has always been that a debate about intervention should focus on externalities.⁷ Write $\Delta\pi^I$ for the change in joint profits of the merging firms (the "insiders"), and ΔW for the change in total welfare associated with a merger. We analyze the next external effect of the merger on rival firms ("outsiders") and on consumers, $\Delta W - \Delta\pi^I \equiv \Delta\pi^O + \Delta CS$. Since any proposed merger is presumably privately profitable, it will also raise welfare if it has a positive external effect—a condition that is, perhaps surprisingly, quite often satisfied. We thus find sufficient conditions for profitable mergers to raise welfare (Proposition 5).

Our emphasis on the external effect also has a great practical advantage. To assess the externality turns out to require much less information than to assess the overall welfare effect, since the effect on insiders' profits

depends on internal cost savings. Such cost savings are often hard to observe, and as a result are given little weight in the Merger Guidelines and in many practitioners' thinking, even though, as Oliver Williamson (1968) stressed, they may well be very important in a merger's overall welfare effect. Thus, Lawrence White (1987, p. 18) wrote:

Efficiencies are easy to promise, yet may be difficult to deliver. All merger proposals will promise theoretical savings in overhead expense, inventory costs, and so on; they will tout "synergies."

Franklin Fisher (1987, p. 36) concurred:

The burden of proof as to cost savings or other offsetting efficiencies, however, should rest squarely on the proponents of a merger, and here I would require a very high standard [of proof]. Such claims are easily made and, I think, often too easily believed.

This emphasis on the externality is our first innovation in merger policy analysis, and it also leads us into our second. Although the externality is easier to work with than the total welfare effect, directly signing the externality is mathematically difficult even in the simplest cases, and completely intractable in general. To overcome this, we introduce differential techniques. The external effect of a merger involving a change ΔX_i in the merging firms' joint output can be calculated as the integrated external effect of *small* changes dX_i . We call such a small change an "infinitesimal merger." It turns out to be easy to sign the external effect of an infinitesimal merger (Proposition 4), and in fairly general circumstances we can sign the external effect of a merger (Proposition 5).

Our paper is organized as follows. In Section I, we recall some results from standard Cournot oligopoly theory. In Section II, we analyze the output and price effects of a merger among Cournot oligopolists, and find broad sufficient conditions for a merger to raise the equilibrium price. In Section III, we calculate the external effect of an infinitesimal merger, and show that it depends only

⁷Of course, mergers typically do not generate externalities in the usual sense of the term. With imperfectly competitive markets, however, a change in the behavior of merging firms does affect the welfare of consumers and other firms. Throughout the paper we take "externality" to mean $\Delta W - \Delta\pi^I$.

on the merging firms' joint market share and on the output responsiveness and market shares of nonparticipant firms. We then give conditions under which privately profitable mergers raise total welfare. Finally, in Section IV, we use our methods to provide welfare analyses of two other oligopoly problems: commitment in Cournot competition, and the optimal level of an import quota. A conclusion follows.

I. Cournot Oligopoly

We use the traditional model of Cournot oligopoly with homogeneous goods. Demand is given by $p(X)$, where p is price, X is industry output, and $p'(X) < 0$; we write ϵ for the absolute value of the elasticity of demand, $\epsilon(X) \equiv -p(X)/Xp'(X)$. The number of firms, n , is exogenous (although of course it changes with a merger), reflecting some important barriers to entry.⁸ We denote firm i 's cost function by $c^i(x_i)$, where x_i is firm i 's output. For notational ease, we write $c^i \equiv c^i(x_i)$ for firm i 's total cost and $c_x^i \equiv c_x^i(x_i)$ for firm i 's marginal cost. Importantly, we permit the firms to differ in efficiency.

In the Cournot equilibrium, each firm i picks its output x_i to maximize its profits, given its rivals' outputs. Writing $y_i \equiv \sum_{j \neq i} x_j = X - x_i$ for aggregate output of all firms other than firm i , firm i 's profits are $\pi^i(x_i, y_i) \equiv p(x_i + y_i)x_i - c^i(x_i)$. Firm i 's first-order condition, $\partial \pi^i / \partial x_i = 0$, is

$$(2) \quad p(X) + x_i p'(X) - c_x^i(x_i) = 0, \\ i = 1, \dots, n.$$

A Cournot equilibrium is a vector (x_1, \dots, x_n) such that equation (2) holds for all n firms. We denote firm i 's market share by $s_i \equiv x_i/X$.

⁸Our analysis can easily accommodate entry by, or the existence of, price-taking fringe firms, if we reinterpret the demand curve $p(X)$ as the residual demand curve facing the oligopolists that we model. What we are ruling out is entry by additional large firms that behave oligopolistically.

Comparing two firms i and j , the Cournot equilibrium conditions, (2), tell us that $x_i > x_j$ if and only if $c_x^i < c_x^j$. In equilibrium, larger firms have lower marginal costs. In any Cournot equilibrium in which different firms produce different quantities, marginal costs differ across firms, so that costs are not minimized given the aggregate output level, and consequently aggregate output, X , is *not* in general a sufficient statistic for welfare.

Throughout the paper, we make two weak assumptions on the Cournot equilibrium; we require both to hold throughout a relevant range, as will become clear below. First, we assume that each firm's reaction curve slopes downward. Equivalently, an increase in rivals' output, y_i , lowers firm i 's marginal revenue:

$$(3) \quad p'(X) + x_i p''(X) < 0, \quad i = 1, \dots, n.$$

Inequality (3) is a very weak assumption and is standard in Cournot analysis; see Avinash Dixit (1986) and Shapiro (1989). It holds if the industry demand curve satisfies $p'(X) + Xp''(X) < 0$.⁹

Second, we assume that each firm's residual demand curve, $p(\cdot + y_i)$, intersects its marginal cost curve from above. This is equivalent to

$$(4) \quad c_{xx}^i(x_i) > p'(X), \quad i = 1, \dots, n.$$

Condition (4) is surely met if marginal cost is nondecreasing, i.e., if $c_{xx}^i \geq 0$. It is among the weaker known stability conditions for Cournot equilibrium (Dixit, 1986).

We now note some comparative-statics properties of Cournot equilibria that will be important below. Consider the effect of a change in rivals' aggregate output, y_i , on firm i 's output. From equation (2), the slope of firm i 's reaction schedule is given by

$$\frac{dx_i}{dy_i} \equiv R_i = - \frac{p' + x_i p''}{2p' + x_i p'' - c_{xx}^i}.$$

⁹If $p'' \leq 0$, then (3) surely holds. If $p'' > 0$, then $p' + Xp'' > p' + x_i p''$.

From condition (3) and firm i 's second-order condition, $R_i < 0$; with (4) we also have $-1 < R_i < 0$. That is, if its rivals jointly expand production, firm i contracts, but by less than its rivals' expansion. From $dx_i = R_i dy_i$ we have $dx_i(1 + R_i) = R_i(dx_i + dy_i) = R_i dX$, or

$$(5) \quad dx_i = -\lambda_i dX,$$

where

$$\lambda_i \equiv -\frac{R_i}{1 + R_i} = -\frac{p'(X) + x_i p''(X)}{c_{xx}^i(x_i) - p'(X)}.$$

Below, it will prove easier to work with λ_i instead of R_i . Under conditions (3) and (4), $\lambda_i > 0$. The λ_i 's are important below, so we pause here to show how they can be expressed in terms of more familiar elasticities. Write $E \equiv -Xp''(X)/p'(X)$ for the elasticity of the slope of the inverse demand curve, and $\mu_i \equiv x_i c_{xx}^i/c_x^i$ for the elasticity of firm i 's marginal cost, c_x^i , with respect to its output, x_i .¹⁰ Then

$$(6) \quad \lambda_i = \frac{s_i - s_i^2 E}{s_i + \mu_i(\varepsilon - s_i)}.$$

With constant elasticity of demand ε , $E = 1 + 1/\varepsilon$, so λ_i can be expressed solely in terms of s_i , μ_i , and ε . With linear demand and constant marginal costs, $E = 0 = \mu_i$, so $\lambda_i = 1$.

Finally, we prove in the Appendix and record here a fact¹¹ about the response of all other firms to an output change by one firm:

LEMMA: *Consider an exogenous change in firm 1's output, and let the other firms' outputs adjust to re-establish a Cournot equilibrium among themselves. If firms' reaction curves slope downward (condition (3)), and if the stability condition (4) holds, then aggregate output moves in the same direction as firm 1's output, but by less.*

Note that for the Lemma to hold it is *not* necessary that firm 1 behave as a Cournot oligopolist.

II. Price Effects of Horizontal Mergers

We model a merger as a complete combination of the assets and of the control of the merging firms, whom we call the "insiders." After the merger, a new Cournot equilibrium is established between the merged entity M and the nonparticipant firms, whom we call the "outsiders." In this section, we examine the effect of a merger on aggregate output, X . This is, of course, the central question if merger analysis is concerned only with consumer welfare (ignoring consumers' ownership of profits). As we shall see in Section III, it is also an essential component of an analysis of overall economic welfare, W .

Mergers differ enormously in the extent to which productive assets can usefully be recombined, and in the extent to which output decisions can usefully, or anticompetitively, be coordinated. At one extreme, consider a production technology in which all firms have constant and equal marginal costs, and the merged entity has the same costs. In this special case, mergers are purely anticompetitive: there is no other motive. For a slightly rosier view, recall that by equation (2), firms' marginal costs typically differ in Cournot equilibrium, so that a merger may offer an opportunity to *rationalize production*—that is, without changing total output, to shift output to the facility with lower marginal cost. A still sunnier view is that mergers may create synergies. For example, two firms that own complementary patents may combine and produce much more efficiently than either could alone (without a licensing agreement).

For a theory of horizontal mergers to be useful for policy purposes, it should be general enough to allow for all these possibilities, which can be captured in assumptions about the relationship between the merged entity's cost function, $c^M(\cdot)$, and those of the insiders. Our theory is very general in

¹⁰Note that μ_i is the inverse of the elasticity of firm i 's competitive supply curve.

¹¹Dixit (1986) notes this property of Cournot equilibrium.

this regard, as we make no a priori assumptions on $c^M(\cdot)$ beyond those implied by conditions (3) and (4). Our first proposition gives a necessary and sufficient condition on $c_x^M(\cdot)$ for equilibrium output to fall with the merger.

PROPOSITION 1: *A merger of a group of firms in Cournot oligopoly raises price if and only if M 's markup would be less than the sum of the pre-merger markups at its constituent firms, were M produce just as much as its constituent firms together did before the merger.*

PROOF:

See the Appendix. \square

In the typical situation where two firms (1 and 2) are merging, price will fall if and only if $p - c_x^M > (p - c_x^1) + (p - c_x^2)$, where p is the pre-merger price, c_x^1 and c_x^2 are measured at pre-merger output levels, \bar{x}_1 and \bar{x}_2 , and c_x^M is measured at output $\bar{X}_M \equiv \bar{x}_1 + \bar{x}_2$. Equivalently, price will fall if and only if

$$(7) \quad c_x^2 - c_x^M > p - c_x^1.$$

By condition (7), M must enjoy *substantially* lower marginal costs than did its constituent firms, if price is to fall. Furthermore, the required reduction in marginal costs is larger, the larger were the pre-merger markups of the merging firms; and those markups were, by (2), proportional to pre-merger market shares.

Using equation (2), we can express (7) in terms of pre-merger variables that are relatively easy to observe:

$$c_x^M < p \left(1 - \frac{s_1 + s_2}{\varepsilon} \right).$$

This version shows how much less than current prices the merged firm's marginal costs must be, if price is to fall. We now present some illustrative calculations to draw out some of the implications of condition (7) and Proposition 1.

A. Mergers with No Synergies

In many mergers, the insiders (firms $i \in I$) cannot recombine assets to improve their *joint* production capabilities. After the merger, the combined entity M can perhaps better allocate outputs across facilities ("rationalization"), but M 's production possibilities are no different from those of the insiders (jointly) before the merger. In this case we say that the merger "generates no synergies." Formally, with no synergies,

$$(8) \quad c^M(x) \equiv \min \left\{ \sum_{i \in I} c^i(x'_i) \mid \sum_{i \in I} x'_i = x \right\}.$$

This holds, for instance, in the constant average cost model of Steven Salant, Sheldon Switzer, and Robert Reynolds (1983) and in the quadratic cost model of Martin Perry and Robert Porter (1985).

PROPOSITION 2: *If a merger generates no synergies, then it causes price to rise.*

PROOF:

See the Appendix. \square

When will our no-synergies condition, (8), apply? For illustration, write firm i 's cost function as $c^i(x_i) = \theta_i \phi(x_i, k_i)$, where $\phi(\cdot, \cdot)$ is a short-run variable cost function, k_i measures the amount of a possibly fungible capital good employed at firm i , and θ_i inversely measures "knowledge" at firm i .

With this form of the cost function we can distinguish three types of cost savings from a merger. (1) Participants may rationalize output across their facilities (recall that pre-merger marginal costs are typically unequal); this consists of changing the x_i 's but not the k_i 's or the θ_i 's. (2) They may shift capital across their facilities, changing the distribution of the k_i 's (but not the total capital stock available). (3) They may learn from each other, that is, share techniques, patents, or management skills; learning will change the θ_i 's.

Proposition 2 tells us that efficiencies of the first kind, while of course desirable, can-

not suffice to make output rise instead of fall.¹² Efficiencies of the second kind apply only in the “short run”—that is, when capital cannot be brought in from the outside. These efficiencies are further limited by the observation that equation (8) holds even with mobile capital, if the firms produce according to the same long-run cost function (i.e., they have the same level of knowledge) and if this common technology exhibits constant returns to scale. In that case, even if capital can be moved, there is no point in doing so: equivalent results can be achieved by reallocating output across facilities. This proves

PROPOSITION 3: *Suppose that a merger involves no learning. Then, in the long run, the merger will raise price. In the short run, it will raise price (1) if capital is immobile across facilities, or (2) if all merging firms are equally efficient and their long-run production function exhibits constant returns to scale.*

Thus, a merger can raise output and make consumers better off only if it permits the merging firms to exploit economies of scale or if the participants learn from it. In the next two subsections, we illustrate how large these effects must be for a merger to lower price.

B. Mergers with Economies of Scale

Consider the possibility that some form of “capital” may be best recombined after merger. In particular, suppose that it pays—and is possible—to bring together all the new entity’s capital rather than leaving it divided among its plants (formerly, firms) in its pre-merger configuration. This will always be desirable if there are economies of scale; whether capital is mobile is of course a technical question. How much economies of scale are needed for a merger to increase output and reduce price?

Consider for illustration a merger between two *ex ante* identical firms each with a pre-

merger market share of s . There is a common variable cost function $c(x, k)$, and each firm owns an amount k of capital prior to the merger. The merger permits the firms to combine their capital as well as reallocate outputs; and with economies of scale, M will do so. The one combined facility will then produce with the variable cost function $c(\cdot, 2k)$.

We show in the Appendix that the merger will lower price if and only if

$$(9) \quad c_x(2x, 2k) \leq \left[1 - \frac{s}{\varepsilon - s}\right] c_x(x, k).$$

To illustrate (again, see the Appendix for details), let $c(x, k)$ be dual to the production function $f(L, k) = k^a L^a$. If $s = 0.2$ and $\varepsilon = 1$, then (9) requires $a \geq 1/\log_2 3$: twice the inputs must produce at least $2^{2/\log_2 3} \approx 2.4$ times the output.

C. Mergers with Learning

A merger may enhance efficiency at some or all of the merging facilities: one facility may learn from its partner’s patents, management expertise, etc. How much such learning is needed for a merger to increase output and reduce price?

Suppose that firms 1 and 2 merge. Suppose for simplicity that capital cannot be reallocated across firms,¹³ so we can write firm i ’s cost function as $\theta_i \varphi(x_i)$; suppose further that marginal costs are nondecreasing. In the Appendix we show that, for price to fall, the merger must either reduce θ_1 by at least a factor $s_2/(\varepsilon - s_1)$ or reduce θ_2 by at least a factor $s_1/(\varepsilon - s_2)$. In the symmetric case with $s_1 = s_2 = 0.2$ and $\varepsilon = 1$, this means that at least one plant must achieve a reduction in θ of at least 25 percent as a result of merger if price is to fall.

D. Policy Implications

Propositions 1 to 3, and our illustrative calculations in subsections B and C, support

¹²After our paper was revised, we became aware of the work of F. William McElroy (1988), who proves a result similar to our Proposition 2.

¹³Equivalently, consider the long run, in which case capital does not appear in the cost function at all.

the presumption that an oligopolistic merger will reduce aggregate industry output, and point to the nature and degree of synergies or scale economies that are required to overturn this presumption. If the merger changes the behavioral mode in the industry—from Cournot behavior to something more collusive—then the presumption is even stronger that price will rise.

We have identified the factors that determine a merger's effect on price in Cournot equilibrium. In particular, the larger are the market shares of the participating firms, or the smaller is the industry elasticity of demand, the greater must be the learning effects or scale economies in order for price to fall. It is perhaps encouraging that these are exactly the factors that the Merger Guidelines instruct antitrust officials to consider.

Some antitrust scholars and practitioners, including the National Association of Attorneys General, argue that the objective of antitrust policy is to maximize consumers' surplus.¹⁴ Under this view, mergers should be blocked if and only if they are expected to raise price. Propositions 1, 2, 3, and our illustrative calculations point to the relevant factors in making such an assessment.¹⁵ We find that rather impressive synergies—learn-

ing, or economies of scale—are typically necessary for a merger to reduce price.

But few economists would accept that antitrust policy should aim to maximize consumer surplus alone, with no weight given to profits. Although direct stock ownership is concentrated among the relatively rich, many people are indirect shareholders, for instance through pension funds, so it seems inappropriate to ignore profits entirely. In a conventional economic view, the proper goal of antitrust policy is to maximize overall market efficiency or welfare, and in this case further analysis is required.

III. Welfare Effects of Horizontal Mergers

We will now analyze the welfare effects of horizontal mergers. Unfortunately, it is hard to compare pre- and post-merger allocations directly, even in the simplest special cases. But we will show that a merger's effect on the welfare of nonparticipating firms and consumers, although generally unavailable in closed form, can be expressed as the integral of a relatively simple integrand, namely the external effect of what we will call an "infinitesimal merger." We obtain some powerful results by examining that integrand.

A merger generally changes all firms' outputs in equilibrium. But consumers care only about the net effect on aggregate output, ΔX , and (in Cournot equilibrium) rivals care only about the change in equilibrium output by the merging ("insider") firms, ΔX_I , not about what caused that change. Therefore, in examining the external effects of a merger, once we know the equilibrium change ΔX_I , we need no information about what went into that change: we can simply ask how outsider firms respond and what is the effect on their profits and on consumer surplus, and in doing so we can treat ΔX_I as exogenous.

Moreover, the effect of ΔX_I can be decomposed into the integral of the effects of infinitesimal changes dX_I that make up ΔX_I . We call such a small change in insiders' output, dX_I , an "infinitesimal merger" if it has the same sign as the change in X_I consequent on the merger among the insiders. For mathematical convenience, we can think of a

¹⁴For example, in *Proctor & Gamble*, the Supreme Court stated that "...Possible economies cannot be used as a defense to illegality. Congress was aware that some mergers which lessen competition may also result in economies, but it struck the balance in favor of protecting competition." See the discussion in Williamson (1968), for instance.

¹⁵Note that under this view of antitrust policy, rival firms should not have standing to sue to block a proposed merger, since their interests are diametrically opposed to consumers'. Indeed, when rival firms contest a proposed merger, arguing (ostensibly with commendable unselfishness) that the proposed merger would reduce output and should be forbidden, we should perhaps infer that they believe the opposite, and that the merger would probably benefit consumers! Sometimes rivals adopt a theory of "incipient predation," arguing that the merged entity will increase output, but only as a form of predation against them. This too is unconvincing since predation can be directly, and profitably, fought under the treble-damages provisions of the antitrust laws.

merger as the composite of many such infinitesimal mergers.¹⁶

A. External Welfare Effects of Mergers

Consider an infinitesimal merger, dX_I . Because the marginal gross benefits of output are measured by the market price p , we have

$$(10) \quad dW = pdX_I - dc^I + \sum_{i \in O} [p - c_x^i] dx_i,$$

where c^I is the insiders' total cost and O is the set of outsider firms. These outsiders' output responses are given by (5), $dx_i = -\lambda_i dX$, and their markups are given by (2), $p - c_x^i = -x_i p'(X)$. Adding and subtracting $X_I dp$, and making these substitutions, we can rewrite (10) as

$$(11) \quad dW = (pdX_I + X_I dp - dc^I) - X_I p'(X) dX + \sum_{i \in O} p'(X) \lambda_i x_i dX.$$

In (11), the first three terms constitute the change in the insiders' joint profits, $d\pi^I$. Clearly, $d\pi^I$ involves the cost term dc^I , which is very hard for antitrust enforcement officials to observe, and any attempt to put numbers or signs to equation (11) is consequently difficult. But this troublesome term drops out when we examine the *external* welfare effect. From (11),

$$dW - d\pi^I = -X_I p'(X) dX + \sum_{i \in O} p'(X) \lambda_i x_i dX,$$

¹⁶An infinitesimal merger may correspond to an economic event, such as the transfer of a small amount of capital from one firm to another or the purchase by one firm of a small ownership stake in another firm. In Farrell and Shapiro (1990) we explore these changes in the ownership of assets in oligopoly. But here we use infinitesimal mergers strictly as a mathematical construct.

or

$$(12) \quad dW - d\pi^I = \left(\sum_{i \in O} \lambda_i x_i - X_I \right) \times p'(X) dX.$$

Defining

$$(13) \quad \eta \equiv \sum_{i \in O} \lambda_i x_i - X_I,$$

the net externality from an infinitesimal merger that induces an overall output change dX is $\eta p'(X) dX$, which has the sign of η if $dX < 0$. Converting η into market shares, we have:

PROPOSITION 4: *Consider any change in behavior by a subset of firms, "insiders," in an oligopolistic industry. The net external effect of this change on other firms, "outsiders," who are Cournot oligopolists, and on consumers depends only on the equilibrium change in the insiders' output, X_I . A small reduction in X_I has a net positive welfare effect on outsiders and consumers if and only if $\sum_{i \in O} \lambda_i s_i > s_I$.*

Proposition 4 underlines the importance of nonparticipant firms' responses to the change in X_I .¹⁷ If they did not respond, that is, if $\lambda_i = 0$ for $i \in O$, then every output reduction would be bad for rivals and consumers jointly: rivals would benefit, but consumers would lose by more. This often seems to be implicitly assumed in merger policy. But in fact, as Proposition 4 shows for small arbitrary changes and Proposition 5 will show for mergers, many output-reducing changes benefit rivals more than they hurt consumers.

We now use Proposition 4 to establish our central result: a sufficient condition for a privately profitable merger to be socially beneficial. Evidently, this suggests a "safe

¹⁷Salant, Switzer, and Reynolds (1983) emphasize this effect in their very special model, although they do not explicitly discuss the externality. We will return to their case below.

harbor" provision for merger policy: if Cournot behavior is thought to be a good description of industry behavior and if the conditions of Proposition 5 hold, then any proposed merger should be permitted.

If a merger, or other discrete change, causes the insiders' equilibrium output to change from X_I^{initial} to X_I^{final} , we have

$$(14) \quad \Delta W - \Delta \pi^I = \int_{X_I^{\text{initial}}}^{X_I^{\text{final}}} \left(\frac{dW}{dX_I} - \frac{d\pi^I}{dX_I} \right) dX_I,$$

where for each X_I , the integrand is evaluated assuming a Cournot equilibrium among outsiders given X_I . For expositional simplicity, and as suggested by Propositions 1–3, we focus on output-reducing mergers, for which $\Delta X_I < 0$. With X_I falling, we rewrite (14) as

$$(15) \quad \Delta W - \Delta \pi^I = \int_{X_I^{\text{final}}}^{X_I^{\text{initial}}} - \left(\frac{dW}{dX_I} - \frac{d\pi^I}{dX_I} \right) dX_I.$$

Using (12) and (13), we can rewrite (15) as

$$(16) \quad \Delta W - \Delta \pi^I = \int_{X_I^{\text{final}}}^{X_I^{\text{initial}}} \eta(X) [-p'(X)] \frac{dX}{dX_I} dX_I.$$

This shows that the net externality is a weighted integral of η along a path from X_I^{initial} to X_I^{final} . Consequently, if we can sign η throughout such a path, then we can sign the total external effect, $\Delta W - \Delta \pi^I$.

In Proposition 5, we give conditions sufficient for η to increase as X_I falls. When those conditions hold, it follows that if $\eta \geq 0$ before a merger, and if the merger will involve a reduction in insiders' output, then the total externality $\Delta W - \Delta \pi^I$ is surely positive.¹⁸

¹⁸Moreover, under the conditions of Proposition 5, if $\eta < 0$ after the merger, that is, at $X_I = X_I^{\text{final}}$, then we know that consumers and rival firms jointly are harmed by the merger.

PROPOSITION 5: *Consider a proposed merger among firms $i \in I$, and suppose that their initial (joint) market share s_I does not exceed $\sum_{i \in O} \lambda_i s_i$. Suppose further that p'' , p''' , and c_{xx}^i are all nonnegative and c_{xxx}^i is nonpositive in the relevant ranges and for all nonparticipating firms i . Then, if the merger is profitable and would raise the market price, it would also raise welfare.*

PROOF:

See the Appendix. \square

We pause now to discuss the conditions of Proposition 5. The requirement that $p'' \geq 0$ is met, for example, by all constant elasticity demand curves and of course by linear demand. The condition that marginal costs not decrease, $c_{xx} \geq 0$, is also quite widely met. For moderate sized mergers (those involving modest changes in X_I), these second-order terms will dominate the calculations, so the conclusion of Proposition 5 is likely to hold even if p''' may be negative or c_{xxx}^i may be positive. For a large merger, we need conditions on third derivatives in order to sign its external effect. This is not surprising. For example, if the outsiders face capacity constraints, so $c_{xxx} > 0$, their ability to increase output will be limited and the λ_i 's must fall as the merger proceeds and their output rises. Likewise, if $p''' < 0$ then outsiders' responses to decreases in X_I may diminish as X_I does. Because these third-order effects inevitably appear, and because a merger is inherently a lumpy, non-marginal change, we believe that Proposition 5 may be the strongest "clean" result available for general cost and demand functions.

B. Implications for Antitrust Policy

As we remarked in the Introduction, most economists believe that interventions in the economy should be based on analysis of externalities. We might therefore hope that our explicit analysis of the externalities from a merger in Cournot oligopoly will help us discuss proper policy toward mergers.

Privately unprofitable mergers will not be proposed, so proposed mergers should be permitted unless their external effects are

“sufficiently” bad to outweigh their private profitability. In particular, if a proposed merger would have a *beneficial* external effect, then it should be allowed.¹⁹ We might be able to identify such cases using equation (15) or (16). In particular, if $\eta > 0$ at pre-merger equilibrium then there is some reason to expect the net externality $\Delta W - \Delta\pi^I$ to be positive, unless we have reason to expect η to change sign over the course of the merger (regarded as a gradual sequence of fictitious infinitesimal mergers). More rigorously, Proposition 5 gives sufficient—but far from necessary—conditions for this inference to be valid. Thus, when the conditions of Proposition 5 hold, any privately profitable merger should be permitted.

But a policy that allows only mergers with positive net external effects will be too restrictive to maximize overall economic surplus. Many profitable mergers that involve a negative “wedge” ($\Delta W - \Delta\pi^I < 0$) will nevertheless increase total surplus ($\Delta W > 0$). Although we have no formal results for this case, we hope that our externality technique may nevertheless be useful in informing merger policy, as the following diagrammatic framework may illustrate.²⁰ In Figure 1, a merger is represented by a point whose horizontal component is the net wedge, $\Delta W - \Delta\pi^I$, that it would generate, and whose vertical component is the private profitability, $\Delta\pi^I$. Ideally, we would like to find a policy that would permit exactly those mergers “northeast” of the negatively sloped 45° line through the origin—that is, those in regions A, B, and C; those are the “socially beneficial” mergers (those with $\Delta W > 0$).

In Figure 1, we can see why a policy of allowing only mergers with positive net wedges is too restrictive. For if such a policy were implemented, then only mergers in the northeast quadrant (region B) of Figure 1

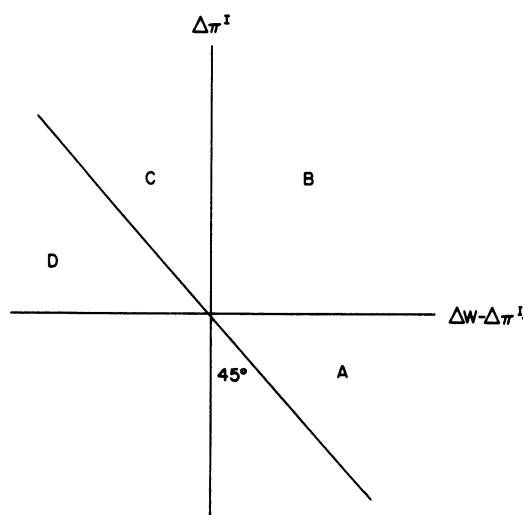


FIGURE 1

will take place (be proposed and approved). It seems worth considering, therefore, whether there is a policy that might allow more of the desirable mergers without also permitting others that are profitable but socially undesirable. Evidently, no policy that does not involve compulsion or subsidies to merger—both of which would be enormous changes from our antitrust policy—can hope to get mergers in region A implemented. We focus, therefore, on whether a policy might be found that would permit us to distinguish proposed mergers in regions B and C (profitable and socially beneficial) from those in region D (profitable but socially harmful).

Our analysis suggests the following two-part procedure for evaluating a proposed merger. First, using (16), estimate the net wedge, $\Delta W - \Delta\pi^I$. Second, if the net wedge appears to be negative, estimate the profit effect, $\Delta\pi^I$. This might be done by observing how the aggregate stock market valuation of the participants changes with the news of the proposed merger. These estimates of the net wedge and of the profit effect can then be plotted on Figure 1, and the merger should be approved if and only if the point so plotted lies in the northeast half-plane in Figure 1.

¹⁹Note that, since nonparticipant firms' and consumers' interests concerning insiders' output are strictly opposed, a merger will never generate a Pareto improvement; we are aggregating nonparticipants' and consumers' surplus into a single measure.

²⁰We are especially indebted to Steven Salop for suggesting this treatment.

We stress, however, that such a procedure would involve a number of problems, some severe. Estimating the net wedge involves estimating structural parameters such as the λ_i 's, and how they will change in equilibrium with changes in X_I . The estimate of the private profitability $\Delta\pi'$, which is needed only if the net wedge appears to be negative, is also problematic. Using the stock-market response to the announcement of the proposal assumes that no significant insider trading predates the announcement and that the stock market can accurately value the proposed change. More fundamentally, this rule would seem to create a multiple-equilibrium problem for proposed mergers that in fact lie in region C. If the market believes that the merger will be approved, then the increase in stock-market value will so impress the regulators that they will indeed approve it. If, however, the market expects regulatory opposition, then the value will rise by little, and the regulators will indeed oppose the merger!

C. Linear Demand and Constant Costs

In this subsection and the next, we illustrate Proposition 5 by recalling the cases analyzed by Salant et al. (1983) and by Perry and Porter (1985) and R. Preston McAfee and Michael Williams (1988).

Salant et al. assume symmetric Cournot competition among n firms, which have identical constant marginal cost c and face a linear demand curve. They consider the private and social benefits of a merger of $m+1$ of those firms—that is, a shift from Cournot to collusive behavior within the group.

For constant marginal costs and linear demand, $\lambda_i = 1$ for all i , and $\eta = X_O - X_I$. Hence, a merger among $m+1$ out of the n firms will have $\eta \geq 0$ all along the path from X_I^{initial} to X_I^{final} if the *pre-merger* outputs satisfy $X_O - X_I \geq 0$, that is, if $n - m - 1 \geq m + 1$, or $m + 1 \leq n/2$. In this model, then, any profitable merger involving at most half the industry is socially desirable.

This may seem puzzling, since, with constant unit costs, welfare declines with any merger. Salant et al. resolve this puzzle by pointing out that outsiders' responses may

make the merger unprofitable, especially when m/n is small.²¹ They also emphasize, however, that internalized savings (for example, fixed costs, which are perfectly consistent with constant marginal costs) may reverse that unprofitability—and hence also reverse the welfare undesirability.

Similarly, the net external effect of a merger among $m+1$ firms is surely negative if the *post-merger* outputs satisfy $X_I \geq X_O$, that is, when $1 \geq n - m - 1$, or when the number of merging firms is at least $n - 1$ (merger to duopoly). This does not necessarily mean that no such merger should ever be approved, but there is certainly a case to answer.

Observe that in this model it does not matter for the externality how X_O is divided amongst the outsiders, for λ_i is independent of firm i 's output or market share. In our next example we shall see that this is not generally the case.

D. Linear Demand and Quadratic Costs

For our second illustration, consider the quadratic cost and linear demand functions used by Perry and Porter (1985), and by McAfee and Williams (1988). Demand is given by $p(X) = A - X$, and costs are given by $c(x, k) = \frac{1}{2}x^2/k$ at all firms.²² This variable cost function, which is dual to the Cobb-Douglas production function $x = \sqrt{Lk}$, exhibits constant returns to scale (it is homogeneous of degree one in capital and output). Hence, by Proposition 3, every merger reduces output. Proposition 5 applies, and simple calculations show that $\lambda_i = x_i/p = s_i/\epsilon$. Consequently, a merger surely has a positive externality if

$$(17) \quad s_I < \frac{1}{\epsilon} \sum_{i \in O} s_i^2,$$

²¹Gérard Gaudet and Salant (1988) study how large a group of firms must be for coordinated output reductions by the group to be profitable, despite the increased output by rivals that such reductions will induce.

²²The analysis would be identical in a model with $p = A - bX$ and $c(x, k) = cx + ex^2/2k$, but the parameters b , c , and e would clutter our formulas.

where these are pre-merger market shares. Thus a merger is more likely to help rivals and consumers jointly if s_i is small—this conforms with standard merger analysis—and if the rest of the industry is *more* concentrated!

The reason for this surprising conclusion is that, in this model, λ_i is larger for larger firms i ($\lambda_i = x_i/p$). In economic terms, if there are large outsiders, then any reduction in X_i will induce an especially large output response by them—which is just where increases in output are socially most valuable. Concentration among outsiders therefore makes their output response more socially beneficial. If λ_i were smaller at larger firms, as can easily happen,²³ then the outsiders' output response might either be weakened or less socially beneficial in a more concentrated market. The Salant-Switzer-Reynolds model is a borderline case: $\lambda_i = 1$ for all i , so that the distribution of market shares among nonparticipants does not matter there.

Another interesting feature of condition (17) is that a merger is *more* likely to have a negative external effect, and hence to require careful antitrust scrutiny, when demand is *more* elastic. The reason is that with elastic demand outsiders' markups are small, so little welfare benefit is to be had from their increased output (while consumers still suffer from any price increase caused by the merger).

In the linear-quadrant model, unlike the constant marginal cost model, mergers may increase welfare even absent fixed-cost savings. In a previous version of this paper, we showed, by viewing infinitesimal mergers as transfers of small amounts of capital, that welfare rises ($\Delta W > 0$) if the merging firms are small and the rest of the industry is highly concentrated.²⁴ This is so even though there are no synergies, output falls, and con-

centration increases! Small merging firms have small price-cost margins, so a reduction in their output has little social cost, whereas large nonparticipant firms increase their output and this is socially valuable. In terms of condition (1), the increase in concentration, dH/H , outweighs the reduction in output, dX/X , yielding an *increase* in welfare. Thus, as hinted in the Introduction, structural changes such as mergers can lead to changes in H and in W that have the same sign, even without synergies. That this can happen more generally, with or without synergies, is shown by Proposition 5.

E. Constant-Elasticity Demand and Constant Costs

For constant marginal costs and constant-elasticity demand, $\lambda_i = 1 - s_i(1 + 1/\epsilon)$. The net externality from an infinitesimal merger that raises price is positive if and only if

$$2(s_1 + s_2) < 1 - \left(1 + \frac{1}{\epsilon}\right) \sum_{i \in 0} s_i^2.$$

In this example, although the $p''' \geq 0$ condition of Proposition 5 does not hold, we can nonetheless show directly that any merger for which the above inequality holds at pre-merger shares generates a positive net externality. To establish this result, it is sufficient to show that the above inequality continues to hold as x falls. Differentiating totally, this is equivalent to $\sum_{i \in 0} \lambda_i ds_i \geq 0$. Since all the λ_i are positive, it is enough to show that s_i increases as X falls. But firm i 's first-order condition requires that $s_i = \epsilon(1 - c_x^i/p)$; since ϵ and c_x^i are constants, s_i must rise with p .

We therefore have derived, in an important special case, a sufficient condition in terms of the pre-merger market shares and the elasticity of demand, for every profitable merger to improve welfare. This result illustrates that the sufficient conditions of Proposition 5 are not necessary.

F. Extending Beyond Cournot

We have used the Cournot assumption in this section in two ways: (a) to measure

²³For example, if firm i has constant unit cost c_i and if $p'' > 0$, then λ_i will be inversely related to x_i .

²⁴McAfee and Williams (1988) also construct examples of welfare-improving mergers in this model, using computer simulation.

outsiders' markups, $p - c_x^i$, by $-x_i p'(X)$, and (b) to compute outsiders' responses as measured by the λ_i 's.²⁵ But within the class of homogeneous-goods models, our qualitative results extend well beyond Cournot behavior. Whatever the behavior of the non-participating firms, there are two external effects of a change in X_I . First, the price changes, causing a loss of $X_I dp$ to consumers.²⁶ Second, outsider firms are induced to change their outputs by some amounts, say $\psi_i dX_I$, and the social value of this is $\sum_{i \in O} [p - c_x^i] \psi_i dX_I$. Consequently, the externality is positive if and only if

$$(18) \quad \sum_{i \in O} [p - c_x^i] \psi_i dX_I - X_I dp > 0.$$

Equation (11) and Proposition 4 are a special case of equation (18). More generally, the ψ_i must be recalculated for each oligopoly theory, but the basic equation (18) persists.

To illustrate, we sketch here how our results would differ with nonzero conjectural variations. Suppose that firm i believes that its rivals will respond to its output changes with $dy_i/dx_i = v_i$.²⁷ Then firm i 's equilibrium markup is $p - c_x^i = -x_i p'(X)(1 + v_i)$, and its equilibrium responsiveness is

$$\hat{\lambda}_i = - \frac{p' + x_i p''(1 + v_i)}{c_{xx} - p'(1 + v_i)}.$$

The condition in Proposition 4 for a positive externality becomes

$$\sum_{i \in O} \hat{\lambda}_i s_i (1 + v_i) > s_I.$$

²⁵We also have used price to measure marginal gross benefits. This procedure is valid provided only that the good is homogeneous and buyers have no market power.

²⁶The price effect on outsider firms' output, $X - X_I$, is merely a transfer from consumers to outsiders.

²⁷The parameter v_i is called firm i 's "conjectural variation." Although the conjectural variation model is logically flawed, it is a useful way of parameterizing the degree of competition among oligopolists. Such behavior might, for example, arise as the equilibrium of an (unmodeled) dynamic oligopolistic game.

We now ask how $\hat{\lambda}_i(1 + v_i)$ varies with v_i . This will tell us how the external effect of a small reduction in output by (say) firms 1 and 2 in an industry with a given vector of market shares (s_1, s_2, \dots, s_n) , varies with the industry's competitiveness. In other words, given the observed market shares, how will the external effect of the merger depend upon the behavior that led to the observed pre-merger equilibrium? This is the relevant question for policy: it determines whether high or low perceived v_i 's would make a proposed merger more socially acceptable, given the observed market shares.

Straightforward calculations show that $\hat{\lambda}_i(1 + v_i)$ is increasing in v_i if and only if

$$(1 + v_i)^2 x p' p'' - (p' + 2(1 + v_i) x p'') c_{xx} > 0.$$

This condition certainly holds if demand is linear and $c_{xx} \geq 0$: in that case, more collusive behavior (an increase in v_i) raises $\hat{\lambda}_i(1 + v_i)$, so an output reduction by firms 1 and 2 becomes more attractive to the rest of society, given all the firms' market shares. Perhaps surprisingly, the more tacit collusion that exists in pre-merger equilibrium and would exist after the merger (we assume the two are the same), the more likely it is that a privately profitable merger is socially desirable.

IV. Other Applications of the Externality Technique

The externality formula of Proposition 4 has applications that extend well beyond horizontal merger policy. We sketch two such applications here: (1) investment or commitment by an oligopolist, and (2) an import quota in an industry with a domestic oligopoly.

A. Investment or Commitment in Oligopoly

What is the welfare effect of a small, observable, unilateral change by an oligopolist that shifts outwards its reaction function? In particular, what external effects on consumers and other firms result from an oligopolist's decision to invest in new capi-

tal, or from an increase in its ability to commit to a high output level, for example, through long term contracts? As above, we can analyze these external effects in terms of an infinitesimal (but in this case a positive) change in X_I , where the subset I of “insiders” consists of the investing firm alone.²⁸ Applying Proposition 4 gives

PROPOSITION 6: *The external effect of a small outward shift in firm 1's reaction function has the sign of*

$$(19) \quad s_1 - \sum_{i=2}^n \lambda_i s_i.$$

Expression (19) is most likely to be negative if firm 1's rivals have large market shares and therefore high price-cost margins, and if their equilibrium outputs are sensitive to expansion by firm 1. In such a market, firm 1 “steals” a great deal of valuable business from its rivals when it invests. This negative externality imposed on rivals must be compared against the transfer to consumers of the price change on firm 1's output.²⁹ Clearly, if firm 1 is sufficiently small, (19) is negative. Thus, although investment by a small firm lowers price and benefits consumers, it harms rival firms by more.

The impact on rivals and on consumers of allowing one firm in a Cournot oligopoly some power of pre-commitment is given also by Proposition 6. If we move from a régime of less commitment power to one of more, we have $dX_I > 0$. Consequently, the external welfare effects of allowing more output commitment by one firm are beneficial if and only if that firm has a sufficiently large market share. And it is certainly profitable for the firm with the commitment power. Hence, if one firm has a market share in Cournot oligopoly that exceeds the λ -weighted sum

of all others', then it is socially desirable to give that firm some Stackelberg power. And if the conditions of Proposition 5 hold, it follows that it is desirable to go all the way to give the firm complete Stackelberg power.

As an illustration, take the simplest possible model. There are two firms, each with zero marginal costs, and $p(X) = 1 - X$. Recall that $\lambda_i = 1$ with this specification, so that, in the (symmetric) Cournot equilibrium, $s_1 = \frac{1}{2} = \lambda_2 s_2$. In Cournot equilibrium, each firm produces output $1/3$, and makes profits of $1/9$, while consumer surplus is $2/9$. But if firm 1 is Stackelberg leader, then $x_1 = 1/2$ and $x_2 = 1/4$, giving firm 2 profits of $1/16$ and consumers surplus of $9/32$. Since $(9/32 + 1/16) > (2/9 + 1/9)$, firm 2 and consumers jointly benefit from the shift to Stackelberg behavior, as Proposition 5 predicted. Of course, firm 1 also gained.

B. Import Quotas in Oligopolistic Industries

We now show how Propositions 4 and 5 illuminate the domestic welfare consequences of quotas when the domestic industry consists of a Cournot oligopoly (plus perhaps a competitive fringe). Thinking of foreign suppliers as the “insiders,” we can view a tightening of a quota as a reduction in X_I . Moreover, the welfare of outsiders (domestic firms) and consumers is simply domestic welfare. Proposition 4 therefore gives us:

PROPOSITION 7: *Consider the effect of an import quota in an industry where domestic firms are Cournot oligopolists, perhaps including a competitive fringe. Slightly tightening the quota raises domestic welfare if and only if the share of imports is less than the λ -weighted sum of domestic producers' shares.*

Proposition 5—when its assumptions hold—implies that a small enough import sector should be excluded altogether!³⁰ Although

²⁸Additional applications of Proposition 4 to investment are presented in our (1990) paper.

²⁹The price effect on other firms' outputs is merely a transfer from rivals to consumers. Marius Schwartz (1988) discusses some similar examples in a special model.

³⁰Janusz Ordover and Robert Willig (1986) obtain a similar result in a model with linear demand and constant unit costs.

this would hurt consumers, it would help domestic producers by more. For instance, in the constant-cost, linear-demand case, Proposition 5 states that if imports have no more than half the market in Cournot equilibrium, domestic welfare would be greater if imports were completely excluded! To illustrate, take the same model as in the previous subsection: demand is $p = 1 - X$ and there are two firms (one now foreign), each with zero marginal costs. Domestic welfare in Cournot equilibrium is $2/9 + 1/9 = 1/3$. Excluding the foreign firm creates a domestic monopoly, which will produce output of $1/2$, yielding (domestic) welfare of $(1/8 + 1/4) = 3/8 > 1/3$.

V. Conclusions

We have studied the output and welfare effects of mergers in Cournot oligopoly, using cost and demand functions restricted only by standard stability assumptions.

We found that mergers do indeed, typically, raise price. In particular, any merger that generates no synergies, in the sense of equation (8), raises price. And for a merger to lower price requires considerable economies of scale or learning. This result is only strengthened if the merger causes behavior in the industry to shift from Cournot to something less competitive. Furthermore, and consistent with the approach taken by the merger guidelines, we found that the economies of scale or learning effects necessary for a merger to lower price are greater, the larger are the merging firms' market shares and the less elastic is industry demand.

In our policy analysis, we focused on the external effects of a merger rather than trying directly to sign its overall welfare effect. This approach, as well as being consistent with market-oriented policy analysis in general, has a great practical advantage: the information required is much more readily available. Looking at the external effect would also allow antitrust authorities to make use of the fact that only privately profitable mergers are proposed, and it would permit them to give cost savings a larger role in merger policy (as Williamson's (1968) cal-

culations suggest they deserve) without demanding either information or credulity about alleged synergies.

The information that *is* needed in our analytical scheme is in two separable parts. First, to test whether a merger will indeed reduce output requires information only on participants' (pre-merger) marginal-cost functions and on that of the merged entity. Although this information may be hard to obtain, our observation does indicate exactly what the relevant information is. Information about market demand and other firms' costs is not relevant for determining whether price will rise or fall, so long as Cournot behavior applies both before and after the merger. Second, to sign the external effect of an output-reducing merger—whether it benefits or harms rival firms and consumers jointly—requires information only on market shares and the output responsiveness parameters (the λ_i) of nonparticipants.

Inevitably, for general results one requires information not only at the pre-merger equilibrium, but along a path from pre-merger to post-merger equilibrium. But Proposition 5 gives conditions—general enough to cover the special cases previously studied and many more—under which no such global inquiry is required: one need merely compare the participants' pre-merger joint market share with the λ -weighted sum of their rivals'. We can thus give surprisingly general (sufficient) conditions under which all privately profitable mergers raise welfare, and presumably all proposed mergers should be approved.

Our techniques have a wide range of application in oligopoly theory, as Propositions 6 and 7 show. Quite generally, our results imply that many output-reducing changes should not be grudgingly tolerated, but should be positively welcomed by the rest of society. The counterintuitive, almost paradoxical, fact is that, in Cournot and similar theories of oligopoly with homogeneous goods, the presence of small firms with little market power is not desirable. Their output, produced at a marginal cost that almost consumes its gross social benefit, also displaces or discourages output at (larger) firms with lower marginal costs (this displacement effect is, of course, absent in perfect compe-

tion). Consequently, it often enhances economic welfare—defined in the usual way—to close down small or inefficient firms, or, failing that, to encourage them to merge so that they produce less output. This observation may call for some rethinking of our views on policy toward competition, including horizontal merger policy.

We believe that our paper provides useful tools and insights for horizontal merger policy. But our analysis is limited in two major ways, both of which are inherent in the standard Cournot model (and its conjectural-variations extensions). First, we analyze a homogeneous-goods industry; our results may not apply well to markets in which product differentiation is substantial. Applying techniques such as those presented here to differentiated-product industries is an important topic for future research. Second, by assuming that the firms behave as Cournot competitors, both before and after the merger, we ignore any effect of a merger on the probability or nature of explicit collusion. We believe that the Cournot model captures the notion of *tacit* collusion fairly well, but we are well aware that many antitrust practitioners are more concerned with the possibility of *explicit* collusion than with the nature of *tacit* collusion. Unfortunately, there is no fully satisfactory model of the probability of successful explicit collusion in oligopoly. Such a model could be used to estimate a merger's effect on the probability of collusion.

APPENDIX

The Herfindahl Index in Cournot Oligopoly

Consider a Cournot oligopoly in which firm i ($i = 1, \dots, n$) produces output x_i at cost $c'_i(x_i)$. Total output is $X \equiv \sum_{i=1}^n x_i$, and the market price, $p \equiv p(X)$, measures the marginal gross benefit of output. For an arbitrary change $\{dx_i\}$ in firms' outputs,

$$dW = \sum_{i=1}^n (p - c'_i) dx_i.$$

In Cournot equilibrium, firm i 's first-order condition gives $p - c'_i = -p'(X)x_i$. Hence,

$$dW = -p'(X) \sum_{i=1}^n x_i dx_i.$$

Now this change in welfare is related to the Herfindahl index, H , since

$$\begin{aligned} \sum_{i=1}^n x_i dx_i &= \frac{1}{2} d \left[\sum_{i=1}^n x_i^2 \right] \\ &= \frac{1}{2} d [X^2 H] = XH dX + \frac{1}{2} X^2 dH. \end{aligned}$$

Thus we have

$$dW = -X^2 H p'(X) \left\{ \frac{dX}{X} + \frac{1}{2} \frac{dH}{H} \right\}.$$

With downward-sloping demand, $p'(X) < 0$, so dW has the sign of

$$\frac{dX}{X} + \frac{1}{2} \frac{dH}{H}.$$

See Keith Cowling and Michael Waterson (1976) or Robert Dansby and Robert Willig (1979) for related calculations showing the welfare significance of the Herfindahl Index in Cournot oligopoly.

Proof of Lemma

Write Δx_1 and ΔX for the changes in firm 1's output and in aggregate output. It is enough to prove the Lemma for an infinitesimal change dx_1 , since $0 < dX/dx_1 < 1$ implies $0 < \Delta X/\Delta x_1 < 1$. For any firm $i \neq 1$, $dx_i = -\lambda_i dX$. Adding up for $i \neq 1$ we have $dy_1 = -\sum_{i \neq 1} \lambda_i dX$. Adding dx_1 to this equation gives $dX = -\sum_{i \neq 1} \lambda_i dX + dx_1$, or $dX(1 + \sum_{i \neq 1} \lambda_i) = dx_1$. With conditions (3) and (4), each λ_i is positive, so dX has the same sign as dx_1 but is smaller in magnitude. \square

Proof of Proposition 1

By our Lemma, we need only sign the effect on the insiders' total output. And in order to do that, we need only find whether the new firm M would increase or decrease its output if the nonparticipant firms held their outputs constant at pre-merger levels.

Denote pre-merger outputs by \bar{x}_i and \bar{X} , and call the insiders' aggregate pre-merger output \bar{X}_M . At pre-merger output levels, M 's marginal revenue is $p(\bar{X}) + \bar{X}_M p'(\bar{X})$. So M will reduce its output if and only if $c'_M(\bar{X}_M) > p(\bar{X}) + \bar{X}_M p'(\bar{X})$, or $p(\bar{X}) - c'_M(\bar{X}_M) \leq -\bar{X}_M p'(\bar{X})$. Thus it is enough to show that $-\bar{X}_M p'(\bar{X})$ is equal to the sum of the merging firms' pre-merger markups. But, for each firm $i \in I$, $c'_i(\bar{x}_i) = p(\bar{X}) + \bar{x}_i p'(\bar{X})$, or $p(\bar{X}) - c'_i(\bar{x}_i) = -\bar{x}_i p'(\bar{X})$. Adding this up over $i \in I$, we find that

$$\sum_{i \in I} [p(\bar{X}) - c'_i(\bar{x}_i)] = -\bar{X}_M p'(\bar{X}),$$

as required. \square

Proof of Proposition 2

We give the proof for a two-firm merger, but it extends to multi-firm mergers. The proof proceeds in four steps.

Step 1. First, by the Lemma, the change in total output has the same sign as the change in the insiders' output. Therefore, it is enough for us to show that the insiders will want to reduce their aggregate output, if the outsiders' output is held constant at \bar{X}_O . For notational simplicity, since the outsiders' output is fixed, we shall write $p(x_1 + x_2)$ instead of $p(x_1 + x_2 + \bar{X}_O)$.

Step 2. Next, we show by a revealed-preference argument that both firms' outputs cannot rise when the two firms maximize joint profits rather than their own profits (as in Cournot equilibrium).

Denote by \bar{x}_i firm i 's output in pre-merger Cournot equilibrium, and by x_i its output when the two firms maximize joint profits. We will show that it is impossible to have $x_i > \bar{x}_i$ for $i=1,2$. By revealed preference,

$$(20) \quad (x_1 + x_2)p(x_1 + x_2) - c^1(x_1) - c^2(x_2) \\ \geq (\bar{x}_1 + \bar{x}_2)p(\bar{x}_1 + \bar{x}_2) - c^1(\bar{x}_1) - c^2(\bar{x}_2),$$

and

$$(21) \quad \bar{x}_1 p(\bar{x}_1 + \bar{x}_2) - c^1(\bar{x}_1) \\ \geq x_1 p(x_1 + \bar{x}_2) - c^1(x_1),$$

$$(22) \quad \bar{x}_2 p(\bar{x}_1 + \bar{x}_2) - c^2(\bar{x}_2) \\ \geq x_2 p(\bar{x}_1 + x_2) - c^2(x_2).$$

Adding equations (21) and (22), and comparing to (20), yields

$$(23) \quad (x_1 + x_2)p(x_1 + x_2) \\ \geq x_1 p(x_1 + \bar{x}_2) + x_2 p(\bar{x}_1 + x_2),$$

which is inconsistent with both $x_1 \geq \bar{x}_1$ and $x_2 \geq \bar{x}_2$ unless both hold with equality. Without loss of generality, suppose that the output at facility 2 does not rise after the merger: $x_2 \leq \bar{x}_2$.

Step 3. Let x'_1 be the output that maximizes π_1 given outputs x_2 and \bar{X}_O . Since $x_2 \leq \bar{x}_2$, the Lemma tells us that $x'_1 \geq \bar{x}_1$ and $x'_1 + x_2 \leq \bar{x}_1 + \bar{x}_2$.

Step 4. Finally, if $x_2 > 0$, when firm 1 is maximizing joint profits, it will produce strictly less (given firm 2's output) than if it were maximizing only its own, since increases in x_1 reduce the profits earned at facility 2. That is, $x_1 < x'_1$. Using Step 3, we thus have $x_1 + x_2 < \bar{x}_1 + \bar{x}_2$ as was required. If $x_2 = 0$ then $x_1 = x'_1$ and the result follows directly from the Lemma. \square

Economies of Scale

For equilibrium output to increase, Proposition 1 requires that marginal cost at twice the pre-merger capital and output levels must be less than pre-merger marginal cost, by an amount equal to the pre-merger markup. That is,

$$c_x(2x, 2k) \leq c_x(x, k) - [p - c_x(x, k)],$$

where p is the pre-merger price and x is each firm's pre-merger output. Now the pre-merger markup $p - c_x(x, k)$ is equal, by the pre-merger first-order condition, (2), to $s/(\epsilon - s)$ times pre-merger marginal cost $c_x(x, k)$. So output falls unless

$$c_x(2x, 2k) \leq \left[1 - \frac{s}{\epsilon - s}\right] c_x(x, k).$$

If $c(x, k)$ is dual to the generalized Cobb-Douglas production function $f(L, k) = k^a L^b$, then $c_x(x, k) \equiv w/bx^{1/b-1}k^{-a/b}$, where w is the price of the variable input L , so that

$$\frac{c_x(2x, 2k)}{c_x(x, k)} = 2^{1/b-1-a/b}.$$

Hence, (9) states that

$$a + b \geq 1 + b \log_2(\epsilon - s) - b \log_2(\epsilon - 2s).$$

If $s = 0.2$ and $\epsilon = 1$, and if $a = b$, this requires $a \geq 1/\log_2 3$.

Learning. Suppose that firms 1 and 2 merge, capital is immobile and marginal costs are nondecreasing. We prove here that for price to fall, the merger must either reduce θ_1 by at least a factor $s_2/(\epsilon - s_1)$ or reduce θ_2 by at least a factor $s_1/(\epsilon - s_2)$.

At least one of the insiders, call it firm 1, must at least maintain its pre-merger output level if price is not to rise. And, by the envelope theorem, M 's marginal cost will equal firm 1's marginal cost at its post-merger level of output. But, by equation (7), for output to rise we must have $c_v^1 - c_v^M > p - c_x^2$, so firm 1's marginal costs must fall by at least firm 2's markup. However, with $c_{xx}^1 \geq 0$ and firm 1 expanding, firm 1's marginal costs could fall only due to synergies. If price is to fall, therefore, synergies must reduce firm 1's marginal costs by at least the extent of firm 2's markup.

Just how strong a synergistic cost reduction is needed for this condition to be met and thus reverse the presumption that aggregate output will fall with the merger? In the pre-merger equilibrium, we have $(p - c_x^1(x_1))/p = -x_1 p'/p = s_1/\epsilon$. Hence, $(p - c_x^2)/p = s_2/\epsilon$ and $p/c_x^1 = \epsilon/(\epsilon - s_1)$.

The reduction in firm 1's marginal costs required for price to fall is $c_v^1 - c_v^M > p - c_x^2$. In percentage terms, this requires

$$\frac{c_v^1 - c_v^M}{c_v^1} > \frac{p - c_x^2}{p} \frac{p}{c_x^1}.$$

Substituting for the two factors on the right-hand side, we must have

$$\frac{c_v^1 - c_v^M}{c_v^1} > \frac{s_2}{\epsilon} \frac{\epsilon}{\epsilon - s_1} = \frac{s_2}{\epsilon - s_1} = \frac{p}{c_x^1}.$$

Learning must reduce firm 1's marginal costs by at least $s_2/(\epsilon - s_1)$ for price to fall. \square

Proof of Proposition 5

We will show that the conditions given imply that $d[\lambda_i, x_i]/dX \leq 0$ for $i \in O$. Since an output-reducing merger involves a reduction in X_i , an infinitesimal merger's effect on $\eta \equiv \sum_{i \in O} \lambda_i x_i - X_i$ is therefore unambiguously positive. So, after doing an infinitesimal merger that benefits nonparticipants, we will find that a further infinitesimal merger benefits them by even more, and so on until the merger is complete. Evidently, this implies (it is much stronger than) Proposition 5.

Recall that we are considering an exogenous change dX_i , which induces changes dx_i by outside firms i , and hence induces a change dX in aggregate output and changes $d\lambda_i$ in the λ 's. Since $d[\lambda_i, x_i] = \lambda_i dx_i + x_i d\lambda_i$, and since $dx_i = -\lambda_i dX$, we have

$$d[\lambda_i, x_i] = -\lambda_i^2 dX + x_i d\lambda_i.$$

Now think of $\lambda_i \equiv -p'(X) + x_i p''(X)/c'_x(x_i) - p'(X)$ as a function of two variables, X and x_i . Thus we have

$$\frac{d[\lambda_i, x_i]}{dX} = -\lambda_i^2 + x_i \left(\frac{\partial \lambda_i}{\partial X} - \lambda_i \frac{\partial \lambda_i}{\partial x_i} \right).$$

Substituting in for λ_i and its partial derivatives, we get

$$\begin{aligned} (24) \quad & (c_{xx} - p')^2 \frac{d[\lambda_i, x_i]}{dX} \\ &= -(p' + x_i p'')^2 - x_i^2 p''' (c_{xx} - p') \\ &\quad + x_i c_{xxx} \frac{(p' + x_i p'')^2}{c_{xx} - p'} \\ &\quad - x_i p'' [c_{xx} + p' + 2x_i p'']. \end{aligned}$$

For quadratic $p(\cdot)$ and $c(\cdot)$ functions, we can ignore the terms in c_{xxx} and in p''' , and then $d[\lambda_i, x_i]/dX$ has the sign of

$$\begin{aligned} & -x_i p'' [c_{xx} + p' + 2x_i p''] - (p' + x_i p'')^2 \\ & \equiv -x_i p'' c_{xx} - \left(p' + \frac{3}{2} x_i p'' \right)^2 - \frac{3}{4} x_i^2 (p'')^2, \end{aligned}$$

which is negative provided that p'' and c_{xx} are nonnegative (actually, provided that their product is nonnegative). The proposition follows by inspection of equation (24), since p''' and c_{xxx} enter with unambiguous signs. \square

REFERENCES

- Cowling, Keith and Waterson, Michael, "Price-Cost Margins and Market Structure," *Economica*, August 1976, 43, 267-74.
- Dansby, Robert E. and Willig, Robert D., "Industry Performance Gradient Indexes," *American Economic Review*, March 1979, 69, 249-60.
- Demsetz, Harold, "Industry Structure, Market Rivalry, and Public Policy," *Journal of Law and Economics*, April 1973, 16, 1-10.
- , "Two Systems of Belief About Monopoly," in Harvey Goldschmid et al., eds., *Industrial Concentration: The New Learning*, Boston, MA: Little Brown, 1974, 164-84.
- Deneckere, Raymond and Davidson, Carl, "Incentives to Form Coalitions with Bertrand Competition," *Rand Journal of Economics*, Winter 1985, 16, 473-86.
- Dixit, Avinash, "Comparative Statics for Oligopoly," *International Economic Review*, February 1986, 27, 107-22.
- Farrell, Joseph and Shapiro, Carl, "Horizontal Mergers: An Equilibrium Analysis," John M. Olin Program for the Study of Economic Organization and Public Policy, Discussion Paper No. 17, Princeton University, June 1988.
- and ———, "Asset Ownership and Market Structure in Oligopoly," *Rand Journal of Economics*, March 1990, 21, forthcoming.
- Fisher, Franklin M., "Horizontal Mergers: Triage and Treatment," *Journal of Economic Perspectives*, Fall 1987, 1, 23-40.
- Gaudet, Gérard and Salant, Stephen, "The Profitability of Exogenous Output Contractions in Cournot Equilibrium: Implications for Strikes, Mergers and Export Subsidies," University of Michigan, May 1988.
- McAfee, R. Preston and Williams, Michael A., "Horizontal Mergers and Antitrust Policy," unpublished manuscript, U.S. Department of Justice, February 1988.
- McElroy, F. William, "Price and Welfare Effects of Oligopolistic Mergers and Partial Acquisitions," unpublished manuscript, Georgetown University, November 1988.
- Ordover, Janusz A. and Willig, Robert D., "Perspectives on Mergers and World Competition," in R. Grieson, ed., *Antitrust and Regulation*, Lexington, MA: Lexington Books, DC Heath, 1986, 201-18.
- Perry, Martin K. and Porter, Robert H., "Oligopoly and the Incentive for Horizontal

- Merger," *American Economic Review*, March 1985, 75, 219–27.
- Salant, Stephen, Switzer, Sheldon and Reynolds, Robert**, "Losses Due to Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium," *Quarterly Journal of Economics*, May 1983, 98, 185–199.
- Schwartz, Marius**, "Investments in Oligopoly: Welfare Effects and Tests for Predation," *Oxford Economic Papers*, 1989, 41, forthcoming.
- Shapiro, Carl**, "Theories of Oligopoly Behavior," in R. Schmalensee and R. Willig, eds., New York: *The Handbook of Industrial Organization*, Elsevier North-Holland, 1989.
- U.S. Department of Justice**, "Merger Guidelines," 2 Trade Reg. Rep. (CCH), 4491–95, 1984.
- White, Lawrence J.**, "Antitrust and Merger Policy: A Review and Critique," *Journal of Economic Perspectives*, Fall, 1987, 1, 13–22.
- Williamson, Oliver E.**, "Economies as an Antitrust Defense: The Welfare Tradeoffs," *American Economic Review*, March 1968, 58, 18–36.