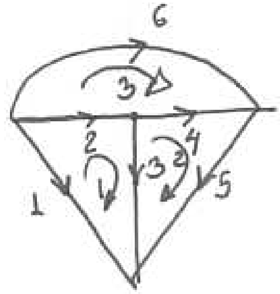
Examen

Problema 1

grafo orientado



matriz de mallas

$$M = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{bmatrix} \quad M^T = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

matriz impedancia de ramos

$$Z_b = \begin{bmatrix} R_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & SL_1 & 0 & -SM & 0 & 0 \\ 0 & 0 & \frac{1}{SC_1} & 0 & 0 & 0 \\ 0 & -SM & 0 & SL_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{SC_2} \end{bmatrix}$$

$$J_S = \begin{bmatrix} -I_f \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad V_S = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \alpha i_{L_1} \\ 0 \end{bmatrix} \quad Z_b J_S - V_S = \begin{bmatrix} -R_1 I_f \\ 0 \\ 0 \\ 0 \\ -\alpha i_{L_1} \\ 0 \end{bmatrix}$$

$$Z_b M^T = \begin{bmatrix} -R_1 & 0 & 0 \\ SL_1 & -SM & -SL_1+SM \\ \frac{1}{SC_1} & -\frac{1}{SC_1} & 0 \\ -SM & SL_2 & SM-SL_2 \\ 0 & R_2 & 0 \\ 0 & 0 & \frac{1}{SC_2} \end{bmatrix}$$

$$Z_M = \begin{bmatrix} R_1+SL_1+\frac{1}{SC_1} & -SM-\frac{1}{SC_1} & -SL_1+SM \\ -SM-\frac{1}{SC_1} & \frac{1}{SC_1}+SL_2+R_2 & SM-SL_2 \\ -SL_1+SM & SM-SL_2 & SL_1-2SM+SL_2+\frac{1}{SC_2} \end{bmatrix}$$

finalmente

$$M(Z_b J_S - V_S) = \begin{bmatrix} R_1 I_f \\ -\alpha i_{L_1} \\ 0 \end{bmatrix}$$

$$i_{L_1} = (I_1 - I_3)$$

$$\begin{bmatrix} R_1+SL_1+\frac{1}{SC_1} & -SM-\frac{1}{SC_1} & SM-SL_1 \\ -SM-\frac{1}{SC_1}+\alpha & \frac{1}{SC_1}+SL_2+R_2 & SM-SL_2-\alpha \\ SM-SL_1 & SM-SL_2 & SL_1+SL_2-2SM+\frac{1}{SC_2} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} R_1 I_f \\ 0 \\ 0 \end{bmatrix}$$

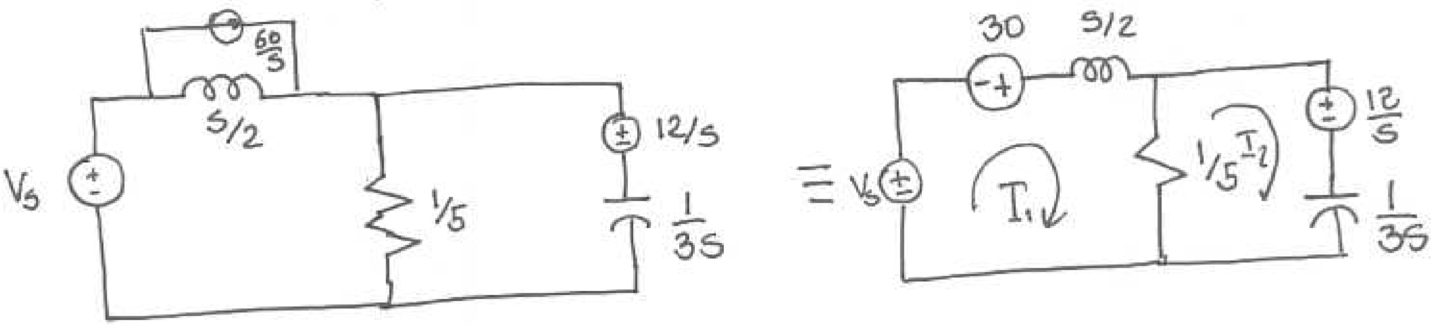
Examen

Problema 2

condiciones iniciales $\qquad i_L(0) = \dfrac{12}{1/5} = 60\ A$

$$V_c(0) = 12\ V$$

Red en Laplace para método de las mallas



$$\begin{bmatrix} \frac{s}{2} + \frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{3s} + \frac{1}{5} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_s + 30 \\ -\frac{12}{s} \end{bmatrix}$$

b) RESC

$$\begin{bmatrix} \frac{s}{2} + \frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{3s} + \frac{1}{5} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix} \qquad \Delta = \left(\frac{s}{2} + \frac{1}{5}\right)\left(\frac{1}{3s} + \frac{1}{5}\right) - \frac{1}{25}$$

$$\Delta = \frac{1}{6} + \frac{s}{10} + \frac{1}{15s}$$

$$\Delta_2 = V_s/5 \qquad I_2 = \frac{V_s/5}{\frac{s}{10} + \frac{1}{6} + \frac{1}{15s}} \quad \begin{matrix} /\cdot 10s \\ /\cdot 10s \end{matrix}$$

$$I_2 = \frac{2sV_s}{s^2 + \frac{5}{3}s + \frac{2}{3}} \qquad V_c = \frac{1}{3s} I_2$$

$$V_c(s) = \frac{2/3\, V_s}{s^2 + \frac{5}{3}s + \frac{2}{3}} \qquad V_s(s) = \frac{4}{(s+2)}$$

$$V_c(s) = \frac{8/3}{(s+2)(s+1)(s+2/3)} = \frac{A}{(s+2)} + \frac{B}{(s+1)} + \frac{C}{(s+2/3)}$$

$$A = \frac{8/3}{(-2+1)(-2+2/3)} = \frac{8/3}{4/3} = 2$$

$$B = \frac{8/3}{(-1+2)(-1+2/3)} = \frac{8/3}{-1/3} = -8$$

$$C = \frac{8/3}{(-2/3+2)(-2/3+1)} = \frac{8/3}{4/9} = 6$$

$$V_c(t) = \left[2e^{-2t} - 8e^{-t} + 6e^{-2/3\,t}\right]u(t)$$

c) RENC

$$\begin{bmatrix} \frac{s}{2}+\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{3s}+\frac{1}{5} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 30 \\ -\frac{12}{s} \end{bmatrix}$$

$$\Delta_2 = -\frac{12}{s}\left(\frac{s}{2}+\frac{1}{5}\right) + 6 = -6 - \frac{12}{5s} + 6 = -\frac{12}{5s}$$

$$I_2 = \frac{-\frac{12}{5s}}{\frac{s}{10}+\frac{1}{6}+\frac{1}{15s}} \; /10s \Rightarrow I_2(s) = \frac{-24}{s^2+\frac{5}{3}s+\frac{2}{3}} \qquad V_c(s) = \frac{1}{3s}I_2(s) + \frac{12}{s}$$

$$V_c(s) = \frac{-8}{s(s+1)(s+2/3)} = \frac{A}{(s+1)} + \frac{B}{(s+2/3)} + \frac{C}{s}$$

$$A = \frac{8}{-1\cdot(-1/3)} = -24$$

$$B = \frac{8}{-2/3\cdot 1/3} = +36$$

$$C = \frac{8}{1\times 2/3} = -12$$

$$V_c(t) = \left(-24e^{-t} + 36e^{-2/3\,t} + \right)u(t)$$

otra forma   sin cambiar de posición R con C

$$\begin{bmatrix} \frac{S}{2}+\frac{1}{3S} & -\frac{1}{3S} \\ -1/3S & \frac{1}{3S}+\frac{1}{5} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_S + 30 - \frac{12}{S} \\ \frac{12}{S} \end{bmatrix}$$

b) RESC

$$\begin{bmatrix} \frac{S}{2}+\frac{1}{3S} & -\frac{1}{3S} \\ -\frac{1}{3S} & \frac{1}{3S}+\frac{1}{5} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_S \\ 0 \end{bmatrix} \quad \Delta = \left(\frac{S}{2}+\frac{1}{3S}\right)\left(\frac{1}{3S}+\frac{1}{5}\right) - \frac{1}{9S^2}$$

$$\Delta = \frac{S}{10} + \frac{1}{6} + \frac{1}{15S}$$

$$\Delta_2 = \frac{1}{3S} V_S \qquad I_2 = \frac{\Delta_2}{\Delta} = \frac{\frac{1}{3S} V_S}{\frac{S}{10} + \frac{1}{6} + \frac{1}{15S}} \begin{matrix} /\cdot 10S \\ /\cdot 10S \end{matrix}$$

$$I_2(S) = \frac{\frac{10}{3} V_S}{S^2 + \frac{5}{3}S + \frac{2}{3}} = \frac{\frac{40}{3}}{(S+2)(S+1)(S+2/3)} \qquad V_R = V_C = \frac{1}{5} I_2(S)$$

$$\Rightarrow V_C(S) = \frac{8/3}{(S+2)(S+1)(S+2/3)} \Rightarrow V_C(t) = [2e^{-2t} - 8e^{-t} + 6e^{-2/3t}]u(t)$$

c) RENC

$$\Delta_2 = \left(\frac{S}{2}+\frac{1}{3S}\right)\cdot\frac{12}{S} + \frac{1}{3S}\left(30 - \frac{12}{S}\right) = 6 + \frac{4}{S^2} + \frac{10}{S} - \frac{4}{S^2}$$

$$\Delta_2 = 6 + \frac{10}{S}$$

$$I_2 = \frac{6 + 10/S}{\frac{S}{10} + \frac{1}{6} + \frac{1}{15S}} \begin{matrix} /\cdot 10S \\ /\cdot 10S \end{matrix} \Rightarrow I_2 = \frac{60S + 100}{S^2 + \frac{5}{3}S + 2/3} \qquad V_R = V_C = \frac{1}{5} I_2(S)$$

$$V_C(S) = \frac{12S + 20}{(S+1)(S+2/3)} = \frac{A}{(S+1)} + \frac{B}{(S+2/3)}$$

$$A = \frac{-12+20}{-1/3} = -24$$

$$B = \frac{-8+20}{1/3} = 36$$

$$V_C(t) = (-24e^{-t} + 36e^{-2/3t})u(t)$$