A Decision-Support System that Helps Retailers Decide Order Quantities and Markdowns for Fashion Goods

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We developed MARK, a stochastic dynamic-programming model-based decision-support system, specifically to help retail-store buyers of fashion goods decide on optimal merchandise order quantities and markdown prices. We implemented MARK as an interactive software program that runs on users' personal computers and can be integrated with the retailers' point-of-sale (POS) and other information systems. As we show in an actual-case illustration, MARK improved profitability and managers' understanding of the decision problems. MARK also provides valuable insights into the interplay between order-quantity and dynamic-pricing decisions. For example, higher markdown percentages do not necessarily mean lower profits.

The bottom-line profitability of fashion-goods retailers, for example, department stores, specialty stores, and catalog retailers, is affected by the inventory-order-quantity and pricing decisions buyers and merchandising managers make. Fashion (or seasonal-style) goods have short selling seasons, uncertain and nonstationary demand (price response) functions, and little salvage value at the end of their selling seasons. Ideally buyers would order inventories of such goods in response to observed demand. Unfortunately, in many instances, long delivery lead times or other supply constraints rule out the possibility of reorders and quick

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replenishment of inventory during the season [Hammond 1990; Iyer and Bergen 1997]. Consequently buyers often must decide and order inventory for the entire season well before the item goes on sale, relying on uncertain forecasts of demand at planned retail prices [Eppen and Iyer 1997].

In practice, lacking quick resupply options, buyers tend to buy stocks of merchandise according to their optimistic forecasts while planning to mark down potential excess inventory to stimulate demand and sell out the excess by the end of the season. (Here, we are referring specifically to clearance or permanent markdowns and not temporary markdowns that are offered in a special sale period, for example, Mother's Day, after which the retail price returns to its original level.) Often demand falls short of projections, and the buyer must take the markdowns necessary to clear out the stocks as profitably as possible. Like the original order-quantity decision, determining the optimal timing and magnitude of sequential markdowns is complicated by uncertain and nonstationary demand functions and the costs associated with selling out the inventory too quickly or too late.

These decision challenges have increased during the last decade as demand unpredictability has grown because product variety has exploded on the supply side and consumer tastes have diversified on the demand side [Fisher et al. 1994; Pashigian 1988; Pashigian and Bowen 1991]. This is reflected in the steep rise in the ratios of markdown amounts (dollars foregone on units sold at the reduced price) to net sales in US department and specialty stores since the 1960s. On average, markdown amounts have grown from around six percent of net sales in the mid-1960s to over 26 percent by the mid-1990s with markdown percentages as high as 30 to 40 percent in apparel and dress departments [Fisher et al. 1994; Levy and Weitz 1995]. The surge in clearance sales by both large and small retailers has received much press attention [Associated Press 1997]. However, while the discounts boost sales, they also erode profits [*Wall Street Journal* 1997]. Most retailers know they need better decision aids to cope with such unpredictable selling situations [Pearson 1994].

We developed MARK, a computer model-based decision-support system (DSS), that can help fashion-retail buyers make more judicious opening buy and markdown decisions that account for demand-function uncertainty and nonstationarity. Typically, store policies specify a set of discrete price-off levels (price points) at which an item can be sold. For example, the store policy may be that the item can be sold at the full or regular (zero percent off) price or at markdowns of either 25 or 50 percent off the regular selling price during the season. Considering these permissible price points, MARK can be used to decide the optimal order quantity along with the markdown plan and budget for the upcoming season. The markdown plan and budget are important inputs for planning merchandise sales and for negotiating with vendors for markdown allowances [Levy and Weitz 1995; *Wall Street Journal* 1998]. Further, during the selling season buyers can use MARK at the start of each period to review and determine the optimal markdown strategy over the remaining season based on revised demand estimates and the current inventory position. In deriving these solutions MARK can take into account a number of complicating supply-side factors, for example, inventory carrying costs, stockout costs, costs of sales returns, and anticipated costs of shrinkage (losses from theft or damage). In this article, we focus on MARK's use to address preseason order-quantity and markdown-planning issues.

Several recent papers concern the problem of marking down style goods. Gallego and Van Ryzin [1994] formulated a continuous-time model involving a current price-dependent Poisson demand-arrival process and applied intensity-control theory to determine the optimal price path as a function of the stock level and length of the horizon. Feng and Gallego [1995] used a similar model and analytical approach to determine the optimal timing and duration of a single price change (markdown or markup). Bitran and Mondschein [1997] developed a continuous-time model in which a seller faces a nonhomogeneous Poisson arrival of customers whose arrival rate varies with the way a store conducts business over the season, combined with a Weibull probability distribution of reservation prices. They used this model to derive and compare optimal continuous-pricing policies as functions of time and inventory with more realistic periodic pricing polices. Smith and Achabal [1998] developed and investigated optimal clearance prices and end-of-season inventory management policies that take into account the impact of reduced assortment and seasonal changes on sales rates. All of these writers focused on the form of the optimal pricing policies for selling out a fixed amount of inventory of a style good. MARK is an operational DSS that can be used to address this within-season problem of optimal markdown pricing, and also the preseason problem of determining the optimal order quantity in conjunction with any planned markdown strategies.

Model Formulations

MARK comprises two stochastic dynamic optimization modules with Module 1 nested in Module 2. Following Bellman [1957] and Howard [1960] in Module 1 of MARK, we treat the retail buyer's dynamic-pricing problem at the start of any period n of an N-period horizon as determining the pricing policy (sequence of prices) that maximizes the (discounted) sum of period n's expected profit and the maximized expected total profit derivable over the remainder of the horizon $n+1, \ldots, N$. This is a stochastic dynamic programming problem. We call the solution to this problem when the selling price is free to go up after being marked down as the optimal unconstrained-pricing (*OU*) policy, while the optimal markdown-only (*OM*) policy is the solution subject to the constraint $P_n \leq P_{n-1}$ for any period $n = 2, \ldots, N$ where P_{n-1} is the price applied in period $(n - 1)$. Once the probability distributions of demand at different permissible price levels in each period are calibrated (see below), MARK evaluates the expected profit at each permissible price in a period by means of numerical integration. The stochastic dynamic-programming problem is formulated and solved as shown in the appendix.

Module 2 of MARK enables the determination of the optimal total inventory (*OI*) in conjunction with the dynamically optimal (*OU* or *OM*) pricing plan. Specifically, the optimize-inventory—optimal unconstrained pricing (*OI-OU*) process nests the Module 1 model. The latter is iteratively solved at each point of a discretized range of order quantity to determine the corresponding *OU* policy and expected profit outcomes. The *OI-OU* process then determines the cumulative expected profit-maximizing order quantity from the given range. In a similar way, the optimize-inventory—optimal-markdown-only (*OI-OM*) process of Module 2 solves the optimal-order-quantity problem subject to the constraint that price can only be marked down. In addition, the optimal order quantity corresponding to any fixed pricing policy can be determined by a third optimize-inventory—given-prices (*OI-GP*) process of Module 2. MARK has the capability to derive these solutions taking into account the vendor's single price or quantity-discount schedule as well as any fixed merchandise procurement budget constraint.

The optimization models utilize measurements of the probabilistic period-by-period demand functions. We conceptualize the demand in any period as a multiplicative function of the following influences:

(1) The seasonal effect, given by the fraction of the estimated total season demand or sales potential that is likely to materialize in that period.
(2) The price applied in that period.
(3) A random disturbance representing the net effects of other possible influences on demand, such as the effects of current prices of competitors and other goods or unpredictable weather. We assume that these period-specific random disturbances are independent of the price charged in that period and intertemporally independent. The detailed demand function specification is shown in the Appendix.

Probabilistic Demand Model Calibration

In general, forecasting fashion demand well in advance of the selling season is largely a subjective process because little objective data exists beyond historical patterns of sales of similar styles. Relying on such data alone is often impractical and unwise because consumers' tastes can change significantly from one season to the next. Also past sales data on similar items are typically inadequate for estimating the new item's potentially time-varying price elasticities for several reasons:

(1) For any time period, there is usually no objective information about what sales would have been at several different price levels other than the one actually charged.
(2) Recorded sales are less than actual demand when initial order quantities are inadequate and there is no scope for replenishment.
(3) Retailers tend to report aggregate unit sales from different price levels [Smith and Achabal 1998].

To develop total-season and period-by-period forecasts and merchandise plans, experienced buyers usually gather additional information from such fashion barometers as *Women's Wear Daily* and *California Apparel News*, from trade associations (such as the National Retail Federation), and from conferences with vendors, fashion coordinators, and so forth. Given baseline seasonal-demand estimates from historical data, the subjective judgements of informed buyers can be used to calibrate the probability density functions of the demand distributions at various prices [Lodish 1980]. Lilien, Kotler, and Moorthy [1992] provide a good review of the use of managers' judgments in calibrating marketing-decision models.

MARK's interactive software program offers two approaches to demand-function calibration: a direct approach and a demand model-based approach. In the direct approach, buyers see graphical displays of the category's past seasons' sales and prices and growth trends. After they study these data, they are asked to enter three estimates of demand (the minimum or most pessimistic level, the modal or most likely level, and the maximum or most optimistic level) at each permissible price level in each period. These estimates can be used to parameterize triangular approximations of the probability distributions of demand. The direct approach, however, can become cognitively difficult and tedious as the number of price levels and periods in an application increase. An alternative approach is to elicit the essential information needed to calibrate the multiplicative demand model specification (Appendix) in the following steps:

Buyers must often order inventory for the entire season. While discounts boost sales, they erode profits. Optimizing the timing and magnitude of markdowns can compensate for buying errors.

Step 1: The buyer enters the most likely estimate of the total season demand at full price. We denote this parameter as M.
Step 2: The buyer enters the minimum or low and maximum or high estimates of M such that the true value has a 95-percent chance of falling within the range of these low-high estimates.
Step 3: The buyer enters the expected percent distribution of total demand by month for the style classification. In most cases, this information is not subjective but based on historical data.

The kinds of data elicited in Steps 1 and 3 are quite familiar to buyers, because they are common inputs to seasonal-merchandise-sales planning at most retailers [Levy and Weitz 1995]. With a little explanation, a buyer can also grasp the essence of the idea of placing a 95-percent confidence interval around his or her estimates. The data effectively determine the most likely demand per period (*sell-through*) [Mason and Mayer 1987] at the full price, assuming no uncertainty.

The next two steps elicit data from the buyers that lead to estimates of the period-specific disturbance variances and the price-off elasticities (denoted $\gamma(n)$ for $n = 1, \ldots N$):
Step 4: Given the period-by-period estimates of modal demands at full price, the buyer enters the minimum or low and maximum or high estimates of these demands such that there is only a five-percent chance that the demands will fall outside these ranges. With these new inputs, MARK can derive estimates of the means and variances of the period-specific disturbance terms.

Step 5: The buyers enters his or her perception of the sensitivity of demand with respect to price reductions from full price by responding to the following question. "What do you feel will be the percentage increase from the most likely demand at full price in period n if the price were reduced by 25 percent?" (This question may be repeated for another price-off level, for example, 50 percent off, to get an additional data point.) Given these inputs, deriving estimates of each period's price-off elasticity is straightforward.

An Application

This application took place at a local store of a large US retail chain that has decentralized markdown-planning responsibility. A group of four executives at this store (the store manager, the menswear department manager, and two assistant managers) were reviewing the merchandise plan for a new style of men's walking shorts to be sold over the upcoming eight-month (January—August) spring season. The planned full selling price was $19.99. The set of permissible levels for the item's selling price in any month included the full price of $19.99 (zero percent off) and reduced prices of $14.99 (25 percent off), $9.99 (50 percent off), and $5.99 (70 percent off). Based on past store-level sales data and the latest information on men's fashion trends, the chain's corporate buyer had recommended that the local store order 1,000 units of these shorts to be supplied in one lot at the start of the season at a cost of $4.00 per unit. This retail chain held a temporary storewide promotion in all stores during June of every year. However, at the start of each month, the local store managers were free to set and change prices among the permissible levels for all seasonal merchandise as they saw fit. The close-of-season date for the walking shorts was the end of August. After that date, any remaining inventory of walking shorts would be removed to make room for the next season's merchandise. Although the store could later donate these leftover units to charity and receive a tax deduction, the store managers treated unsold goods at season's end as having zero salvage value when deciding on markdowns.

Concerned about a trend of rising markdown percentages, the four store executives were debating the adoption of one of the following policies as a way of regulating assistant managers' recourse to markdowns:

(1) An automatic-markdown policy would specify both the timing and magnitude of markdowns during the season. Two alternatives were on the table. The first was motivated by the well-known strategy followed by the Filene's Basement Store [Bitran and Mondschein 1997; Shuch 1988]. Under this Filene's markdown (*FM*) policy, the store would sell men's walking shorts at the regular price, that is, zero percent off, in Months 1 and 2, then 25 percent off in Months 3 and 4, 50 percent off in Months 5 and 6, and 70 percent off in Months 7 and 8 of the selling season. The second alternative was a half-price sale after half the season (H/H) policy. Under this policy, the item would be sold at regular price in Months 1 through 4 and then at 50 percent off for the rest of the season.

(2) A constant price (*CP*) strategy would mean that the merchandise would be sold

at one fixed price throughout the season. One of the managers felt strongly that the best way to control markdowns was not to allow them, and proposed that the men's walking shorts be sold either at the full price of $19.99 throughout the season (*CPO*) or at $14.99, the 25-percent price-off level throughout the season (*CP25*).

(3) A two price-level policy would permit markdowns but the permissible price levels would be reduced from the current four to two, either zero percent off or 50 percent off.

The managers were not sure whether any of these policies were optimal for the recommended order quantity of 1,000 units, or whether 1,000 units was the optimal order quantity. Therefore, they readily accepted our proposal to use MARK to investigate these issues.

Input Data

The only cost input we considered was the item's purchase cost per unit ($4.00). Traditionally this retailer evaluated its assistant managers based simply on the cumulative (undiscounted) gross margin contributions (sales revenues less purchase cost of the units sold) of their merchandise lines. Consequently, the store manager asked us to ignore discounting and zero out the other potential costs MARK allowed for: the costs of holding inventory, stockouts, customer returns, and executing markdown actions.

As for demand input data, we used managers' judgments to calibrate the probability distributions of demand at the four permissible price levels over the season. Further, we took a consensus approach since all four executives wished to participate, were familiar with the merchandise category, and had seen samples of the new item (via satellite TV broadcasts from the corporate buying office). They were also informed about the usual seasonal-demand pattern for this style classification in their store and in other stores in the chain.

Given the managers' differing positions in the store, we used MARK's option of a direct approach to obtain consensus demand estimates via the Delphi group method [Dalkey and Helmer 1962; Jolson and Rossow 1971]. The group's estimates of the minimum, most likely, and maximum demand at each permissible price level in each month converged after two Delphi rounds, and the whole exercise took about one hour. These were unfettered demand estimates, that is, we instructed the managers to provide them, assuming sufficient stock on hand to meet any level of demand in any month of the season. Based on these data, we parameterized triangular approximations of the probability distributions of demand and used them in the analyses (Table 1).
The managers perceived the seasonal demand for walking shorts as growing steadily from January to a peak in June and then rapidly falling off by August. The most likely estimate of the total demand for the season at full price is 698. Some markdowns would be needed to sell out the inventory of 1,000 units by the end of the season (Figure 1).

Analyses

First, we used the processes of Module 1 of MARK to derive optimal pricing plans, markdown budgets, and percentages for the recommended opening inventory of 1,000 units of shorts and full price of

Month	Price-off	Min	Mod	Max	Triangular mean	Triangular std. dev.
1	0%	8	10	15	11.00	1.47
1	25%	10	17	30	19.00	4.14
1	50%	20	32	45	32.33	5.10
1	70%	35	50	75	53.33	8.25
2	0%	15	25	38	26.00	4.71
2	25%	25	38	57	40.00	6.57
2	50%	47	60	82	63.00	7.22
2	70%	82	110	137	109.67	11.23
3	0%	55	77	115	82.33	12.39
3	25%	88	120	162	123.33	15.15
3	50%	125	160	210	165.00	17.44
3	70%	222	282	345	283.00	25.11
4	0%	67	92	135	98.00	14.04
4	25%	105	144	192	147.00	17.79
4	50%	147	190	252	196.33	21.55
4	70%	255	340	415	336.67	32.68
5	0%	140	180	222	180.67	16.74
5	25%	188	252	302	247.33	23.33
5	50%	265	337	417	339.67	31.04
5	70%	460	612	755	609.00	60.23
6	0%	170	215	255	213.33	17.36
6	25%	222	295	345	287.33	25.25
6	50%	327	400	505	410.67	36.53
6	70%	590	742	912	748.00	65.76
7	0%	47	63	70	60.00	4.81
7	25%	77	100	102	93.00	5.67
7	50%	105	145	174	141.33	14.14
7	70%	225	285	362	290.67	28.04
8	0%	22	32	38	30.67	3.30
8	25%	42	60	65	55.67	4.94
8	50%	65	85	102	84.00	7.56
8	70%	112	160	215	162.33	21.04

Table 1: Store managers' consensus estimates of the minimum (Min), most likely (Mod), and maximum (Max) unit demand for men's walking shorts at permissible price levels in each of the eight months of the item's selling season are displayed. The four permissible price levels are the full price or zero percent off (0%), 25 percent off (25%), 50 percent off (50%), and 70 percent off (70%). Based on these data inputs, we calibrated triangular probability distributions of demand, and the estimated means and standard deviations indicate the majority of the distributions are positively skewed and managers' uncertainty about demand increases as the markdown magnitude increases.

\$19.99. The policies we considered were the optimal markdown-only policy with four permissible price levels (*OM*); the optimal *OM*(2) policy when pricing is restricted to either the full price or a 50-percent markdown; and the optimal

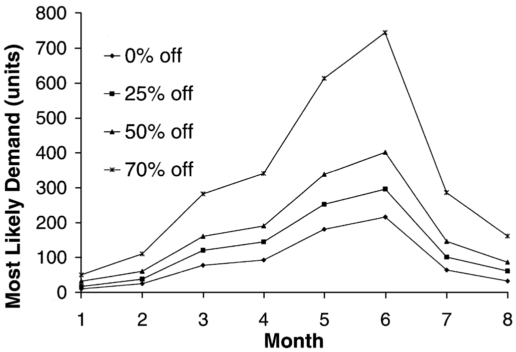

Figure 1: The most likely or modal estimates of demand at the four price levels over the eight months of the season reveal a seasonal demand pattern that peaks in June. The peak in demand in June is attributable to the time of season and to the annual storewide promotion held in that month.

unconstrained-pricing (*OU*) policy. (As the store typically did not allow markups after markdown, the optimal unconstrained-pricing policy results serve here simply as benchmarks for assessing markdown policy results.) We also applied MARK's given-inventory—given-prices (*GI-GP*) process to evaluate the proposed fixed-price policies with the given inventory of 1,000 units and compare their expected outcomes with those of the optimal pricing policies (Table 2).

The results for the anticipated *OM* policy called for just one 25-percent markdown in Month 3 (that is, reducing the price to $14.99 at the beginning of the third month and keeping it there for the rest of the eight-month season). Under this policy, the cumulative expected sales quantity is 990 units and the cumulative expected profit is $11,035. The corresponding total markdown amount (dollars foregone on units sold at the reduced price) is $4,765, implying a markdown percentage of about 32 percent. The markdown percentage is given by the ratio of the cumulative markdown dollar amount (here, $4,765) to the realized cumulative sales revenues (here $15,035). In contrast, the *OM*(2) policy calls for the markdown to be taken in Month 7. However, only 840 units are sold under this policy and although the markdown percentage is lowered to 16 percent, cumulative expected profits are five percent lower than the level under the *OM* policy. On the other hand, the cumulative expected net profit under the benchmark optimal unconstrained-pricing policy is $11,200, about 1.5 percent higher, even though the cumulative expected sales quantity is 16 units lower than that under the optimal markdown-only policy. Also, the cumulative markdown percentage under the *OU* policy is actually lower despite there being more off-price periods than under the *OM* policy. Therefore, more flexible pricing that follows the anticipated shape of seasonal demand is clearly advantageous. However, given their policy of taking only permanent markdowns, the management group found it reassuring that the optimal markdown-only policy had a net expected profit that was not much lower than that of the optimal unconstrained-pricing policy while achieving a higher sales quantity.

Turning to the four fixed-pricing policy options under consideration, the two constant-price policies (*CP0* and *CP25*) perform better than the two automatic-markdown (*FM* and *H/H*) policies. For example, even the simple policy of sticking to the full price throughout the season yields a cumulative expected net profit that is significantly greater than that yielded by the much-discussed Filene's

Price policy	Price path (sequence of markdowns off full price in months 1–8)	Inventory sold	Sales	Profits	Markdown amount	Markdown percentage
Optimal unconstrained-pricing policy (*OU*)	25%, 50%, 25%, 25%, 25%, 25%, 0%, 25%	97.4%	$15,200	$11,200	$4,280	28%
Optimal markdown-only policy based on four permissible price levels (*OM*)	0%, 0%, 0%, 25%, 25%, 25%, 25%, 25%	99%	15,035	11,035	4,765	32%
Optimal markdown-only policy based on two permissible price levels (*OM2*)	0%, 0%, 0%, 0%, 0%, 0%, 50%, 50%	83.6%	14,470	10,470	2,250	16%
Constant full-price policy (*CP0*)	0%, 0%, 0%, 0%, 0%, 0%, 0%, 0%	70.2%	14,040	10,040	0	0%
Constant 25-percent-off policy (*CP25*)	25%, 25%, 25%, 25%, 25%, 25%, 25%, 25%	100%	15,000	11,000	5,000	33%
Filene's markdown policy (*FM*)	0%, 0%, 25%, 25%, 50%, 50%, 70%, 70%	100%	11,720	7,720	8,280	71%
Half-price sale after half the season policy (*H/H*)	0%, 0%, 0%, 0%, 50%, 50%, 50%, 50%	100%	12,170	8,170	8,170	64%

Table 2: Given 1,000 units of opening inventory, we derived the cumulative expected sales, profits, and markdown percentages of seven different pricing policies with the proposed DSS. The results indicate that unless markdown timing and magnitude are optimally determined, taking no markdowns is a better policy than an arbitrary automatic-markdown policy.

markdown (*FM*) policy. This is the case even though the *FM* policy sells out the entire inventory, that is, 30 percent more units than the amount sold with the constant full-price (*CP0*) policy.

We drew two important practical lessons from the tabulated results: First, a no-markdown store policy is likely to be better, that is, to yield higher revenues and profits than an automatic-markdown policy that is applied without reference to the anticipated seasonal pattern of demand. However, second, if the timing and magnitudes of markdowns are determined optimally, a store policy that allows markdowns has greater profit potential than one that rules them out. These insights were reinforced by the profit-versus-markdown-percentage results of all the policies shown in Table 2. Retailers tend to correlate higher markdown percentages with lower net profits. However, as we found, policies calling for higher markdown percentages can yield higher cumulative profits. Again, the import of these results is that the retailer does not need to control the opportunity to take markdowns but rather when they take the markdowns and how large they are.

The next question we addressed was whether 1,000 units were the optimal order quantity for the store. Observing that the optimal markdown-only policy at 1,000 units called for only one markdown of just 25 percent, two executives conjectured that the store could sell more and make higher net profits with a larger order quantity and a more aggressive markdown policy, while the other two disagreed. To gain more insight into this issue, we used MARK to determine the optimal opening-inventory levels (order quantities) corresponding to the optimal unconstrained-pricing (*OU*) policy, the optimal markdown-only (*OM*) policy, and the four fixed-pricing policies (*CP0, CP25, FM, H/H*) considered in the previous analyses. Further, we used MARK to investigate the sensitivity of cumulative expected net profits to deviations from the optimal inventory level under each of the six pricing policies. More specifically, we examined the cumulative expected profits resulting from applying each of the six policies at selected opening-inventory levels in the range of 600 to 1,600 units, including the optimal level for each of the six pricing strategies (Table 3).

Under both the optimal markdown-only (*OM*) and optimal unconstrained-pricing (*OU*) strategies, the optimal opening inventory is significantly less than 1,000 units. Specifically, the optimal inventory following the *OM* policy is 780 units. (With this opening stock, the optimal markdown policy is to stay with the full price until the end of Month 6 and then to sell the remaining stock at 25 percent off

Opening inventory levels (units)	CP0	CP25	FM	H/H	OM	OU
600	$9,600	$6,600	$5,320	$5,770	$9,600	$9,600
709	**11,204**	7,799	5,974	6,424	11,204	11,229
780	10,920	8,580	6,400	6,850	**11,335**	11,435
878	10,528	9,658	6,988	7,438	11,051	**11,538**
1,000	10,040	11,000	7,720	8,170	11,035	11,200
1,027	9,932	**11,072**	7,866	8,332	11,072	11,122
1,198	9,248	10,388	8,348	**9,298**	10,535	10,759
1,502	8,032	9,172	**8,926**	8,092	9,655	9,750
1,600	7,640	8,780	8,618	7,700	9,263	9,358

Table 3: Displayed are the cumulative expected profits starting with different opening-inventory levels, following each of six pricing policies: *CP0* (constant-full-price), *CP25* (constant 25 percent off full price), *FM* (Filene's markdown policy), *H/H* (half-price sale after half the season), *OM* (optimal markdown-only), and *OU* (optimal unconstrained-pricing) policies. With optimal markdown pricing, the optimal opening-inventory level is 48 percent less while the cumulative expected profits are 27 percent greater than optimal inventory and expected profit levels following the automatic Filene's markdown (*FM*) policy.

in Months 7 and 8.) On the other hand, the optimal opening inventory is 878 if the *OU* pricing policy were to be followed, reflecting the store's capacity to sell a greater quantity and make more profit with unconstrained pricing.

The optimized opening-inventory levels for the two automatic-markdown (*FM* and *H/H*) policies are about 1,500 units and 1,200 units respectively, both values significantly greater than the optimal inventory levels corresponding to the two optimized (*OM* and *OU*) pricing strategies. However, cumulative expected profits with the automatic policies are much lower. For example, the profits for the optimized inventory—*OM* policy are nearly 27 percent higher than the profits for the optimized inventory—*FM* policy, despite starting with 48 percent less inventory. Ironically, to maximize expected profits, the poorer-performing *FM* and *H/H* policies call for much larger, not smaller, investments in opening inventory than the optimal pricing policies. Equally dramatic differences appear between the simple constant-full-price (*CP0*) and *FM* policy results. Specifically, the optimal opening inventory under the *FM* policy is more than twice the amount that is optimal under the *CP0* policy, but the *FM* policy yields 20 percent lower cumulative expected profits. Further, in general the best-performing fixed-pricing policy changes as the opening-inventory level is changed (Figure 2). Thus, the constant 25-percent-off-pricing policy dominates the constant-full-price policy at 1,000 units of opening inventory but performs much worse if the opening inventory is lower than 800 units. Similarly, the *FM* policy is dominated by

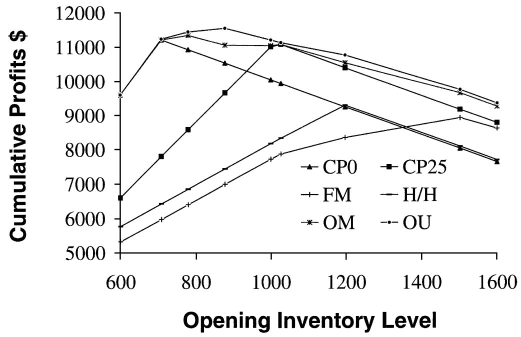

Figure 2: Cumulative expected profits are much more sensitive to order quantity errors when an arbitrary-markdown policy is followed than they are when markdowns are taken optimally.

the constant-full-price policy at opening-inventory levels lower than 1,500 units but yields superior profits at higher opening-inventory levels.

Also, the cumulative expected profit curves for all four fixed-price policies are much more peaked near their optimum order-quantity levels than those corresponding to the optimal pricing policies. In particular, each of the fixed-price policy profit curves falls sharply from its maximum when the opening inventory exceeds the optimum inventory level by small amounts (Figure 2).

Thus, cumulative expected profits with these nonoptimal policies are quite sensitive to deviations of the opening inventory from the optimum level. On the other hand, the optimized pricing policy (*OM* and *OU*) profit curves are rather flat in the region of their optimum order quantities. That is, when prices are set optimally, profits are relatively insensitive to deviations from the optimal opening-inventory levels. In particular, following the optimal unconstrained-pricing policy reduces profits from the optimal level by less than four percent even when the opening inventory is more or less than the optimal level by as much as 17 percent. Thus optimal (model-based) dynamic pricing policies appear to insulate expected profits from errors in setting order quantity.

To explain these results intuitively, we can recast the problem of optimal sequential pricing of a fixed inventory as one of optimal allocation of the given inventory across the four (zero, 25, 50, and 70 percent) permissible price-off points or tiers. That is, how much of the inventory should the store sell at each price point over the season? This is akin to the problem of allocating seat inventory across fare-class buckets in airline yield management [Belobaba 1987; Gallego and Van Ryzin 1994]. Conceptualizing the problem in this way (and ignoring more subtle dynamic considerations), we find that the total expected revenue from any price-off tier is a concave (diminishing returns) function of the units of inventory allocated to that tier. Then the optimal pricing solution is effectively one that allocates the given inventory to each permissible tier such that the marginal expected revenues at these allocations are equal. Further, the optimized inventory solution based on such optimal allocations is effectively that inventory level at which the marginal expected revenue of each tier is equal to the marginal cost of inventory.

Now it is well known in the field of marketing-resource allocation that when the revenue functions are concave, the corresponding profit functions tend to be flat around their optimal allocations, as is the aggregated profit function [Mantrala, Sinha, and Zoltners 1992]. This may explain the flatness of the cumulative expected-profit curves corresponding to the *OU* and *OM* policies in the vicinity of their respective optimal opening-inventory levels (Figure 2). On the other hand, profits are more sensitive to inventory changes around allocations well below or above their optimal levels. Such misallocation of inventory occurs, for example, under the *FM* policy. In effect, the *FM* policy is an arbitrary rule for allocating inventory that overallocates available inventory to the low-margin 50-percent price-off tier (applied in the peak demand Months 5 and 6) and underallocates inventory to the high-margin zero-percent and 25-percent price-off tiers. Consequently, stores require a much larger total inventory to provide optimal allocations to the high-margin tiers. Much of this larger total amount of inventory gets "wasted" in the 50-percent price-off tier under the *FM* pricing rule, but the inventory allocations to the high-margin tiers make compensating contributions. Total profits, therefore, increase up to a point, as the total inventory is increased. As a result, the optimal total inventory under the *FM* policy is almost twice that of the optimal markdown-only pricing policy while yielding significantly lower total expected profits. Further, the total expected profits under the *FM* policy are much more sensitive to deviations from the corresponding optimal total inventory because of the underlying misallocation of inventory to the various price tiers. Similarly, expected total profits are quite sensitive to deviations from the optimal order quantities under the two constant-price policies because, in general, policies that allocate all of the inventory to one price tier are rather inefficient, given the estimated demand functions.

In conclusion, this analysis indicates that the amount of opening stock a store should procure depends on the overall pricing policy to be pursued, and it can obtain the highest profit by jointly deciding order-quantity and pricing policies. These were important insights for the participating managers, who conceded that they usually concentrated on deciding the best markdown plan for a given order quantity without giving much thought to how the optimal order quantity depends on the specified markdown plan.

Managerial Implications

Our analyses showed that even when store managers ordered too much stock, following a no-markdown policy would likely be more profitable than an automatic-markdown strategy applied with no regard to the actual seasonal pattern of demand. However, the optimal pricing policy would likely involve markdowns. This means that pricing policies that lead to high markdown percentages do not necessarily result in low profits. Retailers generally tend to view markdowns as mistakes and often express an urge to control them. Our analyses with MARK show that markdowns are mistakes only when they are wrongly timed or of the wrong magnitudes. On the other hand, optimal markdowns are valuable, and retailers should view them as necessary and welcome strategic mechanisms for coping with an increasingly unpredictable marketplace.

Next, our analyses related to determining the optimal order quantity show that fairly small changes in order quantity can result in significant changes in the optimal timing and magnitude of markdowns. Conversely, apparently minor changes in the timing and magnitudes of planned markdowns can result in significant changes in the optimal order quantity. In particular, optimal order quantities associated with poorly performing aggressive automatic-markdown policies are likely to be much larger than those corresponding to optimal markdown policies, which in turn are likely to be larger than the optimal order quantity corresponding to the constant-full-price strategy. Further, a constant-price strategy that is more profitable than an automatic-markdown policy at a low opening-inventory level may be dominated by the same automatic policy at higher opening-inventory levels, that is, the virtues of any arbitrary-markdown policy change as the total order quantity is changed. Last, cumulative expected profits are quite sensitive to deviations from the optimal order quantities based on arbitrary-markdown policies, but are relatively insensitive to deviations from the optimal order quantities induced by optimal markdown strategies. In this sense, optimizing the timing and magnitude of markdowns can compensate for serious buying errors.

Issues for Future Research and Development

Several specific issues for further research fall in the areas of parameter estimation, model validation, generalizations of qualitative managerial insights, and model enhancements.

First, our model for preseason decision making relies mainly on fashion-savvy managers' subjective estimates of demand functions combined with relevant past seasons' sales data. However, once the selling season begins, there is an opportunity to revise prior demand estimates using actual sales data from point-of-sales (POS) systems. For example, based on the linearized version of the specified demand model, we have implemented a Bayesian approach for period-by-period updating of the prior distribution of the estimated total season demand at full price, taking all other model-parameter estimates as fixed. Updating the total-season-demand parameter is reasonable because typically fashion-merchandise buyers are most uncertain about this parameter, while the style classification's seasonal demand and price sensitivity patterns are usually better established. Under this approach, upcoming periods' forecasts of unit demand at different price levels get modified as the estimate of total-season demand converges to its true value. In turn, these periodic revisions correct optimal markdown paths for the remaining season and thereby improve cumulative expected profits. (We can provide further information on this adaptive model and related simulation results.) We are working on making this Bayesian approach more comprehensive and automated.

Second, how well do buyers actually perform with the help of the decision system over an entire season? How well will model users perform relative to others, everything else being the same? To answer these and related questions, we plan to conduct a model-validation study involving controlled field tests similar to those of the sales-call-planning model CALLPLAN carried out by Fudge and Lodish [1977]. One approach would be to study the performance of matched pairs of fashion buyers for different style classifications of a retailer. These paired individuals would be matched in terms of experience and personal characteristics, as well as the season lengths, demand patterns, sales volumes, and margins for selected merchandise items. We could then randomly assign one individual from each pair to use MARK and compare his or her contribution to that of the other individual who followed the usual practices. A second approach would be to have different buyers manage markdowns of the same item(s) in selected subsets of the stores in the retail chain, some with the help of the proposed model and some without.

Third, we would also like to investigate whether the insights from the application described in this paper are generally applicable. For example, do the insights regarding flat versus peaked profit curves and the relationship between markdown percentages and cumulative profits apply across all classes of fashion goods—displaying different seasonal demand characteristics, and considering various costs that were zeroed out in the case study? To throw light on these issues, we could use MARK to conduct in-depth simulation and sensitivity analyses using demand and cost inputs from retailers of different merchandise categories.

Last, with regard to model enhancements, the current model is intended for use where fashion buyers cannot reorder stocks of a merchandise item after the selling season begins. We would like to extend the model to allow for situations in which delivery lead times are short and reorders are possible. We could also enhance the model to allow demand to be sensitive to the level of on-hand inventory below a certain minimum level. (Smith and Achabal [1998] incorporate such a *fixture-fill effect* in their clearance markdown model but assume there is no uncertainty associated with demand.) However, the challenge would be to incorporate these complexities without sacrificing the model's applicability and utility in the real world. (The MARK DSS software is available commercially under the name "B_Line V3.0" which runs on 32-bit Windows platforms. More information about this package is available at www.fisofex.com)

In general, many other related decision problems remain in the fashion-retailing arena, calling for more research and implementable model-based solutions. We hope to see more such efforts in the future.

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APPENDIX

Stochastic Dynamic-Programming Model Formulation

Let Π_n denote the permissible set of price in period n. Let the stochastic demand for the item in the nth period of the N-period selling season, at the jth permissible price P_{nj}, be denoted by D_{nj} with probability density function $f_{nj}(D_{nj})$ for $j = 1, \ldots, J_n$; $n = 1, \ldots, N$. Further, let

h = holding cost per unit of leftover salable inventory at the end of each period, assumed to be constant per period (based, for example, on the known cost per square foot allocated to the merchandise).

k = stockout cost per unit, that is, the notional monetary cost of lost goodwill per unit of unmet demand due to the good being out of stock (based on executive judgment).

s = salvage price per unit of leftover stock that can be obtained at the end of the season, say, from a deep-discount retailer.

d = fixed fraction of inventory on hand at the start of any period expected to be lost because of damage or poor handling (based on historical experience).

u = fixed fraction of inventory on hand at the start of any period expected to be unaccounted for or lost due to shrinkage, that is, theft or damage (based on historical experience).

r_n = fraction of units sold in period n expected to be returned by customers by the end of that period (based on executive judgment).

Ur_n = fraction of units returned by customers in period n expected to be no longer saleable (based on executive judgment).

A = average cost per markdown event (based on executive experience).

ρ = discounting rate, for example, prevailing interest rate per period.

W_n = discount factor for period n with $W_n = (1 + \rho/100)^{-n}$.

GQH_n = gross quantity of stock on hand at the start of period n (which is unknown, that is, a random variable until the beginning of period n).

$I_n = GQH_n\,(I\text{-}d\text{-}u)$ = net inventory on hand at the start of period n (after accounting for inventory shrinkage and damage anticipated in period n).

Inventory Movement

Suppose we are at the start of some period n, $n = 1, \ldots, N$, and suppose $P_{n-1} \in \Pi_{n-1}$ was the price applied in the previous period, resulting in actual (observed) demand $D_{a,n-1}$. (Hereafter, we use the subscript a to denote realized values of random variables and drop this subscript when the realization has yet to occur.) Then the known gross quantity of stock on hand at the beginning of period n is given by

$$GQH_{a,n} = \{I_{a,n-1} - \text{Min}(I_{a,n-1}, D_{a,n-1})\} + \text{Min}(I_{a,n-1}, D_{a,n-1}) sr_n + Q_n. \quad (1)$$

That is, $GQH_{a,n}$ is the sum of the unsold inventory (before returns) at the end of the previous period, plus the quantity of units sold in period $n - 1$ returned by customers which is still saleable, plus the newly received stock Q_n as per the fixed lot receipt schedule. (The model allows for the single order quantity decided before the beginning of the season to be received in one or more lots over the season.) The unsold inventory in the last period is equal to the net inventory on hand, $I_{a,n-1}$ at the start of that period less the total quantity sold during that period. The latter amount is the lesser of $I_{a,n-1}$ and demand $D_{a,n-1}$ at P_{n-1} as the retailer cannot sell more than what is available.

Single-Period Profit

Consider the application of any of the permissible prices P_{nj}, $j \in \{1, \ldots, J_n\}$ in period n. Denoting the purchase cost per unit by C, the resulting gross contribution, G_{nj}, before subtracting inventory or markdown action costs is given by

$$G_{nj} = [\{\text{Min}\ (D_{nj}, I_{a,n})(P_{nj} W_n - C)\} - \{\text{Min}(D_{nj}, I_{a,n})(P_{nj} W_n - C)\ r_n\} - \{\text{Min}(D_{nj}, I_{a,n})(r_n\ Ur_n) C\}]. \quad (2)$$

The contribution is equal to the revenues (discounted) from all units sold less refunds for units returned by customers less the retailer's purchase cost of all sold units kept by customers and all units returned by customers that are no longer saleable. (Note that while $I_{a,n}$ is known, D_{nj} is probabilistic.)

Next we specify the various other costs that must be subtracted from G_{nj} to arrive at the net profit from applying price P_{nj}, $j \in \{1, \ldots, J_n\}$ in period n. These include

—monetary loss due to anticipated damage and shrinkage in period $n = GQH_{a,n} (d + u)C$;

—inventory holding cost incurred in period $n = \text{Max}((I_{a,,n} - D_{nj}), 0)\ hW_n$;

—inventory stockout cost incurred in period $n = \text{Max}((D_{nj} - I_{a,,n}), 0)\ kW_n$;

—cost of a markdown action $= Ax_n W_{n-1}$, where A denotes the fixed cost incurred every time a markdown is taken, for example, the costs of in-store display rearrangements, putting up sale signs, and so forth, while $x_n = 1$ when $P_{nj} < P_{n-1}$ and $x_n = 0$ when $P_{nj} \geq P_{n-1}$. Such markdown implementation costs can become quite significant when a large number of stores in a retail chain are involved [Robins 1993] and can possibly influence their number, timing, and magnitudes [Van Praag and Bode 1992]. The discount factor W here is subscripted by $n - 1$ assuming that markdown costs are incurred just before the period in which they are applied. Considering these additional costs, the random profit, denoted R_{nj}, from applying some permissible price P_{nj}, $j \in \{1, \ldots, J_n\}$ in period n is then given by

$$R_{nj} = G_{nj} - GQH_{a,n}(d + u)C - \text{Max}((I_{a,n} - D_{nj}), 0) hW_n - \text{Max}((D_{nj} - I_{a,n}), 0) kW_n - Ax_n W_{n-1}. \quad (3)$$

Last in the terminal period $n = N$, there is a possibility that the leftover stock at the end of the season has some salvage value, for example, it can perhaps be returned at some buyback price to the vendor or sold at or below cost in a liquidation sale. This calls for the addition of the following term to the right-hand side of equation (3) when $n = N$:

$$+ ((\mathrm{Max}(I_{a,N} - D_{Nj}), 0) + \mathrm{Min}(I_{a,N}, D_{Nj}) r_N (1 - Ur_n))(sW_N - C).$$

That is, the profit generated through salvage is equal to the total of the unsold inventory and saleable units returned by customers in the last period multiplied by the difference between the (discounted) salvage price per unit and the average cost price per unit.

Buyer's Dynamic Pricing Problem

$$\underset{P_n \in \Pi_n}{\mathrm{Max}}\ V_n = E[R_n + V^*_{n+1}(I_{n+1})] \tag{4}$$

where V_n is the value function to be maximized when choosing the price $P_n \in \Pi_n$, and

$$V^*_{n+1}(I_{n+1}) = \underset{P_{n+1} \in \Pi_{n+1}}{\mathrm{Max}}\ E[R_{n+1} + V^*_{n+2}(I_{n+2})] \tag{5}$$

is the optimal value function associated with future periods. Note that in (4) and (5), if $P_n = P_{nj}$ for $j \in \{I, \ldots, J_n\}$ then $R_n = R_{nj}$ and the inventory level $I_{n+1} = I_{n+1,j}$, which is contingent on the specific price applied in period n. Also, the inventory levels I_t, for $t = n + 1, \ldots, N$ are uncertain at the start of period n.

Probabilistic Demand Model Specification

Let the set of permissible prices in period n be denoted as $\Pi_n = \{P_{n1}, P_{n2}, \ldots, P_{nJn}\}$, where P_{nj} is the jth permissible price in period n, $n = 1, \ldots, N$. Then, we assume for $n = 1, \ldots, N; j = 1, \ldots, J_n$,

$$D_{nj} = \alpha_n M (P_f/P_{nj})^{\gamma(n)}\ \varepsilon_n \tag{6}$$

where

P_f = full retail selling price fixed on price tag at the start of the season.

D_{nj} = demand in units in period n at any price $P_{nj} \in \Pi_n$, given the full price P_f.

M = total season demand expected if price stays fixed at P_f.

α_n = period n seasonal factor, i.e., proportion of total season demand that materializes in period n such that $0 < \alpha_n < 1$; $\Sigma\alpha_n = 1$.

$\gamma(n)$ = elasticity of demand in period n with respect to price applied as a fraction of full price (P_{nj}/P_f).

ε_n = random disturbance term in period n.

The random disturbance terms ε_n for $n = 1, \ldots, N$, are assumed to be independent of the target item's own current price, and intertemporally independent. Further, we assume the disturbance terms are lognormally distributed, following the lead provided in earlier work on fashion demand forecasting by Green and Harrison [1973]. We have found reasonable support for the lognormal distribution assumption in several trial applications of MARK. For example, the positive skewness feature of the lognormal distribution is reflected by a majority of estimates of demand at different price levels (Table 1) that were elicited from fashion-merchandise managers in the application described in the paper.

The selected demand model specification effectively represents how a fashion item's demand per period responds to markdowns from its regular price (e.g., Achabal, McIntyre, and Smith 1990; Smith and Achabal 1998). Its multiplicatively separable form accounts for interactions between the seasonal, price-off, and uncertainty effects. Thus, if the fashion item displays no seasonal pattern, i. e., $\alpha_n = 1/n$, the total season demand at full price, M, is evenly distributed over the selling season (subject of course to the random disturbances in each period). However, if the item's demand does follow a well-recognized seasonal pattern, then the random demand in any period n at full price is $D_{fn} = \alpha_n M \varepsilon_n$ which is increased by a factor $(P_f/P_{nj})^{\gamma(n)}$ when price is marked down to P_{nj}. The parameter $\gamma(n)$ denotes a markdown response elasticity that can vary over time. Similarly, the random disturbance distribution is allowed to vary over time. Thus, the model allows for stochastic and nonstationary demands whose variability depends on the price-off level. As outlined in the text, the specified demand model is parameterized using managers' inputs within the interactive model software.

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