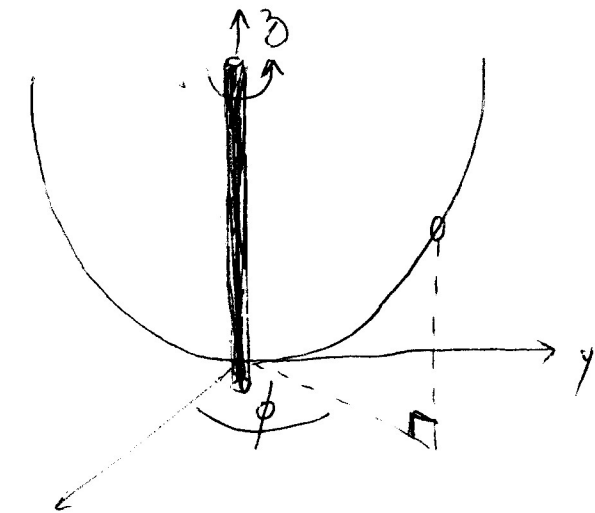


Problema P3-C1



$$\vec{r} = r \hat{r} + z \hat{z}$$

pero $z(r) = \frac{\beta}{2} r^2 \Rightarrow \dot{z} = \frac{2\beta}{2} r \dot{r}$

$$\Rightarrow \vec{v} = \dot{r} \hat{r} + r \dot{\phi} \hat{\phi} + \dot{z} \hat{z}$$

$$= \dot{r} + r \dot{\phi} + \beta r \dot{r} \hat{z}$$

$$\Rightarrow L = \underbrace{\frac{1}{2} m (\dot{r}^2 + r^2 \dot{\phi}^2 + \beta^2 r^2 \dot{r}^2)}_{\text{energía cinética part.}} + \underbrace{\frac{1}{2} I \dot{\phi}^2}_{\text{debido a que la barra tb girará.}} - \underbrace{mg \frac{\beta}{2} r^2}_{mgz}$$

Variable cíclica: ϕ

$$\Rightarrow \frac{\partial L}{\partial \dot{\phi}} = \text{cte} = P_{\phi} = (mr^2 + I) \dot{\phi} \Rightarrow$$

$$\dot{\phi} = \frac{P_{\phi}}{(mr^2 + I)}$$

$$\Rightarrow R = P_{\phi} \dot{\phi} - L = \frac{P_{\phi} \cdot P_{\phi}}{(mr^2 + I)} - \frac{1}{2} m \dot{r}^2 - \frac{1}{2} (mr^2 + I) \dot{\phi}^2$$

$$- \frac{1}{2} m \beta^2 r^2 \dot{r}^2 + mg \frac{\beta}{2} r^2$$

$$= \frac{P_{\phi}^2}{(mr^2 + I)} - \frac{1}{2} \frac{(mr^2 + I) P_{\phi}^2}{(mr^2 + I)^2} - \frac{1}{2} m \dot{r}^2 (1 + \beta^2 r^2) + mg \frac{\beta}{2} r^2$$

$$R = \frac{P_{\phi}^2}{2(mr^2 + I)} - \frac{1}{2} m \dot{r}^2 (1 + \beta^2 r^2) + mg \frac{\beta}{2} r^2$$

$$\Rightarrow \frac{\partial R}{\partial r} = mg\beta r - m\beta^2 r \dot{r}^2 - \frac{P_\phi^2}{(mr^2 + I)^2} \cdot mr$$

$$\frac{\partial R}{\partial \dot{r}} = -m\dot{r}(1 + \beta^2 r^2) \Rightarrow \frac{d}{dt} \left(\frac{\partial R}{\partial \dot{r}} \right) = -m\ddot{r}(1 + \beta^2 r^2) - m\dot{r}\beta^2 \cdot 2r\dot{r}$$

$$\Rightarrow \frac{d}{dt} \left(\frac{\partial R}{\partial \dot{r}} \right) = \frac{\partial R}{\partial r}$$

$$-m\ddot{r}(1 + \beta^2 r^2) - m\dot{r}\beta^2 \cdot 2r\dot{r} = mg\beta r - m\beta^2 r \dot{r}^2 - \frac{P_\phi^2}{(mr^2 + I)^2} \cdot mr$$

$$\Rightarrow \frac{P_\phi^2 \cdot r}{(mr^2 + I)^2} - \beta^2 r \dot{r}^2 - g\beta r = \ddot{r}(1 + \beta^2 r^2)$$

$$\Rightarrow \ddot{r} = \frac{P_\phi^2 \cdot r}{(mr^2 + I)^2 (1 + \beta^2 r^2)} - \frac{(\beta^2 \dot{r}^2 + g\beta) r}{(1 + \beta^2 r^2)}$$

Puntaje

- * L bueno (1 pto)
- * P_ϕ Conservado (1 pto)
- * Routhiano (1 pto)
- * $\frac{\partial R}{\partial r}$ (1 pto)
- * $\frac{d}{dt} \left(\frac{\partial R}{\partial \dot{r}} \right)$ (1 pto)

* Ec. de mov (1 pto)

Error en el lagrangiano de carácter no algebraico, error tipo mal interpretación problema, no usar bien las coordenadas \Rightarrow nota P3: 1.0