

P4. pdq.

$$\sum_{i=0}^{n-1} x^{n-1-i} y^i = \frac{x^n - y^n}{x - y} \quad (1)$$

$$\begin{aligned} \sum_{i=0}^{n-1} x^{n-1-i} y^i &= \sum_{i=0}^{n-1} x^{n-1-i} y^i \left(\frac{x-y}{x-y} \right) = \sum_{i=0}^{n-1} \frac{x^{n-i} y^i - x^{n-i-1} y^{i+1}}{x-y} \\ &= \sum_{i=0}^{n-1} \frac{x^{n-i} y^i}{x-y} - \sum_{i=0}^{n-1} \frac{x^{n-(i+1)} y^{i+1}}{x-y} \\ &= \frac{x^n y^0}{x-y} + \sum_{i=1}^{n-1} \frac{x^{n-i} y^i}{x-y} - \sum_{i=1}^n \frac{x^{n-i} y^i}{x-y} \\ &= \frac{x^n}{x-y} + \sum_{i=1}^{n-1} \frac{x^{n-i} y^i}{x-y} - \sum_{i=1}^{n-1} \frac{x^{n-i} y^i}{x-y} - \frac{x^{n-n} y^n}{x-y} \\ &= \frac{x^n}{x-y} - \frac{y^n}{x-y} = \frac{x^n - y^n}{x-y} \end{aligned}$$