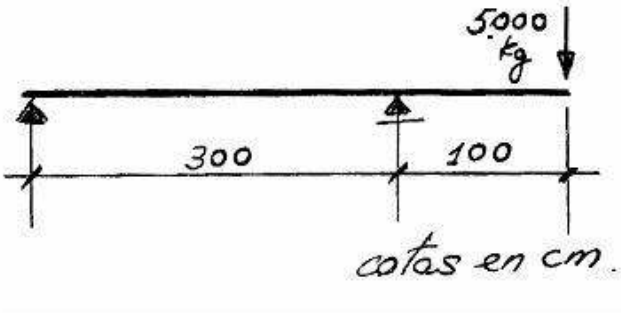
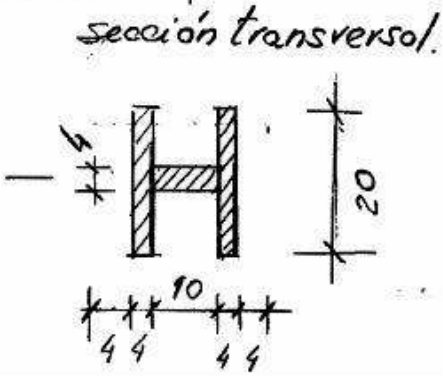
ME46A
Clase auxiliar 5 (2 de Mayo de 2007)

Problema N° 1

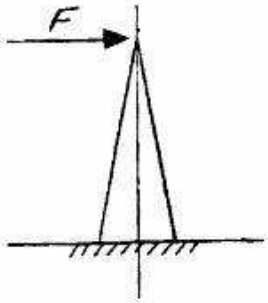
Calcule el máximo esfuerzo de compresión en la viga del esquema





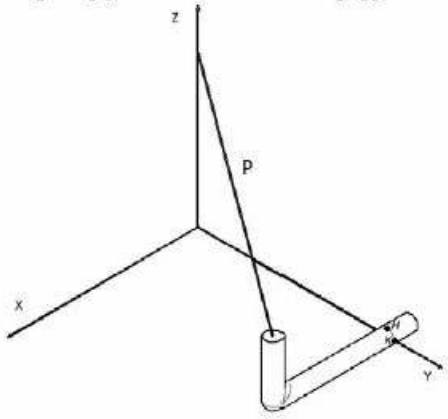
Problema N° 2

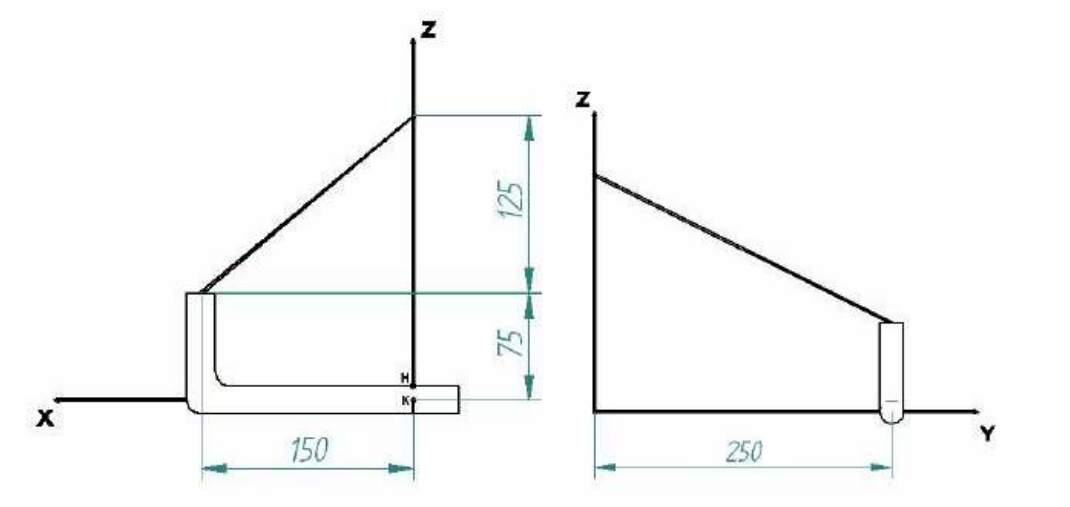
Determine en qué sección se romperá por esfuerzos de flexión la estructura cónica de la figura. No considere los esfuerzos de corte ni el preso propio del cono.
El momento de inercia de una sección circular vale $\frac{\pi D^4}{64}$



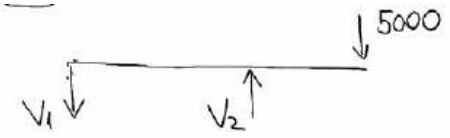
Problema N° 3

P2.-Para el sistema de la figura se pide determinar los esfuerzos de compresión y tracción en los puntos H y K. El diámetro D = 20 [mm] y la fuerza P = 500 [Kg]




Z
125
75
H
K
X
150
Z
Y
250

P1.

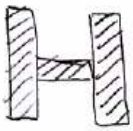


$\Sigma F_y = 0 \Rightarrow V_2 - V_1 = 5000$

$\Sigma M_z = 0 \Rightarrow 300\,V_2 - 400 \cdot 5000 = 0$

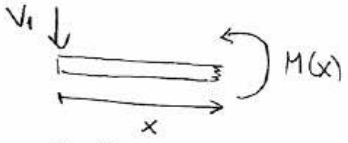
$$V_2 = \frac{20000}{3}$$

$$V_1 = \frac{5000}{3}$$



$$I = \frac{1}{12}\left(2 \cdot 4 \cdot 20^3 + 10 \cdot 4^3\right)$$

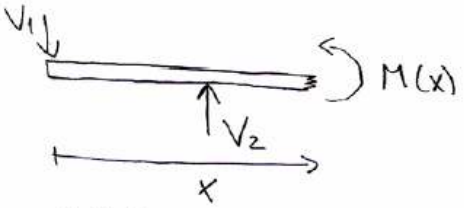
$$I = 5387\ [cm^4]$$



$x \in [0, 300)$

$$M(x) + x\,V_1 = 0$$

$$M(x) = -\frac{5000}{3}\,x$$

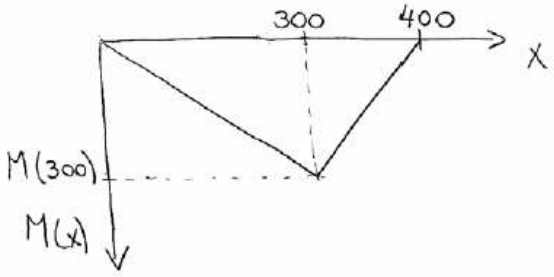


$x \in [300, 400)$

$$M(x) + x V_1 - (x-300)\,V_2 = 0$$

$$M(x) = x(V_2 - V_1) - 300\,V_2$$

$$M(x) = 5000\,x - 2000000$$



$$M_{max} = |M(300)| = |5000 \cdot 300 - 2000000|$$

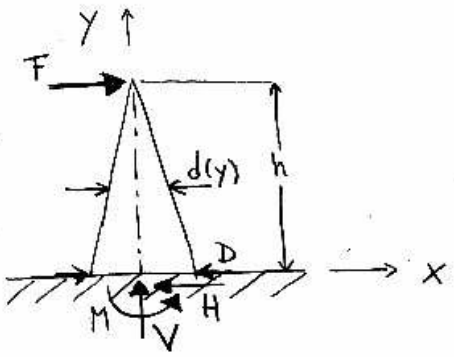
$$M_{max} = 500000\ [kg \cdot cm]$$

$$\Rightarrow \quad \sigma_{max} = -\frac{M_{max} \cdot y_{max}}{I}$$

$$\sigma_{max} = -\frac{500000 \cdot 10}{5387}$$

$$\boxed{\sigma_{max} = -928{,}22\ kg/cm^2}$$

P2.



$\Sigma F_x = 0 \Rightarrow F - H = 0 \Rightarrow \boxed{H = F}$

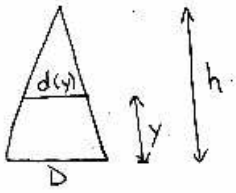
$\Sigma F_y = 0 \Rightarrow \boxed{V = 0}$

$\Sigma M_z = 0 \Rightarrow M - h \cdot F \Rightarrow \boxed{M = h \cdot F}$

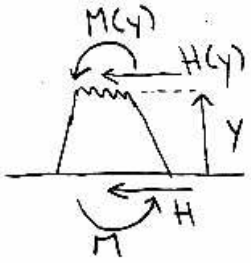
- Por teorema de Thales:

$$\frac{D}{h} = \frac{d(y)}{h-y}$$

$$\boxed{d(y) = D\frac{(h-y)}{h}}$$



- Calculo de M(y):



$$\Rightarrow \quad M + M(y) - y \cdot H = 0$$

$$M(y) = Hy - M$$

$$\Rightarrow \boxed{M(y) = F(y-h)}$$

· Esfuerzo máximo de flexión :

$$\sigma_N = \frac{M \cdot c}{I} \qquad M = F(y-h)$$

$$c = \frac{d(y)}{2} = \frac{D(h-y)}{2h}$$

$$I = \frac{\pi}{64} d(y)^4 = \frac{\pi}{64}\left(\frac{D(h-y)}{h}\right)^4$$

$$\Rightarrow \sigma_N = \frac{F(y-h) \cdot \frac{D(h-y)}{2h}}{\frac{\pi}{64}\left(\frac{D(h-y)}{h}\right)^4}$$

$$\sigma_N = \frac{32\,F(y-h)}{\pi\left(\frac{D(h-y)}{h}\right)^3} = \frac{32\,F \cdot h^3}{\pi D^3 (h-y)^2}$$

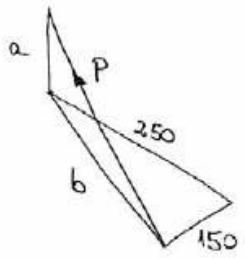
$$\boxed{\sigma_N = \frac{32\,F \cdot h^3}{\pi D^3} \frac{1}{(h-y)^2}} \qquad y \in [0, h]$$

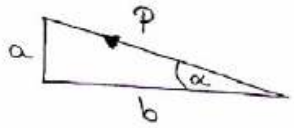
Cuando $y \to h \Rightarrow \sigma_N \to \infty$

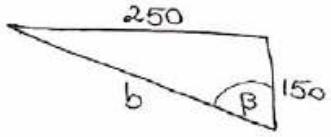
$\therefore \sigma_{N\,máximo} = \sigma_N(y = h - \varepsilon)$ ε muy pequeño

==> *El cono se rompe por flexión en y=h-e*

P3.







$b = \sqrt{150^2+250^2} \Rightarrow b = 291,5 \text{ mm}$

$a = 125 \text{ mm}$

$\alpha = \text{arctg}\left(\frac{a}{b}\right) \Rightarrow \alpha = 23,21°$

$\beta = \text{arctg}\left(\frac{250}{150}\right) \Rightarrow \beta = 59,04°$

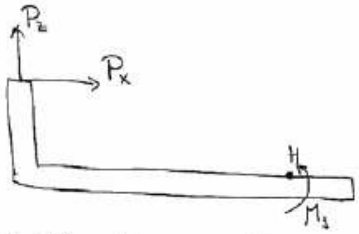
Componentes de P:

$P_x = -P \cdot \cos\alpha \cdot \cos\beta \, \hat{x} \Rightarrow P_x = -236,4 \, \hat{x} \, [Kg]$

$P_y = -P \cdot \cos\alpha \cdot \text{sen}\,\beta \, \hat{y} \Rightarrow P_y = -394,1 \, \hat{y} \, [Kg]$

$P_z = P \cdot \text{sen}\,\alpha \, \hat{z} \Rightarrow P_z = 197,0 \, \hat{z} \, [Kg]$

Plano X - Z



$\Sigma M_y = 0 \Rightarrow \quad M_1 = 75\,P_x + 150\,P_z$

$M_1 = 47286{,}6\ [Kg \cdot mm]$

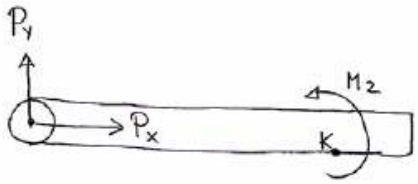
$\sigma_{fH} = -\dfrac{M_1 \cdot R}{\frac{\pi \cdot R^4}{4}} \Rightarrow \quad \sigma_{fH} = -60{,}21\ [Kg/mm^2]$ Esfuerzo flexión en H (compresión)

$\sigma_{AH} = -\dfrac{P_x}{\pi R^2} \Rightarrow \quad \sigma_{AH} = -0{,}753\ [Kg/mm^2]$ Esfuerzo axial en H (compresión)

$\sigma_H = \sigma_{fH} + \sigma_{AH} = -60{,}21 - 0{,}753$

$\Rightarrow \boxed{\sigma_H = -60{,}96\ [Kg/mm^2]}$ Esfuerzo de compresión en H

Plano X-Y



$\Sigma M_z = 0 \Rightarrow M_z = 150\, P_y$

$M_z = 59108{,}3\ [Kg \cdot mm]$

$\sigma_{fK} = -\dfrac{M_z \cdot (-R)}{\frac{\pi \cdot R^4}{4}} \Rightarrow \sigma_{fK} = 75{,}26\ [Kg/mm^2]$ Esfuerzo flexión en K (tracción)

$\sigma_{AK} = -\dfrac{P_x}{\pi \cdot R^2} \Rightarrow \sigma_{AK} = -0{,}753\ [Kg/mm^2]$ Esfuerzo axial en K (compresión)

$\sigma_K = \sigma_{fK} + \sigma_{AK} = 75{,}26 - 0{,}753$

$\rightarrow \boxed{\sigma_K = 74{,}51\ [Kg/mm^2]}$ Esfuerzo de tracción en K.