Flexura de la Litosfera

A

Solución general de la ecuación de flexura unidimensional

$$D\frac{d^4w}{dx^4}+P\frac{d^2w}{dx^2}=q(x)\Leftrightarrow$$

$$D\frac{d^4w}{dx^4}+P\frac{d^2w}{dx^2}+\Delta rgw=0;\, w(x)=w_0e^{lx}\Rightarrow$$

$$w_0e^{lx}\left(Dl^4+Pl^2+\Delta rg\right)=0;\, w_0\neq 0\Rightarrow Dl^4+Pl^2+\Delta rg=0\Rightarrow$$

$$l^2=\frac{-P\pm\sqrt{P^2-4\Delta rgD}}{2D}=-\frac{P}{2D}\left(1\pm\sqrt{1-\frac{4\Delta rgD}{P^2}}\right);\, Sea\quad h\equiv\frac{4\Delta rgD}{P^2}$$

$$l^2=-\frac{P}{2D}\left(1\pm\sqrt{1-h}\right)\Rightarrow\; l=\pm i\sqrt{\frac{P}{2D}\left(1\pm\sqrt{1-h}\right)}$$

$$Caso\quad h\leq 1$$

$$Sea\quad m_{\pm}=\sqrt{\frac{P}{2D}\left(1\pm\sqrt{1-h}\right)}\in\Re_{+}\Rightarrow$$

$$w(x)=a\cdot\cos(m_{+}x)+b\cdot sen(m_{+}x)+c\cdot\cos(m_{-}x)+d\cdot sen(m_{-}x)$$

$$Caso\quad h>1$$

$$l^2=-\frac{P}{2D}\left(1\pm\sqrt{1-h}\right)=-\frac{P}{2D}\left(1\pm i\sqrt{h-1}\right);\quad Sea\quad z_{\pm}=1\pm i\sqrt{h-1}\Rightarrow$$

$$\left|z_{\pm}\right|=\sqrt{1+h-1}=\sqrt{h};\quad z_{\pm}=\left|z_{\pm}\right|\cdot e^{iq_{\pm}}=\left|z_{\pm}\right|\left(\cos(q_{\pm})+i\cdot sen(q_{\pm})\right)\Rightarrow$$

$$\cos(q_{\pm})=\frac{1}{\sqrt{h}};q_{+}>0;q_{-}<0\Rightarrow\quad q_{\pm}=\pm\arccos\left(\frac{1}{\sqrt{h}}\right)$$

$$l^2=-\frac{P}{2D}\sqrt{h}\cdot e^{iq_{\pm}}\Rightarrow\; l=\pm i\sqrt{\frac{P}{2D}\sqrt{h}}\cdot e^{i\frac{q_{\pm}}{2}}$$

$$\cos^2\left(\pm\tfrac{1}{2}\arccos(x)\right)-sen^2\left(\pm\tfrac{1}{2}\arccos(x)\right)=\cos\left(\arccos(x)\right)=2\cos^2\left(\pm\tfrac{1}{2}\arccos(x)\right)-1$$

$$=1-2sen^2\left(\pm\tfrac{1}{2}\arccos(x)\right)=x\Rightarrow$$

$$\cos\left(\pm\tfrac{1}{2}\arccos(x)\right)=\sqrt{\frac{1+x}{2}};\, sen\left(\pm\tfrac{1}{2}\arccos(x)\right)=\pm\sqrt{\frac{1-x}{2}}$$

$$e^{i\frac{q_{\pm}}{2}}=\cos\left(\frac{q_{\pm}}{2}\right)+i\cdot sen\left(\frac{q_{\pm}}{2}\right)=\sqrt{\frac{1+\frac{1}{\sqrt{h}}}{2}}\pm i\cdot\sqrt{\frac{1-\frac{1}{\sqrt{h}}}{2}}=\frac{1}{\sqrt{2}}h^{-\frac{1}{4}}\left(\sqrt{\sqrt{h}+1}\pm i\sqrt{\sqrt{h}-1}\right)$$

$$Sean\quad m=\frac{1}{2}\sqrt{\frac{P}{D}\left(\sqrt{h}+1\right)},n=\frac{1}{2}\sqrt{\frac{P}{D}\left(\sqrt{h}-1\right)}\in\Re_{+}\Rightarrow$$

$$l=\pm n\pm i\cdot m\Rightarrow$$

$$w(x)=e^{n\cdot x}\left(a\cdot\cos(m\cdot x)+b\cdot sen(m\cdot x)\right)+e^{-n\cdot x}\left(c\cdot\cos(m\cdot x)+d\cdot sen(m\cdot x)\right)$$

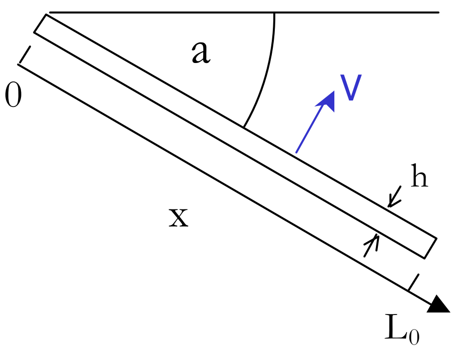
$$\lim_{P \to 0^+} n = \lim_{P \to 0^+} m = \left(\frac{\Delta r \cdot g}{4D} \right)^{\frac{1}{4}}$$, igual que en el caso P=0.

B

Slab oceánico de largo "L_0" subductando en un ángulo "**a**". Suponiendo equilibrio de fuerzas en la dirección perpendicular al slab, calcular el estado de deformación plana del mismo. Suponer que P<0 (extensión por Slab pull).

<u>Sol</u>:

Momento flector "M" punto a punto:



$$dM = \Delta r \cdot gh \cos(a)(L_0 - x)dx \Rightarrow$$
$$M(x) = -\tfrac{1}{2}\Delta r \cdot gh \cos(a)(L_0 - x)^2$$
$$V(x) = \frac{dM}{dx}(x) = \Delta r \cdot gh \cos(a)(L_0 - x)$$
$$\frac{d^2w}{dx^2}(x) = \frac{-M(x=0)}{D}; \frac{d^3w}{dx^3}(x) = \frac{-V(x=0)}{D}$$

C.B.:

$$w(x=0)=0; \quad \frac{dw}{dx}(x=0)=0;$$
$$\frac{d^2w}{dx^2}(x=0) = \frac{\Delta r}{2D} \cdot gh\cos(a)L_0^{\,2}; \quad \frac{d^3w}{dx^3}(x=0) = -\frac{\Delta r}{D} \cdot gh\cos(a)L_0$$

Como P<0, Sea $P_0=-P$, "mu" y "nu" intercambian valores (conservan sus roles):

$$m = \tfrac{1}{2}\sqrt{\tfrac{P_0}{D}\left(\sqrt{h}-1\right)}, n = \tfrac{1}{2}\sqrt{\tfrac{P_0}{D}\left(\sqrt{h}+1\right)}$$

Del problema anterior,

$$w(x) = e^{n \cdot x}\left(a \cdot \cos(m \cdot x) + b \cdot sen(m \cdot x)\right) + e^{-n \cdot x}\left(c \cdot \cos(m \cdot x) + d \cdot sen(m \cdot x)\right) \Rightarrow$$
$$\frac{dw}{dx}(x) = e^{n \cdot x}\left(\cos(m \cdot x)(an + bm) + sen(m \cdot x)(bn - am)\right) +$$
$$e^{-n \cdot x}\left(\cos(m \cdot x)(-cn + dm) - sen(m \cdot x)(dn + cm)\right) \Rightarrow$$
$$\frac{d^2w}{dx^2}(x) = e^{n \cdot x}\left(\cos(m \cdot x)\left(a\left(n^2 - m^2\right) + 2bmn\right) + sen(m \cdot x)\left(b\left(n^2 - m^2\right) - 2amn\right)\right) +$$
$$e^{-n \cdot x}\left(\cos(m \cdot x)\left(c\left(n^2 - m^2\right) - 2dmn\right) + sen(m \cdot x)\left(d\left(n^2 - m^2\right) + 2cmn\right)\right) \Rightarrow$$
$$\frac{d^3w}{dx^3}(x) = e^{n \cdot x}\begin{pmatrix} \cos(m \cdot x)\left(a\left(n^3 - 3m^2n\right) + b\left(-m^3 + 3mn^2\right)\right) + \\ sen(m \cdot x)\left(b\left(n^3 - 3m^2n\right) + a\left(m^3 - 3mn^2\right)\right) \end{pmatrix} +$$
$$e^{-n \cdot x}\left(\cos(m \cdot x)\left(c\left(-n^3 + 3m^2n\right) + d\left(-m^3 + 3mn^2\right)\right) + sen(m \cdot x)\left(d\left(-n^3 + 3m^2n\right) + c\left(m^3 - 3mn^2\right)\right)\right)$$

Evaluando estas funciones en x=0 y las C.B., resulta:

$$a + c = 0$$
$$(a-c)n + (b+d)m = 0$$
$$(a+c)(n^2 - m^2) + 2(b+d)mn = \frac{\Delta r}{2D} \cdot gh\cos(a)L_0^2$$
$$(n^2 - m^2)((a-c)n + (b+d)m) + 2nm(m(c-a) + n(b+d)) = \frac{\Delta r}{D} \cdot gh\cos(a)L_0$$

Resolviendo este sistema lineal de ecuaciones, resulta:

$$m = \tfrac{1}{2}\sqrt{\tfrac{P_0}{D}\left(\sqrt{h}-1\right)}, n = \tfrac{1}{2}\sqrt{\tfrac{P_0}{D}\left(\sqrt{h}+1\right)}$$

$$a = -c = \frac{\Delta r\, ghL_0 \cos(a)}{2P_0 n\sqrt{h}}$$
$$b = \frac{1}{2P_0}\Delta r\, ghL_0 \cos(a)\left(\frac{L_0}{\sqrt{h}-1} - \frac{1}{m\sqrt{h}}\right)$$
$$d = -\frac{1}{2P_0}\Delta r\, ghL_0 \cos(a)\left(\frac{L_0}{\sqrt{h}-1} + \frac{1}{m\sqrt{h}}\right)$$

$$w(x) = e^{n \cdot x}(a \cdot \cos(m \cdot x) + b \cdot sen(m \cdot x)) + e^{-n \cdot x}(c \cdot \cos(m \cdot x) + d \cdot sen(m \cdot x))$$