>

PAUTA Tarea 3

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Inicialización

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> restart;
> with(DEtools) :
> with(plots):
> with(linalg):
> with(PDEtools) :
> with(inttrans) :
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P1 i

> $p1 := diff(y(t), t\$4) - y(t) = \exp(t) + \cos(t);$

$$p1 := \frac{d^4}{dt^4}\, y(t) - y(t) = e^t + \cos(t) \tag{3.1}$$

> $solu1 := rhs(dsolve(p1, y(t)))$

$$solu1 := -\frac{1}{2}\cos(t) + \frac{1}{4}\, e^t\, t - \frac{3}{8}\, e^t - \frac{1}{4}\sin(t)\, t + \_C1\, e^t + \_C2\cos(t) + \_C3\sin(t) + \_C4\, e^{-t} \tag{3.2}$$

>

P1 ii

> $p2 := diff(y(t), t\$2) - 4\cdot diff(y(t), t\$1) + 4\cdot y(t) = (\exp(2\cdot t) + 1)\cdot(\cos(t) + 1);$

$$p2 := \frac{d^2}{dt^2}\, y(t) - 4\left(\frac{d}{dt}\, y(t)\right) + 4\, y(t) = \left(e^{2t} + 1\right)\left(\cos(t) + 1\right) \tag{4.1}$$

> $solu2 := rhs(dsolve(p2, y(t)))$

$$solu2 := e^{2t}\, \_C2 + e^{2t}\, t\, \_C1 + \frac{1}{2}\, t^2\, e^{2t} - e^{2t}\cos(t) + \frac{1}{4} + \frac{3}{25}\cos(t) - \frac{4}{25}\sin(t) \tag{4.2}$$

>

> ?simplify

P1 iii

> $p3 := (1 + t^2)\cdot diff(y(t), t\$2) + 4\cdot t\cdot diff(y(t), t) + (2 + w\cdot(1 + t^2))\cdot y(t) = t\cdot\cos(\mathrm{sqrt}(w)\cdot t)$
;

(5.1)

$$p3 := (1+t^2)\left(\frac{d^2}{dt^2}y(t)\right) + 4t\left(\frac{d}{dt}y(t)\right) + (2+w(1+t^2))\,y(t) = t\cos(\sqrt{w}\,t) \quad (5.1)$$

> $cambio := y(t) = \frac{z(t)}{(1+t^2)};$

$$cambio := y(t) = \frac{z(t)}{1+t^2} \quad (5.2)$$

> $p32 := dchange(\ cambio, p3, [z(t)]\ );$

$$p32 := (1+t^2)\left(\frac{\frac{d^2}{dt^2}z(t)}{1+t^2} - \frac{4\left(\frac{d}{dt}z(t)\right)t}{(1+t^2)^2} + \frac{8\,z(t)\,t^2}{(1+t^2)^3} - \frac{2\,z(t)}{(1+t^2)^2}\right) + 4t\left(\frac{\frac{d}{dt}z(t)}{1+t^2} - \frac{2\,z(t)\,t}{(1+t^2)^2}\right) + \frac{(2+w(1+t^2))\,z(t)}{1+t^2} = t\cos(\sqrt{w}\,t) \quad (5.3)$$

> $p32 := simplify(\ isolate(\ p32, diff(z(t), t)\ ), symbolic, t);$

$$p32 := \left(\frac{d^2}{dt^2}z(t)\right)t^2 + \frac{d^2}{dt^2}z(t) = \left(t\cos(\sqrt{w}\,t) - \frac{(2+w(1+t^2))\,z(t)}{1+t^2}\right)(1+t^2) + 2\,z(t) \quad (5.4)$$

> $solu1 := rhs(dsolve(p323, z(t)))$

$$solu1 := \sin(\sqrt{w}\,t)\,\_C2 + \cos(\sqrt{w}\,t)\,\_C1 + \frac{1}{4}\,\frac{t\left(\cos(\sqrt{w}\,t)\sqrt{w} + \sin(\sqrt{w}\,t)\,w\,t\right)}{w^{3/2}} \quad (5.5)$$

> $solution := \frac{solu1}{1+t^2};$

$$solution := \frac{\sin(\sqrt{w}\,t)\,\_C2 + \cos(\sqrt{w}\,t)\,\_C1 + \frac{1}{4}\,\frac{t\left(\cos(\sqrt{w}\,t)\sqrt{w} + \sin(\sqrt{w}\,t)\,w\,t\right)}{w^{3/2}}}{1+t^2} \quad (5.6)$$

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P1 iv

> $p4 := t^2 \cdot diff(y(t), t\$2) - 3 \cdot t \cdot diff(y(t), t) + 4 \cdot y(t) = \left(\frac{1+\ln(t)+2\cdot(\ln(t))^3}{1+2\cdot(\ln(t))^2}\right)\cdot t^2;$

$$p4 := t^2\left(\frac{d^2}{dt^2}y(t)\right) - 3t\left(\frac{d}{dt}y(t)\right) + 4\,y(t) = \frac{(1+\ln(t)+2\ln(t)^3)\,t^2}{1+2\ln(t)^2} \quad (6.1)$$

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> $solu1 := rhs(dsolve(p4, y(t)))$

$$solu1 := t^2\,\_C2 + t^2\ln(t)\,\_C1 \quad (6.2)$$

$$+\frac{1}{12}t^2\left(2\ln(t)^3-3\ln\left(1+2\ln(t)^2\right)+6\ln(t)\sqrt{2}\arctan\left(\sqrt{2}\ln(t)\right)\right)$$

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P2 i

> p5 := diff(y(x), x$2) − 3·diff(y(x), x) + 2·y(x) = exp(3·x)/(1 + exp(x));

$$p5 := \frac{d^2}{dx^2}y(x)-3\left(\frac{d}{dx}y(x)\right)+2y(x)=\frac{e^{3x}}{1+e^x} \quad (7.1)$$

> solu1 := rhs(dsolve(p5, y(x)))

$$solu1 := \left(\ln\left(1+e^x\right)\left(1+e^x\right)-e^x-1+e^x\_C1+\_C2\right)e^x \quad (7.2)$$

>

P2 ii

> p6 := 4·diff(y(x), x$2) − 4·diff(y(x), x) + y(x) = exp(x/2)/sqrt(1 − x^2);

$$p6 := 4\left(\frac{d^2}{dx^2}y(x)\right)-4\left(\frac{d}{dx}y(x)\right)+y(x)=\frac{e^{\frac{1}{2}x}}{\sqrt{1-x^2}} \quad (8.1)$$

> solu1 := rhs(dsolve(p6, y(x)))

$$solu1 := e^{\frac{1}{2}x}\_C2+e^{\frac{1}{2}x}x\_C1-\frac{1}{4}\frac{e^{\frac{1}{2}x}\left(-1+x^2-\arcsin(x)\,x\sqrt{1-x^2}\right)}{\sqrt{1-x^2}} \quad (8.2)$$

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P2 iii

> p7 := diff(y(x), x$3) + 4·diff(y(x), x$2) = sec(2·x);

$$p7 := \frac{d^3}{dx^3}y(x)+4\left(\frac{d^2}{dx^2}y(x)\right)=\sec(2x) \quad (9.1)$$

> solu1 := rhs(dsolve(p7, y(x)))

$$solu1 := \iint e^{-4x}\left(\int\frac{e^{4x}}{\cos(2x)}\,dx+\_C1\right)dx\,dx+\_C2\,x+\_C3 \quad (9.2)$$

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P2 iv

> p8 := diff(y(x), x$2) − 4·diff(y(x), x) + 4·y(x) = (12·x^2 − 6·x)·exp(2·x);

$$p8 := \frac{d^2}{dx^2} y(x) - 4\left(\frac{d}{dx} y(x)\right) + 4 y(x) = (12 x^2 - 6 x) e^{2x} \quad (10.1)$$

> $solu1 := rhs(dsolve(p8, y(x)))$

$$solu1 := e^{2x} \_C2 + e^{2x} x \_C1 + x^3 (-1 + x) e^{2x} \quad (10.2)$$

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P3 i

> $invlaplace\left(\frac{s}{((s^2 + a^2) \cdot (s^2 + b^2))}, s, t\right);$

$$\frac{\cos(b\,t) - \cos(a\,t)}{a^2 - b^2} \quad (11.1)$$

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P3 ii

> $invlaplace\left(\frac{5 \cdot s + 3}{((s-1) \cdot (s^2 + 2 \cdot s + 5))}, s, t\right);$

$$e^t + \frac{1}{2} (-2 \cos(2\,t) + 3 \sin(2\,t)) e^{-t} \quad (12.1)$$

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P3 iii

> $invlaplace\left(\frac{1}{s^2 + s - 20}, s, t\right);$

$$-\frac{1}{9} e^{-5t} + \frac{1}{9} e^{4t} \quad (13.1)$$

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P3 iv

> $invlaplace\left(\frac{s-1}{s^2 \cdot (s^2 + 1)}, s, t\right);$

$$1 - \cos(t) + \sin(t) - t \quad (14.1)$$

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P3 v

> $laplace(\exp(t) \cdot \cos(3 \cdot t)^2, t, s);$

$$\frac{s^2 - 2s + 19}{(s^2 - 2s + 37)(s - 1)} \tag{15.1}$$

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P3 vi

> $a := laplace(t^2 \cdot \cos(t)^2, t, s);$

$$a := \frac{1}{2(s + 2\,\mathrm{I})^3} + \frac{1}{s^3} + \frac{1}{2(s - 2\,\mathrm{I})^3} \tag{16.1}$$

> $simplify(a)$

$$\frac{2(s^6 + 24 s^2 + 32)}{(s + 2\,\mathrm{I})^3 s^3 (s - 2\,\mathrm{I})^3} \tag{16.2}$$

> $simplify((s + 2 \cdot I) \cdot (s - 2 \cdot I));$

$$s^2 + 4 \tag{16.3}$$

> $a := \frac{2(s^6 + 24 s^2 + 32)}{(s^2 + 4)^3 s^3}$

$$a := \frac{2 s^6 + 48 s^2 + 64}{(s^2 + 4)^3 s^3} \tag{16.4}$$

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