Lagrangeano

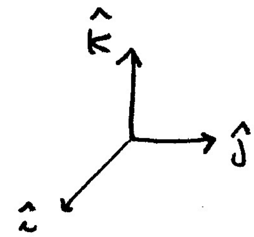
$$L = T - V \quad ; \quad L(q,\dot{q};t) = T(q,\dot{q}) - V(q)$$

$$T = \frac{1}{2} m |\vec{v}|^2$$

E-L:
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

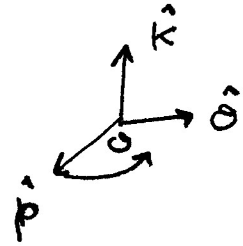
- En cartesianos:

$$\vec{v} = \dot{x}\,\hat{i} + \dot{y}\,\hat{j} + \dot{z}\,\hat{k}$$



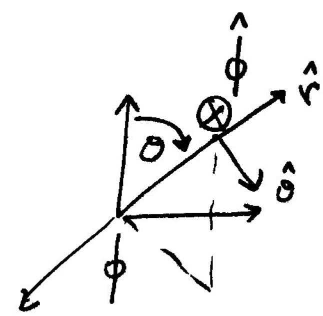
- En polares:

$$\vec{v} = \dot{\rho}\,\hat{\rho} + \rho\dot{\theta}\,\hat{\theta} + \dot{z}\,\hat{k}$$



- En esféricos:

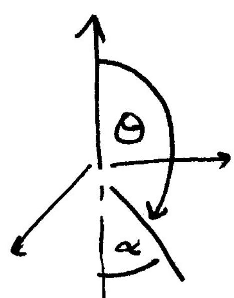
$$\vec{v} = \dot{r}\,\hat{r} + r\dot{\phi}\,\text{sen}\theta\,\hat{\phi} + r\dot{\theta}\,\hat{\theta}$$



ojo: $\theta > \pi/2$

$\Rightarrow$ $\cancel{\theta = \frac{\pi}{2} + \alpha}$

$$\theta = \pi - \alpha$$

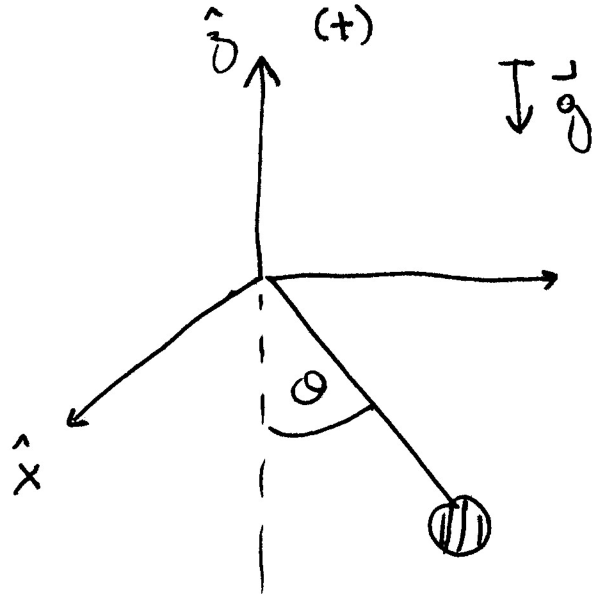


$\Rightarrow$ $\cancel{\text{sen}\theta = \text{sen}\frac{\pi}{2}\cos\alpha + \cos\frac{\pi}{2}\,\text{sen}\alpha}$

$\cancel{\text{sen}\theta = \cos\alpha}$

ó $\quad \text{sen}\theta = \cancel{\text{sen}\pi}^{\,0}\cos\alpha - \text{sen}\alpha\cos\pi$

$\quad \text{sen}\theta = \text{sen}\alpha \quad \Rightarrow$ sirve para θ def. abajo ✓✓



$r = l \Rightarrow \dot{r} = 0$

$z = l\cos\theta$

$\Rightarrow V = -mgl\cos\theta$

$$L = \frac{1}{2} m \left(l^2 \dot{\phi}^2 \sin^2\theta + l^2 \dot{\theta}^2 \right) + mgl\cos\theta$$

$$\Rightarrow \frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta} \quad \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = ml^2\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = ml^2\dot{\phi}^2 \sin\theta\cos\theta - mgl\sin\theta$$

$$\Rightarrow \cancel{m}l^{\cancel{2}}\ddot{\theta} = \cancel{m}l^{\cancel{2}}\dot{\phi}^2 \sin\theta\cos\theta - \cancel{m}g\cancel{l}\sin\theta$$

$$\Rightarrow \ddot{\theta} = \dot{\phi}^2 \sin\theta\cos\theta - \frac{g}{l}\sin\theta$$

ϕ]

$$\frac{\partial L}{\partial \dot{\phi}} = ml^2\dot{\phi}\sin^2\theta \qquad \frac{\partial L}{\partial \phi} = 0$$

$\Rightarrow \frac{\partial L}{\partial \dot{\phi}} = cte \Rightarrow$ cantidad conservada.
pseudo-momentum P_ϕ (angular)

$$\Rightarrow P_\phi = ml^2\dot{\phi}\sin^2\theta$$

$$\Rightarrow \quad \dot{\phi} = \frac{P_\phi}{m l^2 \sin^2\theta}$$

$$\Rightarrow \quad \ddot{\theta} = \frac{P_\phi^2}{m^2 l^4 \sin^4\theta} \sin\theta \cos\theta - \frac{g}{l} \sin\theta$$

$$\boxed{\ddot{\theta} = -\frac{g}{l} \sin\theta + \frac{P_\phi^2 \cos\theta}{m^2 l^4 \sin^3\theta}}$$

$$H = \sum p \dot{q} - L$$

P_ϕ ✓

$P_\theta = ?$

$$\Rightarrow \quad \frac{\partial L}{\partial \dot{\theta}} = m l^2 \dot{\theta} = P_\theta \quad \Rightarrow \quad \boxed{\dot{\theta} = \frac{P_\theta}{m l^2}}$$

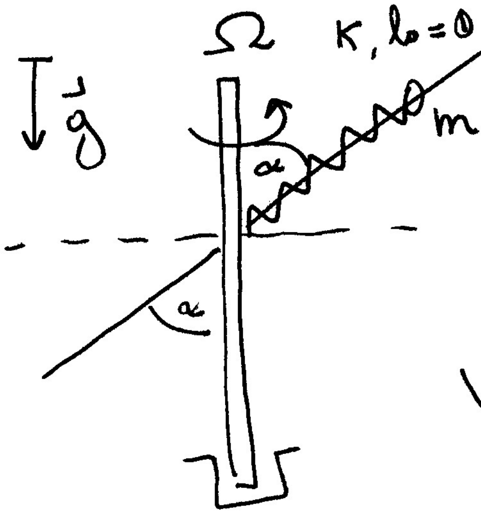
$$\Rightarrow \quad H = P_\theta \cdot \dot{\theta} + P_\phi \dot{\phi} - L$$

$$= \frac{P_\theta^2}{m l^2} + \frac{P_\phi^2}{m l^2 \sin^2\theta} - \frac{1}{2} m \left(l^2 \frac{P_\phi^2}{m^2 l^4 \sin^{\cancel{4}\,2}\theta} \cancel{\sin^2\theta} + l^2 \frac{P_\theta^2}{m^2 l^4} \right)$$

$$- m g l \cos\theta$$

$$\boxed{H = \frac{P_\theta^2}{2 m l^2} + \frac{P_\phi^2}{2 m l^2 \sin^2\theta} - m g l \cos\theta}$$

~~Ecuaciones de Hamilton~~



$\theta = \alpha = \text{fijo} \Rightarrow \dot{\theta} = 0$

$\dot{\phi} = \Omega = \text{fijo} \Rightarrow \phi = \Omega \cdot t$

$r = \text{libre.}$

$$\vec{v} = \dot{r}\hat{r} + r\Omega \operatorname{sen}\alpha\, \hat{\phi}$$

$$V = r\cos\alpha\, mg + \frac{1}{2}k r^2$$

$$\Rightarrow L = \frac{1}{2}m\left(\dot{r}^2 + r^2\Omega^2 \operatorname{sen}^2\alpha\right) - mgr\cos\alpha - \frac{1}{2}kr^2$$

$\Rightarrow \underline{r}\rfloor$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) = m\ddot{r}$$

$$\frac{\partial L}{\partial r} = mr\Omega^2 \operatorname{sen}^2\alpha - mg\cos\alpha - kr$$

$$\Rightarrow m\ddot{r} = mr\Omega^2 \operatorname{sen}^2\alpha - mg\cos\alpha - kr$$

$$\Rightarrow \ddot{r} = r\Omega^2 \operatorname{sen}^2\alpha - g\cos\alpha - \frac{k}{m}r$$

$$\ddot{r} = -\left(\frac{k}{m} - \Omega^2 \operatorname{sen}^2\alpha\right)r - g\cos\alpha$$

$$r_{eq} = \frac{g\cos\alpha}{\Omega^2 \operatorname{sen}^2\alpha - \frac{k}{m}} \Rightarrow r = \tilde{r} + r_{eq}$$

$$\Rightarrow \boxed{\ddot{\tilde{r}} = -\left(\frac{k}{m} - \Omega^2 \operatorname{sen}^2\alpha\right)\tilde{r}}$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{k}{m} - \Omega^2 \operatorname{sen}^2\alpha}}$$

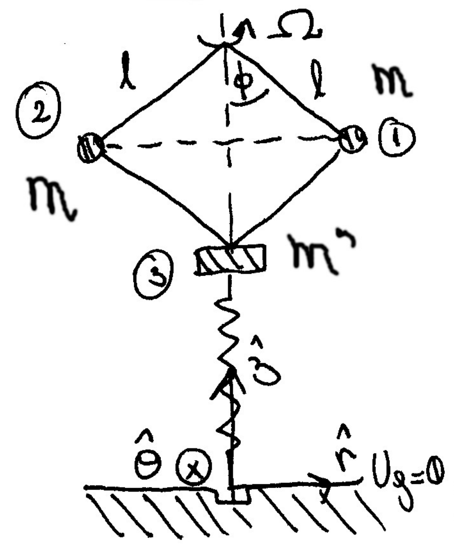
$$P_r = \frac{\partial L}{\partial \dot r} = m\dot r \quad \Rightarrow \quad \dot r = \frac{P_r}{m}$$

$$\Rightarrow \quad H = P_r \cdot \dot r - L$$

$$= P_r \cdot \frac{P_r}{m} - \frac{1}{2} m \left(\frac{P_r^2}{m^2} + r^2 \Omega^2 \operatorname{sen}^2\alpha \right) + mgr\cos\alpha + \frac{1}{2} K r^2$$

$$= \frac{P_r^2}{2m} - \frac{m}{2} r^2 \Omega^2 \operatorname{sen}^2\alpha + mgr\cos\alpha + \frac{1}{2} K r^2$$

$$\boxed{H = \frac{P_r^2}{2m} + \underbrace{mgr\cos\alpha + \frac{1}{2} k r^2 - \frac{m}{2} r^2 \Omega^2 \operatorname{sen}^2\alpha}_{V_{eff}}}$$



En coord. cilíndricos:

$$\vec{v}_{①} = \dot r \hat r + r\dot\theta\hat\theta + \dot z \hat z$$

pero $r = l\,\text{sen}\phi \Rightarrow \dot r = l\cos\phi\,\dot\phi$

$$\dot\theta = \Omega$$

$$z_1 = 2l - l\cos\phi \Rightarrow \dot z = l\,\text{sen}\phi\,\dot\phi$$

$$\Rightarrow \vec{v}_{①} = l\cos\phi\,\dot\phi\,\hat r + l\,\text{sen}\phi\,\Omega\,\hat\theta + l\,\text{sen}\phi\,\dot\phi\,\hat z$$

Notar que: $\vec{v}_{①} = \vec{v}_{②}$

$\vec v_3 = \dot z_3\,\hat z$ donde $z_3 = 2l - 2l\cos\phi$

$$\Rightarrow \dot z_3 = 2l\,\text{sen}\phi\,\dot\phi$$

$$\Rightarrow T = \frac{1}{2} m' 4l^2 \text{sen}^2\phi\,\dot\phi^2 + m\left(l^2\dot\phi^2 + l^2\text{sen}^2\phi\,\Omega^2\right)$$

$$U = 2mg(2l - l\cos\phi) + m'gl(2 - 2\cos\phi) + \frac{1}{2}K(2l - 2l\cos\phi)^2$$

$$\Rightarrow L = m\left(l^2\dot\phi^2 + l^2\text{sen}^2\phi\,\Omega^2\right) + 2m'l^2\text{sen}^2\phi\,\dot\phi^2 - 2mgl(2-\cos\phi)$$
$$- 2m'gl(1-\cos\phi) + 2kl^2(1-\cos\phi)^2$$

Ecs.

$$\frac{\partial L}{\partial \phi} = 2ml^2\text{sen}\phi\cos\phi\,\Omega^2 + 4m'l^2\text{sen}\phi\cos\phi\,\dot\phi^2 - 2mgl\,\text{sen}\phi - 2m'gl\,\text{sen}\phi$$
$$+ 4kl^2(1-\cos\phi)\,\text{sen}\phi$$

$$\frac{\partial L}{\partial \dot\phi} = 2ml^2\dot\phi + 4m'l^2\text{sen}^2\phi\,\dot\phi \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial\dot\phi}\right) = 2ml^2\ddot\phi + 8m'l^2\text{sen}\phi\cos\phi\,\dot\phi^2$$
$$+ 4m'l^2\text{sen}^2\phi\,\ddot\phi$$

$$\Rightarrow \ddot\phi\left(m + 2m'\text{sen}^2\phi\right) = -m'\text{sen}(2\phi)\,\dot\phi^2 + m\,\text{sen}\phi\cos\phi\,\Omega^2 - (m+m')gl\,\text{sen}\phi$$
$$+ 2K(1-\cos\phi)\,\text{sen}\phi$$