CI72A EJERCICIO Nº1 2005
TUNEL
SEPT 2006 (Unidades: m, ton)

$R := 5$ $H := 3$ $h := 0.3$ $qo := -3$ $kb := 20000$

$N1 := 5$ $N2 := 25$ Nº de elementos sin carga $N3 := 12$

$E := 2530000$

Factor de Amplificación del gráfico de desplazamiento

Propiedades de la sección

$A := h$ $I := \frac{h^3}{12}$ $I = 2.25 \times 10^{-3}$

Coordenadas de los nodos

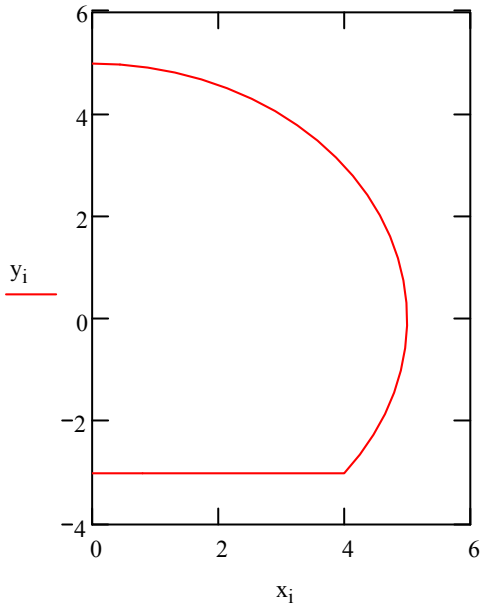
$i := 1 .. N1 + 1$ $x_i := \frac{4}{5}(i - 1)$

$y_i := -H$ $\alpha := \operatorname{asin}\left(\frac{H}{R}\right)$

$i := N1 + 1 .. N1 + N2 + 1$ $\alpha \cdot \frac{180}{\pi} = 36.87$ $\alpha t := \alpha + \frac{\pi}{2}$

$x_i := R \cdot \cos\left[\frac{\alpha t}{N2}(i - N1 - 1) - \alpha\right]$ $y_i := R \cdot \sin\left[\frac{\alpha t}{N2}(i - N1 - 1) - \alpha\right]$

$i := 1 .. N1 + N2 + 1$



Cargas en los elementos

$$e := 1 .. N1 + N2$$

$$q_e := \begin{cases} 0 & \text{if } e \le N3 \\ qo & \text{otherwise} \end{cases}$$

Matriz de Rigidez de los elementos

$$l_e := \sqrt{\left(x_{e+1} - x_e\right)^2 + \left(y_{e+1} - y_e\right)^2} \qquad s_e := \frac{y_{e+1} - y_e}{l_e} \qquad c_e := \frac{x_{e+1} - x_e}{l_e}$$

$$k_e := \begin{cases} kb & \text{if } e \le N1 + 7 \\ 0 & \text{otherwise} \end{cases}$$

Contribución de la flexión

$$\phi_e := \frac{k_e \cdot \left(l_e\right)^4}{E \cdot I} \qquad \lambda_e := \frac{E \cdot I}{\left(l_e\right)^3}$$

$$K(e) := \begin{bmatrix} \left(\frac{E \cdot A}{l_e}\right) & 0 & 0 & -\frac{E \cdot A}{l_e} & 0 \\ 0 & \lambda_e \cdot \left(12 + \frac{13}{35} \cdot \phi_e\right) & \lambda_e \cdot \left(6 \cdot l_e + \frac{11}{210} \cdot \phi_e \cdot l_e\right) & 0 & \lambda_e \cdot \left(-12 + \frac{9}{70} \cdot \phi_e\right) \\ 0 & \lambda_e \cdot \left(6 \cdot l_e + \frac{11}{210} \cdot \phi_e \cdot l_e\right) & \lambda_e \cdot \left[4 \cdot \left(l_e\right)^2 + \frac{1}{105} \cdot \phi_e \cdot \left(l_e\right)^2\right] & 0 & \lambda_e \cdot \left(-6 \cdot l_e + \frac{13}{420} \cdot \phi_e \cdot l_e\right. \\ -\frac{E \cdot A}{l_e} & 0 & 0 & \left(\frac{E \cdot A}{l_e}\right) & 0 \\ 0 & \lambda_e \cdot \left(-12 + \frac{9}{70} \cdot \phi_e\right) & \lambda_e \cdot \left(-6 \cdot l_e + \frac{13}{420} \cdot \phi_e \cdot l_e\right) & 0 & \lambda_e \cdot \left(12 + \frac{13}{35} \cdot \phi_e\right) \\ 0 & \lambda_e \cdot \left(6 \cdot l_e - \frac{13}{420} \cdot \phi_e \cdot l_e\right) & \lambda_e \cdot \left[2 \cdot \left(l_e\right)^2 - \frac{1}{140} \cdot \phi_e \cdot \left(l_e\right)^2\right] & 0 & \lambda_e \cdot \left(-6 \cdot l_e - \frac{11}{210} \cdot \phi_e \cdot l_e\right. \end{bmatrix}$$

Transformación a coordenadas globales

Matrices de rotación:

$$R(e) := \begin{pmatrix} c_e & s_e & 0 & 0 & 0 & 0 \\ -s_e & c_e & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_e & s_e & 0 \\ 0 & 0 & 0 & -s_e & c_e & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad K(e) := R(e)^T \cdot K(e) \cdot R(e)$$

Matriz de rigidez global

$i := 1 .. (N1 + N2 + 1) \cdot 3$

$j := 1 .. (N1 + N2 + 1) \cdot 3$

$Kg_{i,j} := 0$

$i := 1 .. 6 \qquad j := 1 .. 6$

$Kg_{(e-1)\cdot 3+i,(e-1)\cdot 3+j} := Kg_{(e-1)\cdot 3+i,(e-1)\cdot 3+j} + K(e)_{i,j}$

Cargas sobre los elementos (se usaron las mismas que en GTS):

$k := 1 .. 18$

$Pa_k :=$

0.08255
0.192
0.318
0.441
0.543
0.642
0.7365
0.8385
0.9345
1.0095
1.0755
1.137
1.1985
1.2465
1.278
1.3035
1.32
0.66

$k := 1 .. 13 \cdot 3 \qquad P_k := 0$

$k := 14 .. N1 + N2 + 1$

$P_{(k-1)\cdot 3+2} := (-Pa)_{k-13}$

$P_{93} := 0$

Condiciones de apoyo

$Kg_{1,1} := 10^{10} \qquad Kg_{1,1} := 10^{10}$

$Kg_{91,91} := 10^{10} \qquad Kg_{93,93} := 10^{10}$

Solución para los desplazamientos

$$U := Kg^{-1} \cdot P$$

$U =$

	1
78	1.64·10-3
79	2.985·10-4
80	-4.288·10-3
81	1.447·10-3
82	1.255·10-4
83	-4.843·10-3
84	1.167·10-3
85	3.268·10-5
86	-5.275·10-3
87	8.179·10-4
88	1.922·10-7
89	-5.549·10-3
90	4.212·10-4
91	-6.766·10-10
92	-5.643·10-3
93	5.496·10-10

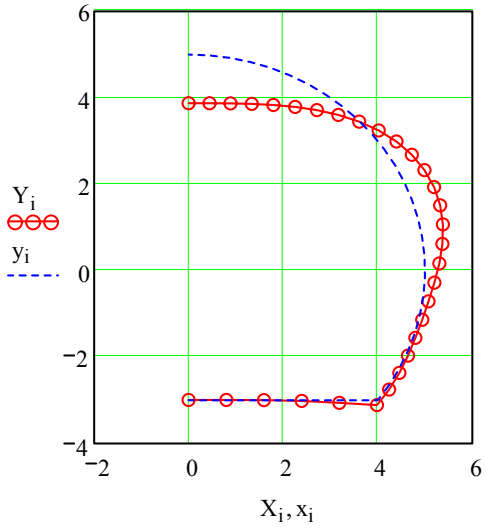
Posición deformada (nótese que la deformación está amplificada por Amp)

$$i := 1 .. N1 + N2 + 1$$

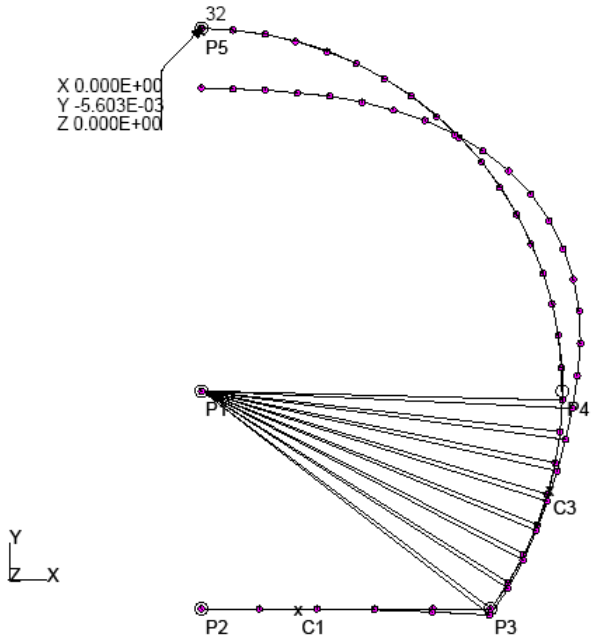
$$Amp := 200$$

$$X_i := x_i + \left[U_{3\cdot(i-1)+1}\right] \cdot Amp$$

$$Y_i := y_i + \left[U_{3\cdot(i-1)+2}\right] \cdot Amp$$



Desplazamientos de acuerdo a modelo en GTSTRUDL



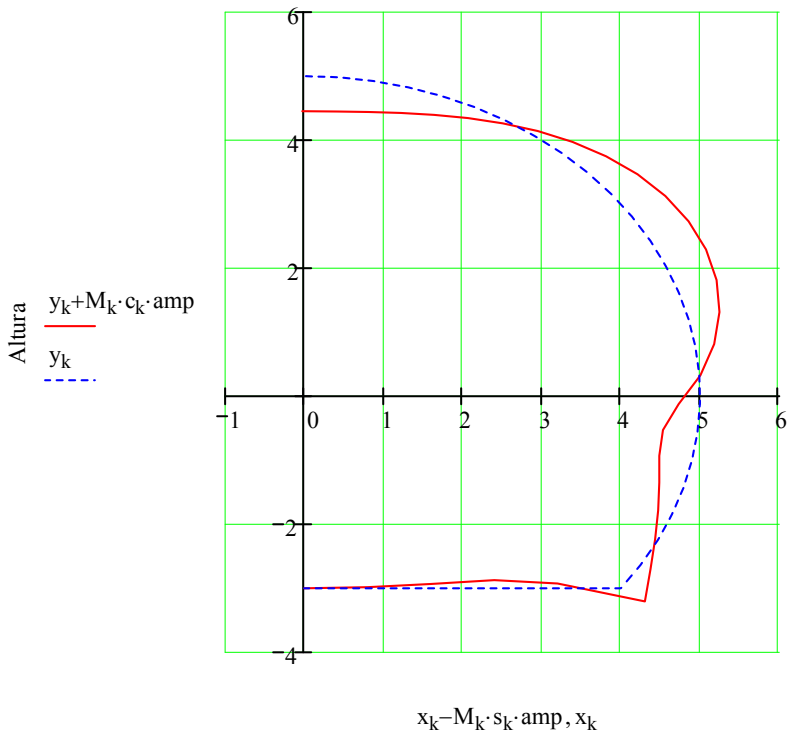
Momentos:

$e := 1 .. N1 + N2 \qquad i := 1 .. 6$

$M_e := \sum_i \left[K(e)_{3,i} \cdot U_{(e-1)\cdot 3+i} \right] \qquad M_{N1+N2+1} := -\sum_i \left[K(N1 + N2)_{6,i} \cdot U_{(N1+N2-1)\cdot}$

$k := 1 .. N1 + N2 + 1$ $c_{31} := c_{30}$ $s_{31} := s_{30}$

amp := 0.1



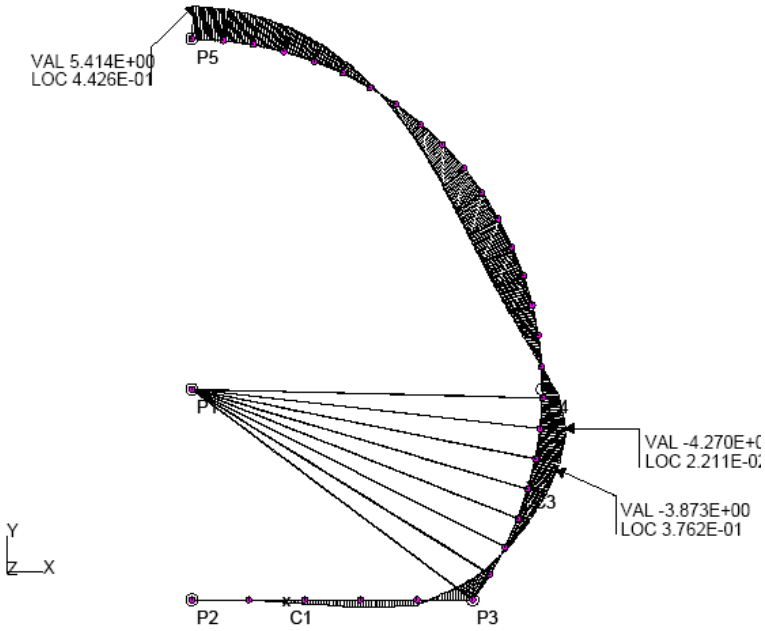
M =

	1
1	0
2	0.157
3	0.658
4	1.256
5	0.732
6	-3.679
7	-1.416
8	0.376
9	1.864
10	3.175
11	4.193
12	4.388
13	2.695
14	-0.162
15	-2.42
16	-4.079

M =

	1
16	-4.079
17	-5.154
18	-5.683
19	-5.713
20	-5.305
21	-4.529
22	-3.469
23	-2.212
24	-0.845
25	0.548
26	1.887
27	3.095
28	4.104
29	4.865
30	5.337
31	5.496

Momentos de acuerdo a GTS



Errores relativos:

Desplazamiento en punto superior: $\frac{5643 - 5603}{5603} = 0.0$

Momento en punto superior: $\frac{5496 - 5414}{5414} = 0.0$

$$
\begin{bmatrix}
 & 0 & & \\
 & \lambda_e \cdot \left(6 \cdot l_e - \frac{13}{420} \cdot \phi_e \cdot l_e\right) & & \\
 & \lambda_e \cdot \left[2 \cdot \left(l_e\right)^2 - \frac{1}{140} \cdot \phi_e \cdot \left(l_e\right)^2\right] & & \\
 & 0 & & \\
 & \lambda_e \cdot \left(-6 \cdot l_e - \frac{11}{210} \cdot \phi_e \cdot l_e\right) & & \\
 & \lambda_e \cdot \left[4 \cdot \left(l_e\right)^2 + \frac{1}{105} \cdot \phi_e \cdot \left(l_e\right)^2\right] & &
\end{bmatrix}
$$