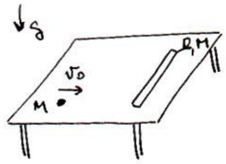
Ejemplo



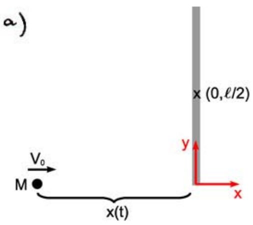
a) Posición del CM del sistema

b) $\vec{v}_{CM}$ antes y después del choque inelástico

c) ω de la barra + masa c/r al CM

sol

a)



$$x_{cm} = \frac{1}{2M}\left[-xM + 0\cdot M\right] = -\frac{x(t)}{2}$$

$$y_{cm} = \frac{1}{2M}\left[0\cdot M + \frac{\ell}{2}M\right] = \frac{\ell}{4}$$

pero $x(t) = -d + v_0 t$

$$\therefore \quad x_{cm}(t) = -\frac{1}{2}\left[v_0 t - d\right] \qquad y_{cm} = \frac{\ell}{4}$$

b) $\vec{v}_{CM} = $ cte porque $\sum \vec{F}_{ext} = 0$ en el plano de la mesa

ANTES

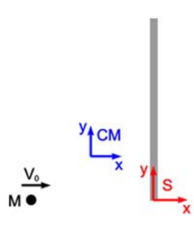
$$v_{CM_x} = \frac{1}{2M}\left[v_0 M + 0\cdot M\right] = \frac{v_0}{2}$$

$$v_{CM_y} = \frac{1}{2M}\left[0\cdot M + 0\cdot M\right] = 0$$

DESPUES

$$\vec{v}_{CM} = \left(\frac{v_0}{2}, 0\right)$$

c) El CM se mueve con velocidad constante. Coloquemos un sistema de referencia en el CM

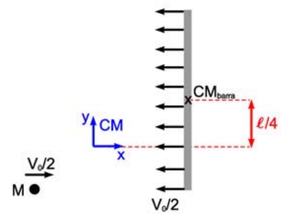


$$\vec{v}_{barra/CM} = \vec{v}_{barra/S} + \vec{v}_{S/CM}$$

$$= \vec{v}_{barra/S} - \vec{v}_{CM/S}$$

$$= (0,0) - \left(\frac{v_0}{2}, 0\right)$$

$$\vec{v}_{barra/CM} = \left(-\frac{v_0}{2}, 0\right)$$



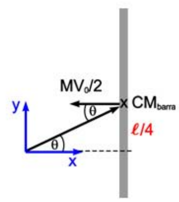
$$\vec{v}_{masa/CM} = \vec{v}_{masa/S} - \vec{v}_{S/CM}$$

$$\vec{v}_{masa/CM} = (v_0, 0) - \left(\frac{v_0}{2}, 0\right)$$

$$\vec{v}_{masa/CM} = \left(\frac{v_0}{2}, 0\right)$$

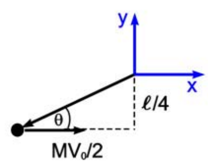
$$L_{barra/CM} = \vec{r}_{CM} \times \vec{P}$$

$$= \frac{\ell}{4} M \frac{v_0}{2} = \frac{\ell M v_0}{8}$$

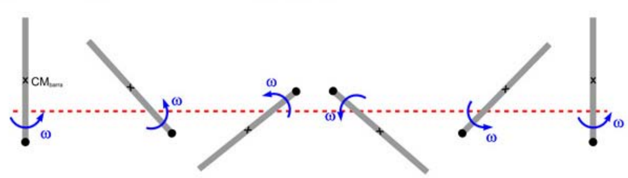


$$L_{masa/CM} = \frac{\ell}{4} \frac{M v_0}{2} = \frac{\ell M v_0}{8}$$

$$\therefore \quad L_{TOTAL/CM} = \frac{\ell M v_0}{4}$$



DESPUÉS DEL CHOQUE :



$$L^{DESPUES}_{TOTAL/CM} = L_{barra/CM} + L_{masa/CM}$$

$$= I_0 \omega + I_0^{masa} \omega$$

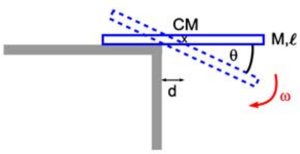
$$= \frac{7}{48} M \ell^2 \omega + \frac{M \ell^2}{16} \omega$$

$$\Rightarrow L^{DESPUES}_{TOTAL/CM} = \frac{5}{24} M \ell^2 \omega$$

$$L_i = L_f \Rightarrow \frac{\ell M v_0}{4} = \frac{5}{24} M \ell^2 \omega$$

$$\omega = \frac{6}{5} \frac{v_0}{\ell}$$

Ejemplo



¿θ crítico para que la barra comience a resbalar por el borde?

$$I = \frac{1}{12} M\ell^2 + Md^2$$

$$E_i = 0$$

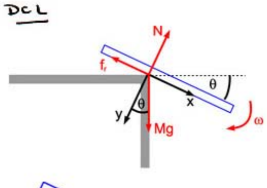
$$E_f = \frac{1}{2} I\omega^2 - Mgh = \frac{1}{2} I\omega^2 - Mg\,d\,\text{sen}\,\theta$$

cons. de energía: $E_i = E_f$

$$0 = \frac{1}{2} I\omega^2 - Mgd\,\text{sen}\,\theta$$

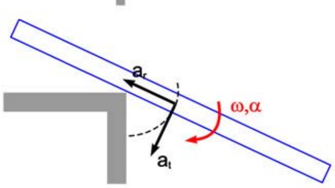
$$\omega = \sqrt{\frac{2Mgd\,\text{sen}\,\theta}{I}}$$

DCL



$$-N + Mg\cos\theta = Ma_{cm_y} \quad (1)$$

$$-f_r + Mg\,\text{sen}\,\theta = -Ma_{cm_x}$$



$$a_{cm_x} = a_r = \frac{v^2}{d}$$

con $v = \omega d$

$$a_{cm_y} = a_t = \frac{dv_t}{dt} = \frac{d}{dt}(\omega d) = \alpha d$$

$$\sum \tau_A = Mg\cos\theta\, d = I\alpha \Rightarrow \alpha = \frac{Mg\cos\theta\, d}{I}$$

entonces

$$a_{cm\,y} = \frac{Mgd^2\cos\theta}{I}$$

$$(1) \longrightarrow N = Mg\cos\theta - \frac{M^2\, gd^2\cos\theta}{I}$$

$$f_r = \mu N \Rightarrow$$

$$Mg\,\text{sen}\,\theta - \mu\left[Mg\cos\theta - \frac{M^2}{I}gd^2\cos\theta\right] = -M\frac{v^2}{d}$$

$$\mu\left[g\cos\theta - M\frac{gd^2\cos\theta}{I}\right] = d\omega^2 + g\,\text{sen}\,\theta$$

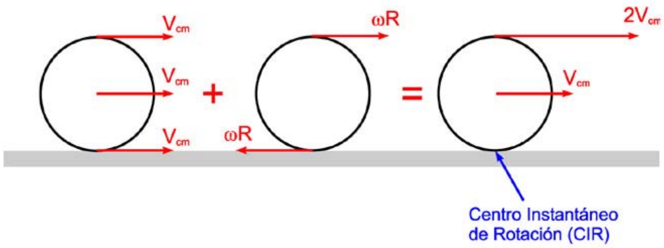
$$\mu\frac{\cancel{g}\cos\theta}{I}\left[I - Md^2\right] = d^2\cdot\frac{2M\cancel{g}\,\text{sen}\,\theta}{I} + \cancel{g}\,\text{sen}\,\theta$$

$$\mu\cos\theta\,\frac{1}{12}Ml^2 = \left[2Md^2 + I\right]\text{sen}\,\theta$$

$$\tan\theta = \frac{\frac{1}{12}Ml^2\mu}{\frac{1}{12}Ml^2 + 3Md^2}$$

$$\boxed{\tan\theta = \frac{\mu l^2}{l^2 + 36\,d^2}}$$

RODAMIENTO SIN RESBALAR (RSR)

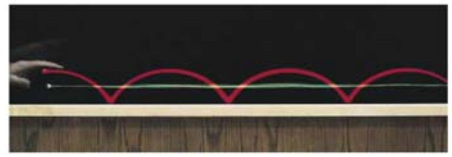


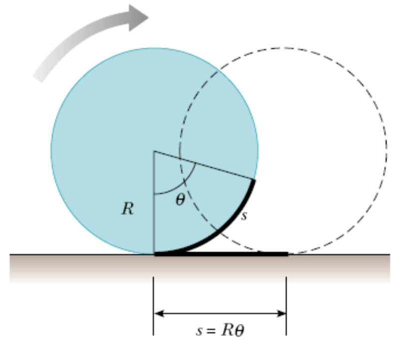
CONDICIÓN PARA RODAR SIN RESBALAR $\boxed{v_{CM} = \omega R}$

Energía cinética

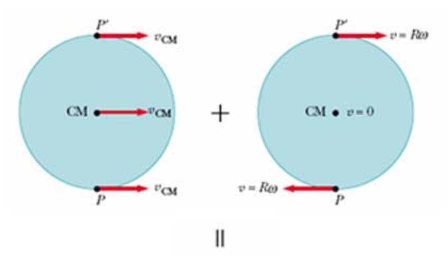
$$K = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I_{CM} \frac{v_{CM}^2}{R^2}$$

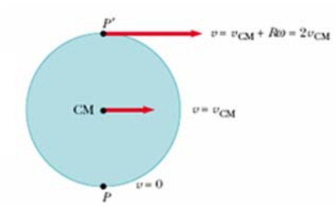
$$K = \frac{1}{2} M R^2 \omega^2 + \frac{1}{2} I_{CM} \omega^2$$



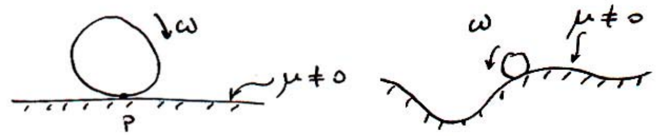


RSR necesita que exista roce, pero $W_{roce} = 0$ porque el punto de contacto está en reposo instantáneo c/r al suelo





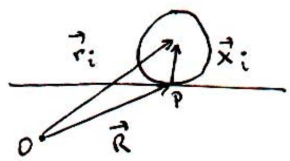
Centro Instantáneo de Rotación



RSR $\Rightarrow$ $\vec{v}_P = 0$

Siempre se puede tomar torque respecto del CIR.

Dem



$$\vec{\tau}_O = \frac{d\vec{L}}{dt}$$ O está fijo

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$$

pero $\vec{r}_i = \vec{R} + \vec{x}_i$

$$\vec{L} = \sum_i \vec{R} \times \vec{p}_i + \sum_i \vec{x}_i \times \vec{p}_i$$

$$\Rightarrow \vec{\tau}_O = \frac{d}{dt}\left\{ \vec{R} \times \sum_i \vec{p}_i + \sum_i \vec{x}_i \times \vec{p}_i \right\}$$

$$\vec{\tau}_O = \frac{d\vec{R}}{dt} \times \sum_i \vec{p}_i + \vec{R} \times \frac{d}{dt} \sum_i \vec{p}_i + \frac{d}{dt} \underbrace{\sum_i \vec{x}_i \times \vec{p}_i}_{\vec{L}_P}$$

pero $$\frac{d\vec{r}_i}{dt} = \frac{d\vec{R}}{dt} + \frac{d\vec{x}_i}{dt}$$

Si el punto P es el CIR $\Rightarrow$ $\frac{d\vec{R}}{dt} = 0$ $\Rightarrow$ $\vec{p}_i = \vec{P}_i$

Entonces

$$\vec{\tau}_0 = \vec{R} \times \sum_i \frac{d\vec{p}_i}{dt} + \frac{d\vec{L}_P}{dt}$$

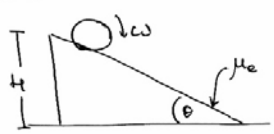
$$\vec{\tau}_0 - \vec{R} \times \sum \vec{F}_i^{ext} = \frac{d\vec{L}_P}{dt}$$

$$\sum_i \vec{r}_i \times \vec{F}_i^{ext} - \vec{R} \times \sum \vec{F}_i^{ext} = \frac{d\vec{L}_P}{dt}$$

$$\sum_i \underbrace{(\vec{r}_i - \vec{R})}_{\vec{x}_i} \times \vec{F}_i^{ext} = \frac{d\vec{L}_P}{dt} \Rightarrow \vec{\tau}_P = \frac{d\vec{L}_P}{dt}$$

QED

RSR

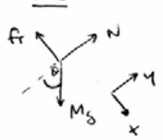


$I_{cm} = \frac{1}{2} M R^2$ (cilindro)

¿v_{cm} al llegar al suelo?

Usando 2a ley de Newton:

DCL



$\hat{x})$ $Mg \operatorname{sen}\theta - f_r = M a_{cm}$ (1)

$\hat{y})$ $N - Mg\cos\theta = 0$

RSR $\Rightarrow$ $v_{cm} = \omega R \Rightarrow a_{cm} = \alpha R$

$\sum \tau_{c/r\,CM} = I\alpha \Rightarrow f_r R = I\alpha$

$f_r = \frac{1}{2} M a_{cm}$

Reemplazando en (1)

$$Mg \operatorname{sen}\theta = \frac{3}{2} M a_{cm} \Rightarrow a_{cm} = \frac{2}{3} g \operatorname{sen}\theta$$

Entonces $v_{cm}^2 - \cancel{v_0^2}^{\nearrow 0} = 2a_{cm}\, d$

$$v_{cm}^2 = \frac{4}{3} g \underbrace{d \operatorname{sen}\theta}_{H} \Rightarrow \boxed{v_{cm} = \sqrt{\frac{4}{3} gH}}$$

La fuerza de roce debe satisfacer: $f_r \le \mu_e N$

$$f_r = \frac{1}{3} Mg \operatorname{sen}\theta \le \mu_e Mg\cos\theta$$

$$\Rightarrow \mu_e \ge \frac{1}{3}\tan\theta$$

μ_e mínimo $= \frac{1}{3}\tan\theta$ para que pueda RSR.

Usando energía:

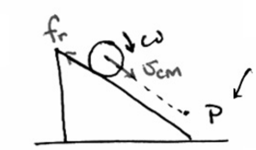
$E_i = MgH$

$E_f = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \left(\frac{v_{cm}}{R}\right)^2$

$E_i = E_f \Rightarrow \frac{3}{4} M v_{cm}^2 = MgH$

$$v_{cm} = \sqrt{\frac{4}{3} gH}$$

Otra manera:



punto fijo a la altura del CM del cilindro

$$\tau_P = \frac{dL_P}{dt} \quad \text{con} \quad L_P = L_{CM} + L_{c/r\,CM}$$

En este caso, $L_{CM} = 0 \wedge L_{c/r\,CM} = I\omega$

Por otro lado, $\tau_P = \underbrace{\tau_N + \tau_{peso}}_{0} + \tau_{roce}$

$$\tau_P = \tau_{roce} = f_r R$$

$$\therefore \frac{dL_P}{dt} = I\alpha = f_r R$$