P6. pdq.

$$\sum_{k=0}^{n}(1-x)^k = \sum_{k=0}^{n}\binom{n+1}{k+1}x^k(-1)^k \tag{1}$$

Solucion El termino de la izquierda puede considerarse una suma geometrica:

$$\sum_{k=0}^{n}(1-x)^k = \frac{1-(1-x)^{n+1}}{1-(1-x)}$$

Usamos binomio de Newton sobre $(1-x)^n$

$$\begin{aligned}
\frac{1-(1-x)^{n+1}}{x} &= \frac{1-\sum_{k=0}^{n+1}\binom{n+1}{k}1^{n+1-k}x^k(-1)^k}{x} \\
&= \frac{1}{x} - \sum_{k=0}^{n+1}\binom{n+1}{k}1^{n+1-k}x^{k-1}(-1)^k \\
&= \frac{1}{x} - \sum_{k=0}^{n+1}\binom{n+1}{k}x^{k-1}(-1)^k \\
&= -\sum_{k=1}^{n+1}\binom{n+1}{k}x^{k-1}(-1)^k \\
&= -\sum_{k=0}^{n}\binom{n+1}{k+1}x^k(-1)^{k+1} = \sum_{k=0}^{n}\binom{n+1}{k+1}x^k(-1)^{k+2} \\
&= \sum_{k=0}^{n}\binom{n+1}{k+1}x^k(-1)^k
\end{aligned}$$