8.5-11 CONT.

$\sigma_x = 0 \quad \sigma_y = 37,730\text{ psi} \quad \tau_{xy} = -9431\text{ psi}$

$\sigma_{1,2} = 18,860\text{ psi} \pm 21,090\text{ psi}$

$\sigma_1 = 39,950\text{ psi} \quad \sigma_2 = -2230\text{ psi}$

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 21,090\text{ psi}$

Maximum tensile stress: $\sigma_t = 39,950\text{ psi}$ ←

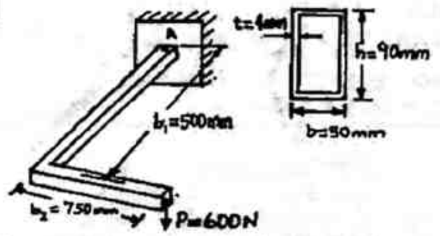
Maximum compressive stress: $\sigma_c = -2230\text{ psi}$ ←

Maximum in-plane shear stress: $\tau_{max} = 21,090\text{ psi}$ ←

Note: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

8.5-12

L-shaped bracket



$P = 600\text{ N}$
$t = 4\text{ mm}$
$b = 50\text{ mm}$
$h = 90\text{ mm}$
$b_1 = 500\text{ mm}$
$b_2 = 750\text{ mm}$

Stress resultants at the support

Torque: $T = Pb_2 = (600\text{ N})(750\text{ mm}) = 450\text{ N·m}$

Moment: $M = Pb_1 = (600\text{N})(500\text{mm}) = 300\text{ N·m}$

Shear force: $V = P = 600\text{N}$

Stresses at point A

$\tau = \frac{T}{2tA_m} \qquad A_m = (b-t)(h-t) = (46\text{mm})(86\text{mm}) = 3956\text{ mm}^2$

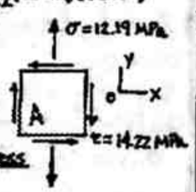
$\tau = \frac{450\text{ N·m}}{2(4\text{mm})(3956\text{mm}^2)} = 14.22\text{ MPa}$

$\sigma = \frac{Mc}{I} \qquad I = \frac{1}{12}bh^3 - \frac{1}{12}(b-2t)(h-2t)^3$

$= \frac{1}{12}(50\text{mm})(90\text{mm})^3 - \frac{1}{12}(42\text{mm})(82\text{mm})^3$

$= 1.1077 \times 10^6\text{ mm}^4$

$\sigma = \frac{(300\text{ N·m})(45\text{mm})}{1.1077 \times 10^6\text{mm}^4} = 12.19\text{ MPa}$



(The shear force V produces zero shear stress at point A)

Principal stresses and maximum shear stress

$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_x = 0 \quad \sigma_y = 12.19\text{ MPa} \quad \tau_{xy} = -14.22\text{ MPa}$

$\sigma_{1,2} = 6.095\text{ MPa} \pm 15.47\text{ MPa}$

$\sigma_1 = 21.57\text{ MPa} \quad \sigma_2 = -9.38\text{ MPa}$

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 15.47\text{ MPa}$

Maximum tensile stress: $\sigma_t = 21.6\text{ MPa}$ ←

Maximum compressive stress: $\sigma_c = -9.4\text{ MPa}$ ←

Maximum shear stress: $\tau_{max} = 15.5\text{ MPa}$ ←

Note: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

8.5-13

Cylindrical pressure vessel

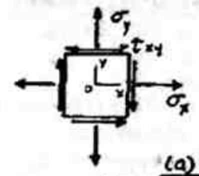
$T = 90\text{ k-ft} = 1080\text{ k-in}$

$r = 12\text{ in.} \quad t = 0.6\text{ in.}$

$p = 360\text{ psi}$

CONT.

8.5-13 CONT.



Stresses in the wall of the vessel

$\sigma_x = \frac{pr}{2t} = 3600\text{ psi} \quad \sigma_y = \frac{pr}{t} = 7200\text{ psi}$

$\tau_{xy} = -\frac{T}{2tA_m} = -\frac{1080\text{ k-in.}}{2(0.6\text{ in.})(\pi)(12\text{ in.})^2} = -1989\text{ psi}$

(a) Principal stresses

$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 5400 \pm 2683\text{ psi}$

$\sigma_1 = 8080\text{ psi} \quad \sigma_2 = 2720\text{ psi} \quad \therefore \sigma_{max} = 8080\text{ psi}$ ←

Maximum in-plane shear stress

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 2680\text{ psi}$ ←

(b) Maximum allowable torque T

$\tau_{allow} = 3000\text{ psi}$ (in-plane shear stress)

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (1)$$

$\sigma_x = 3600\text{ psi} \qquad \sigma_y = 7200\text{ psi}$

$\tau_{xy} = -\frac{T}{2tA_m} = -0.0018421\,T$

Units: τ_{xy} = psi T = lb-in.

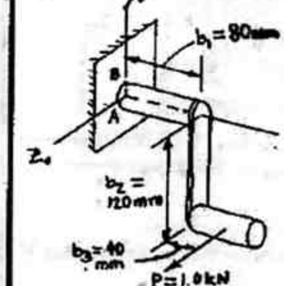
Substitute into Eq. (1):

$3000 = \sqrt{(-1800)^2 + (-0.0018421\,T)^2}$

Solve for T: $T = 1.303 \times 10^6\text{ lb-in.} = 109\text{ k-ft}$ ←

8.5-14

Crankshaft



$P = 1.0\text{ kN} \qquad d = 20\text{mm}$

$b_1 = 80\text{mm} \qquad b_2 = 120\text{mm}$

$b_3 = 40\text{mm}$

Stress resultants at the support:

$M_y = -Pb_3 - Pb_1$
$= -40\text{N·m} - 80\text{N·m} = -120\text{N·m}$

$T_x = -Pb_2 = -120\text{N·m}$

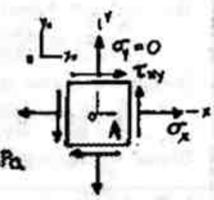
$V_z = P = 1.0\text{ kN}$

a) Stresses at point A

$\sigma_x = \frac{M_y}{S} = \frac{32M_y}{\pi d^3} = \frac{32(-120\text{ N·m})}{\pi(20\text{mm})^3}$

$= -152.8\text{ MPa}$ (compression)

$\tau_{xy} = \left|\frac{16T_x}{\pi d^3}\right| = \frac{16(120\text{ N·m})}{\pi(20\text{mm})^3} = 76.39\text{ MPa}$



(The shear force V produces zero shear stress at point A)

Principal stresses and maximum in-plane shear stress

$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= -76.40\text{ MPa} \pm 108.04\text{ MPa}$

$\sigma_1 = 31.64\text{ MPa} \qquad \sigma_2 = -184.44\text{ MPa}$

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 108.04\text{ MPa}$

Maximum tensile stress: $\sigma_t = 32\text{ MPa}$ ←

Maximum compressive stress: $\sigma_c = -184\text{ MPa}$ ←

Maximum shear stress: $\tau_{max} = 108\text{ MPa}$ ←

Note: Since the principal stresses have opposite signs, the maximum in-plane shear stress is larger than the maximum out-of-plane shear stress.

(b) Stresses at point B $\qquad \sigma_x = 0 \qquad \sigma_y = 0$

$\tau_{xy} = \frac{16T}{\pi d^3} - \frac{4V}{3A} \qquad A = \frac{\pi d^2}{4}$

$\tau_{xy} = 76.39\text{ MPa} - 4.24\text{ MPa}$

$= 72.15\text{ MPa}$

CONT.

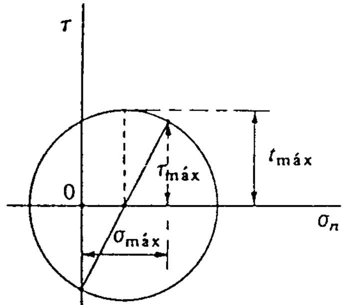


Figura X.2-*b*.

Sustituyendo los valores de $\sigma_{máx}$ y $\tau_{máx}$ en función de los momentos M_F y M_T queda:

$$t_{máx} = \frac{1}{2}\sqrt{\left(\frac{2M_F}{I_0}R\right)^2 + 4\left(\frac{M_T}{I_0}R\right)^2} = \frac{R}{I_0}\sqrt{M_F^2 + M_T^2} = \frac{2}{\pi R^3}\sqrt{M_F^2 + M_T^2}$$

Aplicando el criterio de Tresca, se habrá de verificar

$$\frac{2}{\pi R^3}\sqrt{M_F^2 + M_T^2} \leqslant \frac{\sigma_{adm}}{2}$$

de donde

$$R^3 \geqslant \frac{4}{\pi\sigma_{adm}}\sqrt{M_F^2 + M_T^2} = \frac{4 \times 10^2}{\pi \times 1600}\sqrt{675^2 + 1910^2}\ \text{cm}^3 = 161.20\ \text{cm}^3$$

$$\boxed{R = 55\ \text{mm}}$$

X.3. Un redondo de acero al carbono que tiene la forma y dimensiones indicadas en la Figura X.3-*a* está perfectamente empotrado en un paramento vertical.
Calcular el radio necesario para resistir una carga $P = 250$ kp aplicada en el punto medio del lado paralelo al paramento, si las tensiones máximas admisibles a tracción y cortadura son $\sigma_{adm} = 1200$ kp/cm² y $\tau_{adm} = 720$ kp/cm² respectivamente.

Supondremos la dimensión *b* pequeña en relación a *a*. Esto nos permite considerar la parte *AB* empotrada en sus extremos.

Esta parte trabaja a flexión, mientras que las *BC* y *AD* a torsión principalmente. Los momentos de empotramiento en *A* y *B* son precisamente los momentos torsores que actúan sobre *AD* y *BC*.

Supongamos, pues, una viga recta de longitud $a = 1.20$ m empotrada en sus extremos: y cargada en su punto medio con $P = 250$ kp. Los momentos de empotramiento en *B* y *A* son iguales, por razón de simetría.

De la observación del diagrama de momentos flectores en esta viga (Fig. X.3-*b*), se deduce que los momentos de empotramiento son, en valor absoluto:

$$M_A = M_B = \frac{Pa}{8} = \frac{250 \times 1.20}{8} = 37.5\ \text{m}\cdot\text{kp}$$

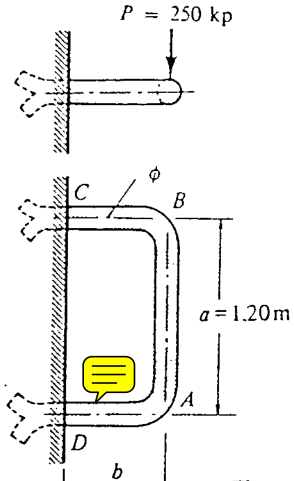


Figura X.3-*a*.

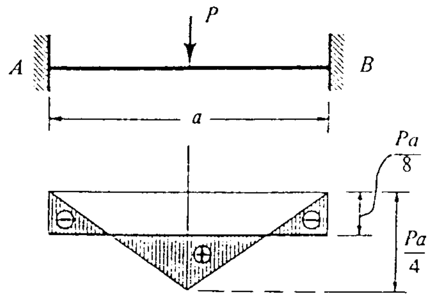


Figura X.3-*b*.

y que el momento flector máximo tiene también este valor.

$$M_{máx} = 3750\ \text{cm}\cdot\text{kp} = W_z \cdot \sigma_{adm} = \frac{\pi r^3}{4}\sigma_{adm}$$

de donde:

$$r^3 = \frac{4 \times 3750}{3.14 \times 1200} = 4\ \text{cm}^3$$

el diámetro del redondo será:

$$\boxed{\Phi = 32\ \text{mm}}$$

Este resultado será válido si no se supera en *AD* y *BC* la tensión admisible a cortadura. Se comprueba, en efecto, que la tensión de cortadura máxima en estos tramos es:

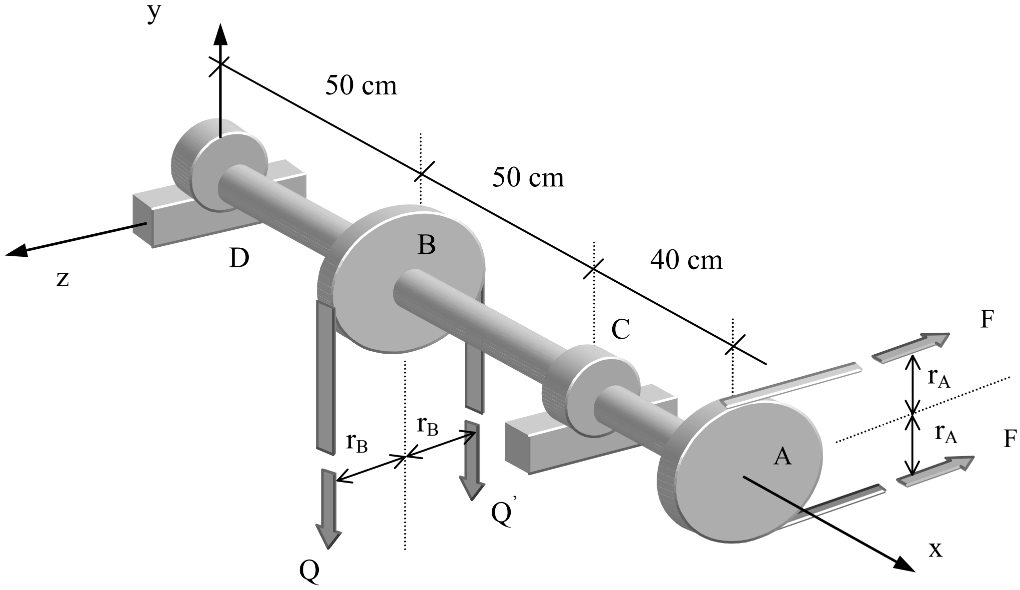
$$\tau_{máx} = \frac{M_T}{W} = \frac{2M_{máx}}{\pi r^3} = \frac{\sigma_{adm}}{2} = 600\ \text{kp/cm}^2 < \tau_{adm} = 720\ \text{kp/cm}^2$$

X.4. Un prisma mecánico de línea media rectilínea y sección recta constante está sometido a flexión y torsión combinadas. En un punto *P* del prisma, se pide:

1.° Determinar la matriz de tensiones.
2.° Calcular las tensiones principales.
3.° Hallar las relaciones que tienen que verificar las componentes de la matriz de tensiones para que el material del prisma en el punto *P* no se plastifique si se toma como criterio:
a) el criterio de la tensión principal máxima.
b) el criterio de Tresca.
b) el criterio de von Mises.

Problema 7.6

Un árbol, de acero, debe de transmitir 120 CV a 600rpm desde la polea A a la B. La tensión cortante admisible para el material del árbol es τ_{adm} = 420 Kg/cm^2 y la tensión normal admisible es σ_{adm}=728 kp/cm^2. Calcular el diámetro del árbol. Datos: F=2·F' , Q=2·Q' , r_A=15 cm , r_B=22 cm. (radios de las poleas).



Resolución:

$$P = M_x \cdot \omega \Rightarrow M_x = \frac{P}{\omega} \quad \left\{ \begin{array}{l} 1\,\text{CV} = 736\ \text{W} \\ 1\,\text{rpm} = 2\pi/60\ \text{rad/s} \end{array} \right\} \quad M_x = \frac{120 \cdot 736}{600 \cdot \frac{2\pi}{60}} = 1405\ \text{Nm}$$

$$M_x = 1405\ \text{Nm} = 14324\ \text{cmKg}$$

$$M_x = F \cdot r_A - F' \cdot r_A = (2F' - F') \cdot r_A = F' \cdot r_A$$

$$F' \cdot 15 = 14324\ \text{cmKg} \quad \Rightarrow \quad F' = \frac{14324}{15} = 955\ \text{Kg}$$

$$F = 2F' = 1910\ \text{Kg}$$

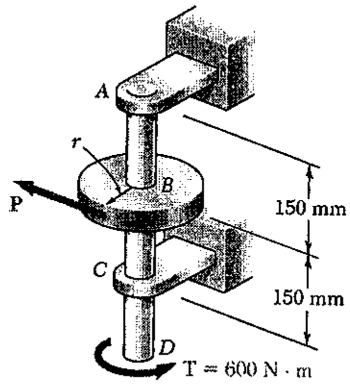
también $$M_x = Q \cdot r_B - Q' \cdot r_B$$

$$Q' \cdot 22 = 14324\ \text{cmKg} \qquad Q' = \frac{14324}{12} = 1193{,}7\ \text{Kg}$$

$$Q = 2 \cdot 1193{,}7 = 2387{,}4\ \text{Kg}$$

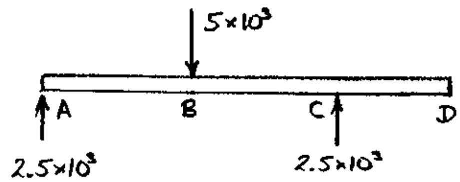
PROBLEM 8.16

8.16 Determine the smallest allowable diameter of the solid shaft *ABCD*, knowing that $\tau_{all} = 60$ MPa and that the radius of disk *B* is $r = 120$ mm



SOLUTION

$\Sigma M_{AD} = 0 \qquad T - Pr = 0 \qquad P = \frac{T}{r} = \frac{600}{120 \times 10^{-3}} = 5 \times 10^3 \text{ N}$

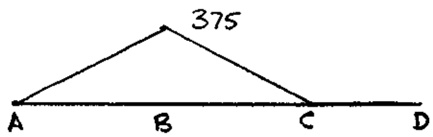


$R_A = R_C = \frac{1}{2}P$

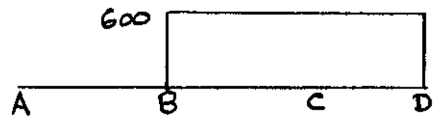
$= 2.5 \times 10^3 \text{ N}$

$M_B = (2.5 \times 10^3)(0.150 \times 10^{-3})$

$= 375 \text{ N·m}$



Bending moment



Torque

Critical section lies at point B $\qquad M = 375 \text{ N·m}, \quad T = 600 \text{ N·m}$

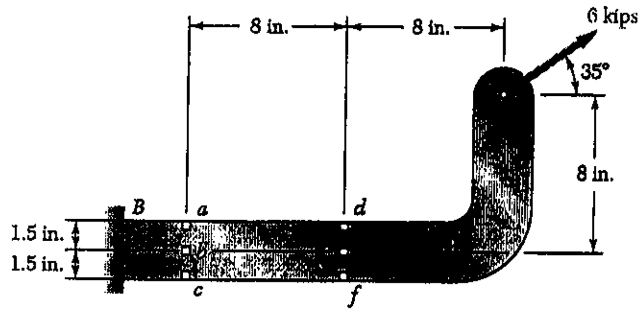
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2 + T^2}\right)_{max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{2}{\pi}\frac{\sqrt{375^2 + 600^2}}{60 \times 10^6} = 7.507 \times 10^{-6} \text{ m}^3$$

$c = 19.58 \times 10^{-3}$ m $\qquad d = 2c = 39.2 \times 10^{-3}$ m $= 39.2$ mm ◄

PROBLEM 8.33

8.33 A 6-kip force is applied to the machine element *AB* as shown. Determine the normal and shearing stresses at (*a*) point *d*, (*b*) point *e*, (*c*) point *f*



SOLUTION

thickness = 0.8 in

At the section containing points d, e, and f

$P = 6\cos 35° = 4.9149$ kips $\qquad V = 6\sin 35° = 3.4415$ kips

$M = (6\sin 35°)(8) - (6\cos 35°)(8) = -11.788$ kip·in.

$A = (0.8)(3.0) = 2.4\ \text{in}^2$

$I = \frac{1}{12}(0.8)(3.0)^3 = 1.80\ \text{in}^4$

(a) At point d $\quad \sigma_x = \frac{P}{A} - \frac{Mc}{I} = \frac{4.9149}{2.4} + \frac{(11.788)(1.5)}{1.8} = 11.87$ ksi ◄

$\tau_{xy} = 0$ ◄

(b) At point e $\quad \sigma_x = \frac{P}{A} = \frac{4.9149}{2.4} = 2.05$ ksi ◄

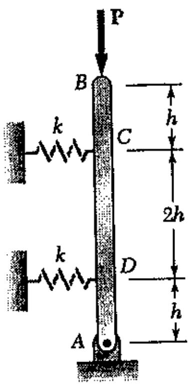
$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\cdot\frac{3.4415}{2.4} = 2.15$ ksi ◄

(c) At point f $\quad \sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{4.9149}{2.4} - \frac{(11.788)(1.5)}{1.8} = -7.78$ ksi ◄

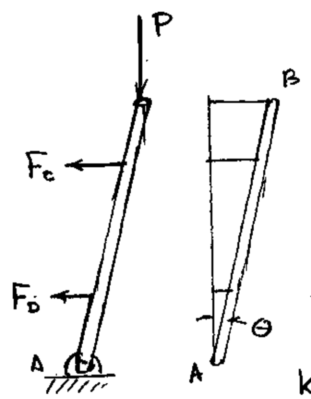
$\tau_{xy} = 0$ ◄

PROBLEM 10.7

10.7 The rigid rod *AB* is attached to a hinge at *A* and to two springs, each of constant $k = 2.0$ kip/in., that can act in either tension or compression. Knowing that $h = 2.0$ ft, determine the critical load.



SOLUTION



Let θ be the small rotation angle

$x_D \approx h\theta$, $x_C \approx 3h\theta$, $x_B \approx 4h\theta$

$F_C = kx_C \approx 3kh\theta$

$F_D = kx_D \approx kh\theta$

$+\circlearrowleft \Sigma M_A = 0 \qquad hF_D + 3hF_C - Px_B = 0$

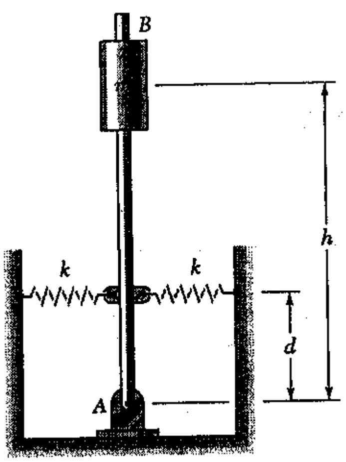
$kh^2\theta + 9kh^2\theta - 4hP = 0, \quad P = \frac{5}{2}kh$

Data: $k = 2.0$ kip/in, $h = 2$ ft $= 24$ in

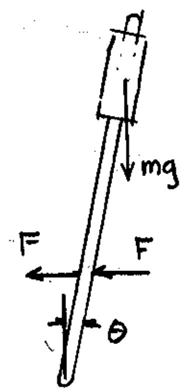
$P = \frac{5}{2}(2.0)(24) = 120$ kips. ◄

PROBLEM 10.8

10.8 If $m = 125$ kg, $h = 700$, and the constant of each spring is $k = 2.8$ kN/m, determine the range of values of the distance *d* for which the equilibrium of the rigid rod *AB* is stable in the position shown. Each spring can act in either tension or compression.



SOLUTION



$h = 700$ mm. $= 700 \times 10^{-3}$ m

Let θ be the small rotation of AB

$x = d\theta \qquad F = kx = kd\theta$

$+\circlearrowleft \Sigma M_A = 0 \qquad 2Fd - mgh\theta = 0$

$2kd^2\theta - mgh = 0$

$d_{cr}^2 = \frac{mgh}{2k}$

$d_{cr} = \sqrt{\frac{mgh}{k}} = \sqrt{\frac{(125)(9.81)(700\times10^{-3})}{(2)(2.8\times10^3)}}$

$= 0.392$ m $= 392$ mm

$d > 392$ mm for stability ◄