Optimal negotiation of maintenance contracts under several failure processes

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Abstract: Nowadays, we observe a dramatic increase of maintenance outsourcing. This is specially true for heavy industries such as mining in which focus in core business, increasing technological complexity and scale economies appear as some of the main reasons to outsource maintenance. In this work, the authors propose an analysis for the optimal negotiation of maintenance service contracts in which the parties involved (service provider and equipment owner) have shared long-run goals in order to maximize their own profits. In these models we study how different type of planned maintenance actions affect aspects of the reliability of equipments having an increasing failure rate. The results show a maximum reduction of about 95% in the surplus generated by the parties, when the aging of the equipment is taken into account. It has been predicted the optimal period between equipment replacements and preventive overhauls, showing the relevant influence of the type of planned actions over the decisions making associated to the settling of the terms and conditions of the maintenance service contract. The model improves previous works by reporting some misconceptions related to the evaluation of the expected number of failure times occurring during a contract horizon. The results show a loss of about 10% in the surplus generated by the parties, due to mistaken decisions arising from such misconceptions. The models are validated by simulations over a test case carried out in a commercial software.

I. INTRODUCTION

Modern industrial equipments require tools and personnel to carry out repairs when they fail. Often it is uneconomical for the owner of the equipment to have such specialist tools and personnel in-house. There are situations in which it is more economical to out-source the maintenance of such equipment. Furthermore, even if it is not more economical, the owner of the equipment could be forced into having a maintenance service contract with an external agent. An example situation occurs when the knowledge to carry out the maintenance and the spares for replacement need to be obtained from the Original Equipment Manufacturer (OEM). In this case, maintenance operations are necessarily carried out by the OEM and the owner of the equipment can be viewed as a client of the OEM for service providing. It can be noted that outsourcing maintenance is only appropriate into an operational level (maintenance implementation) and not into a strategic one (maintenance management). According to Murthy et al. [1], this is because the long-run goals of the service agent and the business could differ when management is outsourced.

There is a vast literature on maintenance service contracts using qualitative approaches. However, the number of papers dealing with mathematical models is small.

Murthy & Padmanabhan [2] developed a model for service contracts as extended warranties. A warranty is an agreement by the manufacturer/seller prior to the sale of a product which allows the customer to seek redress if the product does not perform satisfactorily over a certain period. Then, service contracts can be viewed as a kind of warranties that extend the coverage period of original warranty.

Murthy & Asgharizadeh [3] developed a Stackelberg game theoretic model formulation to obtain the optimal pricing structure with the service provider as the leader and the customer as the follower. The model assumes exponential failure times so that there is no need for preventive maintenance actions. By doing so, no consideration is made to the natural increasing failure rate of real equipments.

Asgharizadeh & Murthy [4] extended their earlier model to include multiple customers attended by a single service channel. This implies that when a unit fails, its repair cannot commence immediately if there are more failed equipments waiting for repair. In this case, the number of customers to service is an extra decision variable (in addition to the pricing structure) which the agent must select optimally.

Jackson & Pascual [5] improved previous models by considering equipments that age with time, so the terms of the contract include clauses related to preventive actions and replacement. The authors also extended previous literature by considering that the pricing was determined by negotiation (bargaining Nash equilibrium), instead of solving a leader-follower game formulation (monopolistic Stackelberg solution).

It has to be noted that the number of failures occurring over a certain period of time, is a relevant quantity since it determines the revenue generated by the equipment. Such quantity is an stochastic process since the unit fails in an uncertain manner. This particular stochastic process is called *counting process*. Rausand & Høyland [6] highlight four failure processes: (i) Homogeneous Poisson Process or HPP, (ii) Renewal Process, (iii) Imperfect Repair, and (iv) Non-Homogeneous Poisson Process or NHPP. Fig. 1 characterizes each of these processes in terms of the failure intensity.

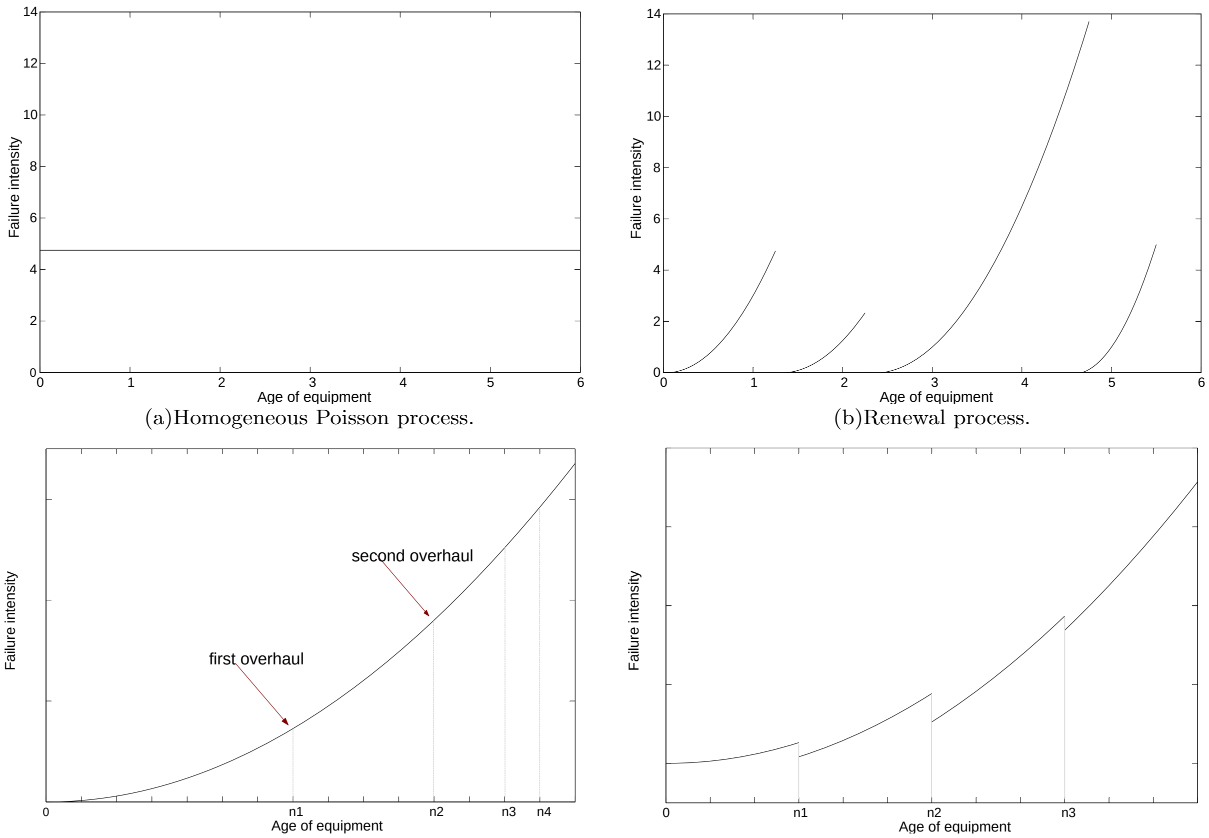
In this paper, we develop a formal model that integrates these processes in order to consider all the aspects involved about the reliability of an equipment that ages with time, so it needs to be subjected to planned actions of preventive maintenance. Under this framework, the fee in the contract is negotiated by solving a non-cooperative game (with symmetric and complete information) where the Nash equilibrium is encountered. Fig. 2 shows such interaction between the parts involved in a maintenance service contract. Once the equilibrium is reached, the surplus generated due to negotiation is maximized by both parties in order to set the rest of the terms and conditions of the contract. At the end of the paper we propose some extensions for future models to research about.

Fig. 1(a) shows the behavior of the failure intensity characterizing an homogeneous Poisson process. In such processes the failure rate remains constant along the contract horizon, so repairs, overhauls and replacements have no impact over the failure process.

Fig. 1(b) shows the behavior of the failure intensity characterizing a renewal process. The discontinuities represent preventive overhauls. Between such preventive actions, failure times occur according to an increasing failure rate. This process is also called *perfect repair process*.

Fig. 1(c) the behavior of the failure intensity characterizing a non-homogeneous Poisson process. Note preventive actions have no impact over the failure process. This process is also called *minimal repair process*.

Fig. 1(d) the behavior of the failure intensity characterizing an imperfect repair process. It is shown that the process corresponds to an intermediate situation between minimal and perfect repair.



(a)Homogeneous Poisson process.
(b)Renewal process.
(c)Non-homogeneous Poisson process.
(d)Imperfect repair process.

Figure 1: System subjected to different failure processes.

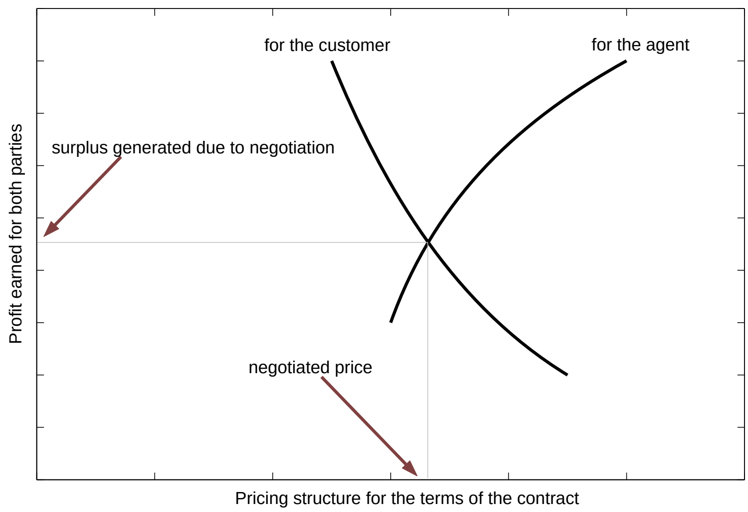


Figure 2: Characterization of the Nash equilibrium in the negotiation.

II. METHODOLOGY

There are different ways to outsource maintenance. In general, the outsourcing involves a service contract, where the terms and conditions include penalty clauses. However, the owner of the equipment could just pay per each repair that his/her machine requires with no contract involved, so the service provider has no penalty costs.

The outsourcing considered in the current work involves a contract. The description of the contract is as follows. For a fixed price, P, the agent offers to carry out maintenance operations (corrective and preventive) over the unit life-cycle. In addition, the contract includes a penalty clause in order to ensure a minimum level, A, of availability. So when the availability falls down the threshold, the service provider incurs into a cost, α, proportional to the downtime in which the minimum level of availability is not reached.

We explore the use of a game theoretic formulation to derive on the equilibrium fee in the contract. The rest of the variables involved (period between replacements and overhauls) are determined using non-linear optimization. The objective function to maximize is defined as the profit per unit time, explicitly expressed once the bargaining condition is achieved. By doing so, both parts maximize their own profits with no conflict of interest (optimal solution for clients corresponds to optimal solution for contractor). Since the objective function emerges from the equilibrium in a negotiation with complete and symmetric information, we will note that the parameters regarding the cost structure of contractor affect the profit of customers, and viceversa.

A. Model formulation

Equipment failures

Each customer owns a single unit which is used to generate revenue for itself. The revenue generated is R per unit time when the equipment is in working state and no income when in failed state. The purchase price of a unit is C_r. We will assume that at the end of the life-cycle the equipment is absolutely out-of-date so that its market recovery value is negligible.

All units are statistically similar in terms of reliability. Such reliability is characterized by an increasing failure intensity due to mechanical wear out. Typically, this kind of aging behavior is well-described by a failure intensity, λ, given by

$$\lambda(t) = \lambda_0 + rt \tag{1}$$

where λ_0 is the initial failure intensity when the equipment is new, at t=0, and r is the aging rate of the equipment. Note that t is the age of the unit, and not the calendar time since the equipment is new.

Equipment repairs

The equipment is subjected to corrective and preventive maintenance. The time to repair is exponentially distributed with repair rate μ. Since preventive maintenance actions are planned, we assume the time taken to carry them out is short compared to the mean time to repair, $1/\mu$, associated to corrective actions.

During corrective actions, the failed unit is returned back to working state by minimal repair. This approach is appropriate for large complex systems where the failure occurs due to a few components failing. As a result, the age of the system after repair is nearly the same as that before since the repaired components have a negligible impact on the system as a whole.

For preventive actions, the unit receives $N-1$ imperfect overhauls during its life ($N \geq 1$, integer). An overhaul improves the equipment in term of its failure intensity λ. Let $\lambda_n(t)$ be the failure intensity after the nth overhaul ($n = 1, ..., N-1$). Zhang & Jardine [7] expressed such failure intensity as

$$\lambda_n(t) = p\lambda_{n-1}(t-T) + (1-p)\lambda_{n-1}(t) \tag{2}$$

where p is an improvement factor that characterizes the quality of the overhauls ($0 \leq p \leq 1$). The *improvement factor method* was introduced by Malik [8]. However, there exists other models such as the *virtual age method* presented by Kijima et al. [9], and the *shock model method* developed by Kijima & Nakagawa [10]. Pham & Whang [11] discuss and summarize various treatment methods concerning imperfect maintenance.

The interval between overhauls, T, is constant. The Nth intervention corresponds to replacement, so the life-cycle of the unit, $L = NT$, is a decision variable too.

Equipment availability

Consider the time, τ, as the maximum delay allowed for the contractor to return the equipment back to operation. Then, the minimum level of availability to be ensured is given by

$$A = \frac{\frac{1}{\lambda}}{\tau + \frac{1}{\lambda}} \tag{3}$$

Note that for a fixed value of τ, the minimum allowable level of availability have to diminish if the mean failure intensity increases. Situation which occurs when equipment ages with time.

Client's profit

For customer j ($j = 1, ..., M$), let the number of failures over $[0,\ NT)$ be F_j. Let Y_{ji} denote the time taken to make the equipment operational after ith failure. Then, the profit to the jth client is given by

$$\omega = R\left(NT - \sum_{i=1}^{F_j} Y_{ji}\right) + \alpha\left(\sum_{i=1}^{F_j} max\{0, Y_{ji} - \tau\}\right) - C_r - P \tag{4}$$

Contractor's profit

Let C_m and C_o be the costs of each corrective repair and preventive overhaul for the contractor, respectively. These costs include material and workforce. Then, the profit to the contractor is given by

$$\pi = \sum_{j=1}^{M}\left[P - C_m F_j - C_o(N-1) - \alpha\left(\sum_{i=1}^{F_j} max\{0, Y_{ji} - \tau\}\right)\right] \tag{5}$$

Expected value for time to wait and repair

The model formulation is identical to a Markovian queue with finite population, M, and a single server attending according to a first-come, first-served rule where the arrival rate is λ and the service rate is μ. Let $f(y)$ denote the steady state density function for Y_{ji}. According to White et al. [12], the expression for such density is given by

$$f(y) = \sum_{k=0}^{M-1} P_k \mu e^{-\mu y}\frac{(\mu y)^k}{k!} \tag{6}$$

where P_k $(k = 0, ..., M-1)$ is given by

$$P_k = \frac{(M-k)(\lambda/\mu)^k\{M!/(M-k)!\}}{\sum_{k=0}^{M-1}(M-k)(\lambda/\mu)^k\{M!/(M-k)!\}} \tag{7}$$

It can be demonstrated that the expected value of Y_{ji} is given by

$$E[Y_{ji}] = \int_0^\infty y f(y)dy = \sum_{k=0}^{M-1}\frac{P_k(k+1)}{\mu} \tag{8}$$

Also, we obtain the expected value of $max\{0, Y_{ji} - \tau\}$ in the following way

$$E[max\{0, Y_{ji} - \tau\}] = \int_\tau^\infty (y-\tau)f(y)dy = \sum_{k=0}^{M-1} P_k \mu^k e^{-\mu\tau}\left[(k+1-\mu\tau)\sum_{l=0}^{k}\frac{\tau^{k-l}}{\mu^{l+1}(k-l)!} + \frac{\tau^{k+1}}{k!}\right] \tag{9}$$

Here we present the formula that allows derive this last expression. The interested reader could demonstrate it by mathematical induction on k

$$\int_\tau^\infty y^k e^{-\mu y}dy = k!e^{-\mu\tau}\sum_{l=0}^{k}\frac{\tau^{k-l}}{\mu^{l+1}(k-l)!} \tag{10}$$

Expected value for number of failure times

Let $H(t)$ be the expected value of the number of failure times in the period $[0,\ t)$ when the unit is not subjected to any overhaul. On the other hand, let $\hat{H}(t)$ denote the number of failure times when there are overhauls that were carried out over $[0,\ t)$. Then, it can be demonstrated that

$$\hat{H}(NT) = \sum_{n=0}^{N}\binom{N}{n}p^{N-n}(1-p)^{n-1}H(nT) \tag{11}$$

where

$$H(nT) = \int_0^{nT}\lambda(t)dt = \lambda_0 nT + r\frac{(nT)^2}{2} \tag{12}$$

Then, combining (11) and (12), we obtain the expected number of failure times over the contract horizon in the following way

$$E[F_j] = \hat{H}(NT) = \lambda_0 NT + rT^2 \frac{N^2(1-p)+Np}{2} \tag{13}$$

Here we present the formulas that allow calculate (13) from (11) and (12). The interested reader could derive them by mathematical induction on N

$$\sum_{n=0}^{N} \binom{N}{n} p^{N-n}(1-p)^{n-1} n = N \tag{14}$$

$$\sum_{n=0}^{N} \binom{N}{n} p^{N-n}(1-p)^{n-1} n^2 = N^2(1-p) + Np \tag{15}$$

Finally, it is easily seen that the mean value of failure intensity is given by

$$\bar{\lambda} = \frac{\hat{H}(NT)}{NT} = \lambda_0 + rT\frac{N(1-p)+p}{2} \tag{16}$$

Client's expected profit

Then, in terms of expected values for the random variables involved, the profit (4) is given by the following expression

$$E[\omega] = R\left(NT - \hat{H}\sum_{k=0}^{M-1}\frac{P_k(k+1)}{\mu}\right) + \alpha\hat{H}\sum_{k=0}^{M-1} P_k \mu^k e^{-\mu\tau}\left(\sum_{l=0}^{k}\frac{\tau^{k-l}\,(k+1-\mu\tau)}{\mu^{l+1}(k-l)!} + \frac{\tau^{k+1}}{k!}\right) - C_r - P \tag{17}$$

Contractor's expected profit

Likewise, in terms of expected values for the random variables involved, the profit (5) is given by the following expression

$$E[\pi] = M\left[P - C_m\hat{H} - C_o(N-1) - \alpha\hat{H}\sum_{k=0}^{M-1} P_k\mu^k e^{-\mu\tau}\left(\sum_{l=0}^{k}\frac{\tau^{k-l}\,(k+1-\mu\tau)}{\mu^{l+1}(k-l)!} + \frac{\tau^{k+1}}{k!}\right)\right] \tag{18}$$

Nash equilibrium

Binmore et al. [13] demonstrated that the bargaining condition involved in a non-cooperative game corresponds to a Nash equilibrium, in which the total surplus generated due to negotiation is divided into equal parts. Then, the fee bargained in the contract corresponds to the one in which the contractor's profit per client equals the client's profit. Note that, since negotiation is carried out in a bilateral manner, the bargaining condition is given by

$$\omega(P^\star, M, N, T) = \frac{\pi(P^\star, M, N, T)}{M} \tag{19}$$

By doing so, the negotiated price value, $P^\star$, is obtained in the following way

$$P^\star = \frac{R}{2}\left(NT - \hat{H}\sum_{k=0}^{M-1}\frac{P_k(k+1)}{\mu}\right) + \alpha\hat{H}\sum_{k=0}^{M-1} P_k\mu^k e^{-\mu\tau}\left(\sum_{l=0}^{k}\frac{\tau^{k-l}\,(k+1-\mu\tau)}{\mu^{l+1}(k-l)!} + \frac{\tau^{k+1}}{k!}\right) - \frac{C_r}{2} + $$
$$+\frac{C_m}{2}\hat{H} + \frac{C_o}{2}(N-1) \tag{20}$$

By replacing $P = P^\star$ on the expression (5), we obtain the expected profit for the agent as a result from negotiation

$$\pi(M,N,T) = M\left[\frac{R}{2}\left(NT - \hat{H}\sum_{k=0}^{M-1}\frac{P_k(k+1)}{\mu}\right) - \frac{C_m}{2}\hat{H} - \frac{C_o}{2}(N-1) - \frac{C_r}{2}\right] \tag{21}$$

Then, using the bargaining value $P^\star$, we may explicitly define the objective function $f(M,N,T)$ in the following way

$$f(M,N,T) = \frac{\pi(M,N,T)}{NT} = M\left[\frac{R}{2}\left(1 - \bar{\lambda}\sum_{k=0}^{M-1}\frac{P_k(k+1)}{\mu}\right) - \frac{C_m}{2}\bar{\lambda} - \frac{C_o}{2}\left(\frac{1}{T} - \frac{1}{NT}\right) - \frac{C_r}{2NT}\right] \tag{22}$$

B. Model validation

Fig. 3 presents the diagram of the process simulation as implemented and run from the commercial software Arena. It is shown the particular case in which there are two customers to service.

Table I indicates the function of the blocks used and the imputs set into each one of these blocks. It should be noted the analytical model deals with a single server, so it should be imposed the capacity of resources equals the requirements of resources associated to carrying out the repair process. Note that failure and repair processes are exponentially distributed since μ is constant and λ was time-averaged.

Previously to begin simulations, it was found that 1000 replications is enough to converge to theoretical values. To do that, it was defined the single customer model as test case since this model implies no queueing so it is only necessary to verify (according to Law of Large Numbers) that obtained averages converge to theoretical mean time to repair, which is $1/\mu$. By doing so, it is ensured that differences encountered between analytical model and computational simulation do not arise from errors associated to lack of data. Then, the optimization is obtained by simulation the situations in which the number of clients to service is 1, 2, 3, ..., M, and so on, until the profit per unit time decreases by first time (determination of the optimal solution) and then until the profit per unit time becomes negative (determination of the space of feasible solutions).

The idea of simulation as validating tool and not as solving tool, in this paper, arises from the fact that developed analytical model allows to save large computational times associated to the simulation in itself and the data post-processing spent each time results are required.

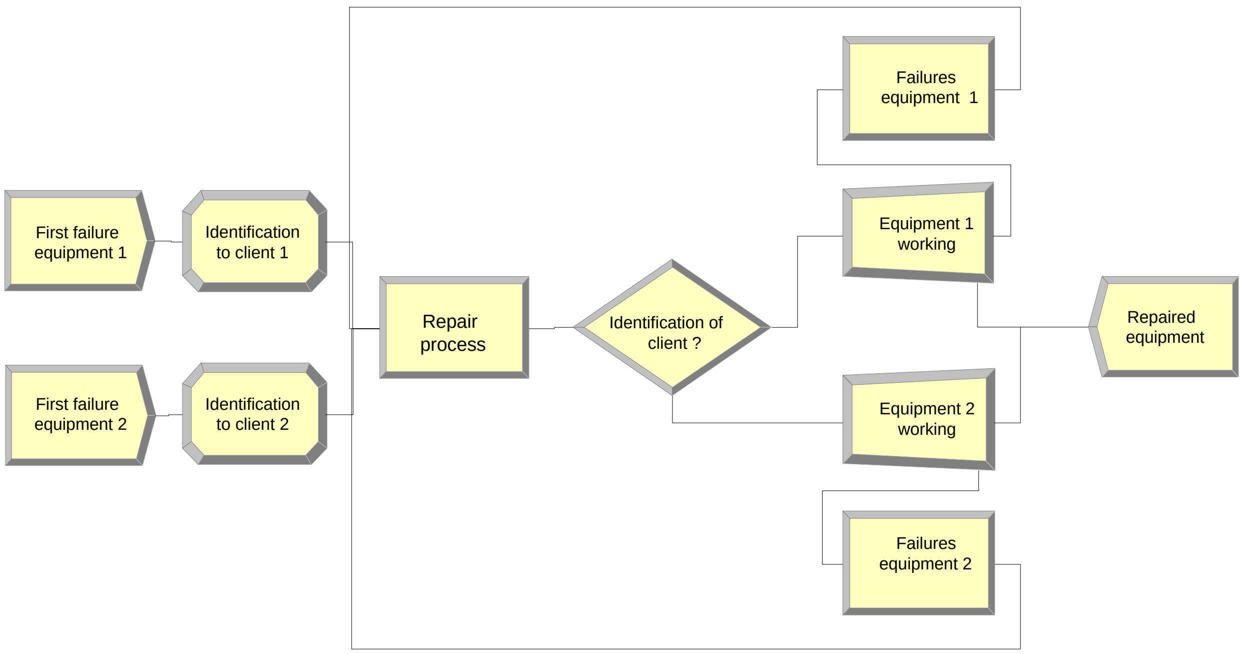


Figure 3: Diagram of the process simulation.

Table I: Definition of the blocks and imputs involved in the simulation

Block	**Type**	**Function**	**Imputs**
First failure equipment i	Create	Creation and delay of equipment i	Arrivals: 1 EXPO($1/\bar{\lambda}$)
Identification to client i	Assign	Assigns equipment i to client i	Type: attribute
Repair process	Process	Delay with requirement of resources	EXPO($1/\mu$) Requirements: 1
Identification of client ?	Decide	Recognizes whose client belongs the repaired equipment	Type: conditional
Equipment i working	Separate	Auxiliary block for the feedback of equipment i	
Failures equipment i	Process	Delay of equipment i due to subsequent failures	EXPO($1/\lambda$)
Repaired equipment	Dispose	Auxiliary block for the failures counting process	

III. RESULTS

The results are obtained using the solver of Excel with the following data: α=0.06(10^2\$/hours), C_m=1(10^3\$), C_o=4(10^3\$), C_r=300(10^3\$), λ_0=0.01(10^{-1}/hours), μ=0.02(1/hours), R=0.15(10^2\$/hours), τ=70(hours). The model integrates four different types of preventive maintenance, according to the existing counting processes. The results for HPPs are obtained by using $r = 0$, meaning that the equipment does not age with time. The other processes are characterized using r=0.01(10^{-5}/hours^2). For renewal processes, the quality of overhauls are perfect in the sense that reliability of the equipment becomes *as good as new* after intervention, so the improvement factor used is given by p=1. For NHPPs, the quality of overhauls are minimal in the sense that reliability of the equipment stays *as bad as old* after intervention, so the improvement factor used is given by p=0. The results for imperfect repair are obtained using p=0.5.

Fig. 4 presents the influence of the type of preventive maintenance over the profit generated by the contractor during the service process. The figure shows an optimal number of customers to service. The reason for its existence is as follows. When the number of clients increases the service provider has more contracts to earn profit from. However, for too many clients, the restriction of capacity derives into penalties due to increase in the mean time to wait and so the decrease of availability under the allowable limit. It can be also noted that as far as the quality of preventive overhauls are improved, the contractor can serve more clients since the mean value for failure times diminishes.

Fig. 5 shows the difference, in the profit generated due to negotiation, between the results obtained according to computational simulation versus the ones obtained according to analytical model developed in the previous section (using r=0 as test case). It is observed that the initial prediction is bad since the decisions maker will commit an error of 10% (if selects 12 customers to service), corresponding to a monetary loss respect to the maximum profit generated (which occurs at a number of 16 customers, according to simulation). It should be mentioned that, for this comparison, a cost of C_m=5(10^3\$) has been used in order to reduce the space of feasible solutions, so the computational time diminishes.

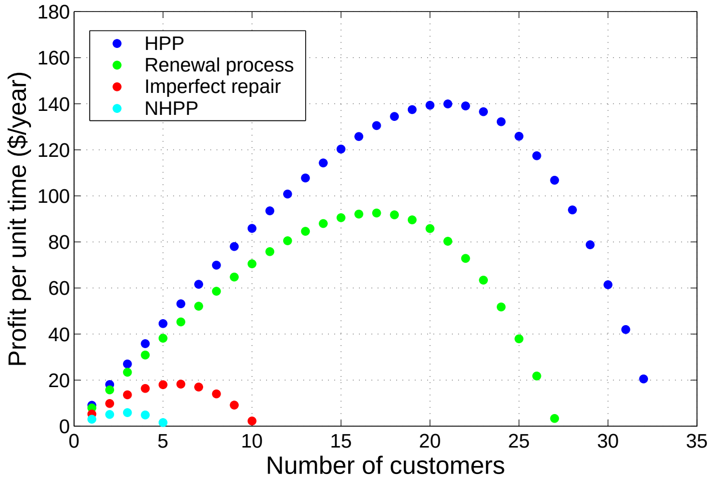


Figure 4: Initial reliability integrated analysis characterizing service process.

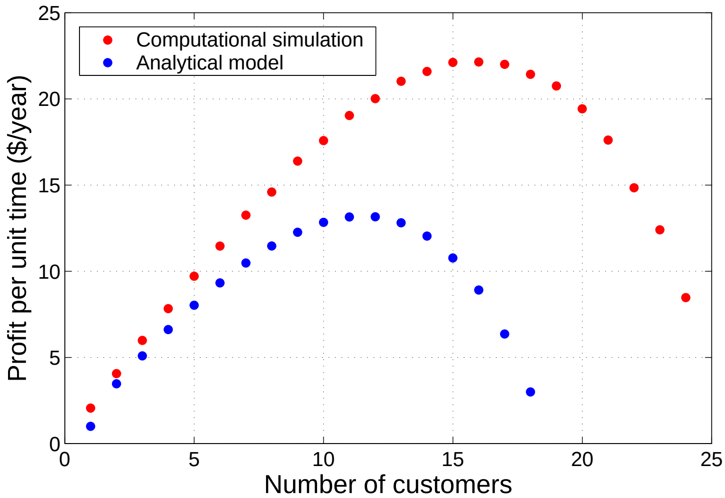


Figure 5: Comparison between the initial analytical model and the simulation.

The mistaken decision associated to the analytical model proposed comes from two sources of error:

1. Evaluation of the expected number of failure times. Fig. 6 shows the analytical model overestimates the expected number of failure times occurring along the contract horizon. The error comes from neglecting the time to service in the evaluation of the mean time between failures, $MTBF$, which results not too appropriate when intervention costs are estimated (specially when there are many customers to service). By correcting this aspect, we will improve the previous papers mentioned in the introduction concerning maintenance service contracts.

2. Assumption of stationarity for the probabilistic density function defining the time to service (repair + wait). This means there is a developed queue at the beginning of the process. Fig. 7 shows the analytical model overestimates the mean time to service, which is used to evaluate the downtime costs. However, it is observed a maximum error not greater than 1.3% respect to computational simulation. No improvement can be made along this aspect to develop an analytical model, but we will show that the improvement concerning the first source of error mentioned above is enough to reach the validation required.

From equation (16) it is observed that the expected number of failure times is given by

$$\hat{H}(NT) = \bar{\lambda} NT = \frac{NT}{\frac{1}{\bar{\lambda}}} = \frac{L}{MTTF} \tag{23}$$

By doing so, no consideration is made about the mean time to service, $MTTS$, which increases as the number of customers increases. Then, the actual expected number of failure times is given by

$$\hat{H}(NT) = \frac{NT}{\frac{1}{\lambda} + \sum_{k=0}^{M-1} \frac{P_k(k+1)}{\mu}} = \frac{L}{MTTF + MTTS} = \frac{L}{MTBF} \tag{24}$$

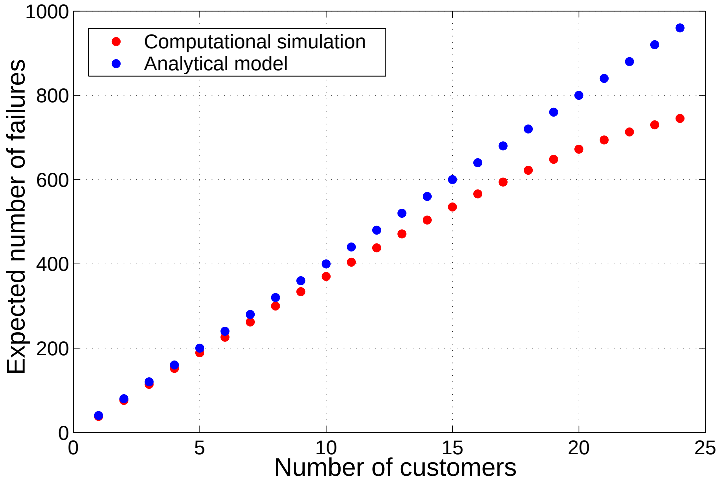


Figure 6: First source of error.

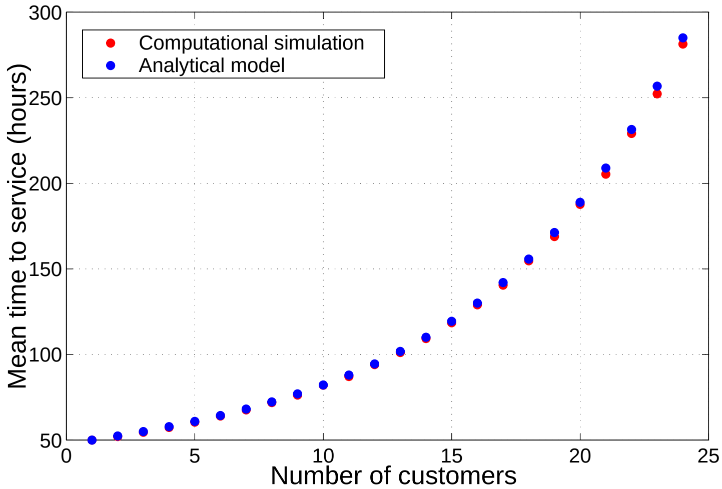


Figure 7: Second source of error.

Fig. 8 shows the impact of the improvement made by using (23) instead of (24). Note the error has been reduced from 10% to 0% since there is no mistaken decisions (the decisions maker selects 16 customers to service as predicted by simulation), as occurred in decisions predicted from Fig. 7. So the proposed improved models are finally validated by the computational simulation carried out in the commercial software Arena.

Fig. 9 presents the results associated to the effect of the quality of planned actions over the service provided by the contractor, when the improvement about the evaluation of the expected number of failure times, occurring along the contract horizon, is taken into account. Note the actual situation is more optimistic than initial predictions from Fig. 6. So the contractor can serve more clients and then earn a greater profit.

Tables II and III report the decision variables involved in the maintenance service contract for the initial models and the improved models, respectively. Note the contract includes no clauses for preventive maintenance (N and T) for the cases of Poisson processes (HPP and NHPP). In the case of HPPs, planned actions have no sense since the equipment does not age with time. In the case of NHPPs, planned actions have no sense since they are minimal so no goal is reached in the sense of delay the failure occurrence. Note that, in addition to the optimal number of customers to service and the profit generated, the equilibrium price and the optimal replacement period also increase when the evaluation of the number of failure times is improved.

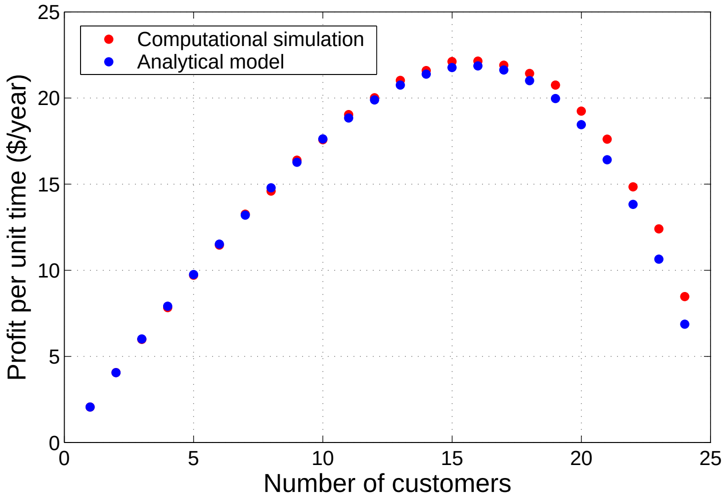


Figure 8: Comparison between the improved analytical model and the simulation.

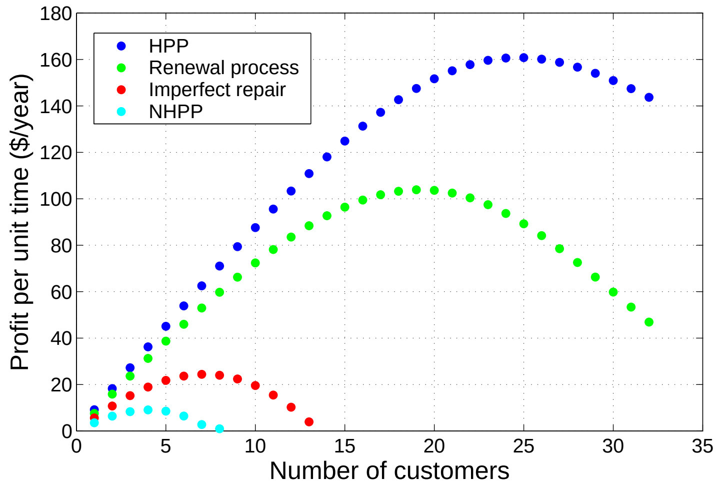


Figure 9: Improved reliability integrated analysis characterizing service process.

Table II: Reliability integrated analysis for the initial models

Model	M	N	T(hours)	A(%)	$P(10^3\$)$	L(hours)	$f(10^3\$/\text{year})$
HPP	21	-	-	93	916.1	70000	139.9
Renewal process	17	20	3500	92	903.1	70000	92.56
Imperfect repair	6	10	6075	84	720.7	60750	18.26
NHPP	3	-	-	79	591.5	57206	5.883

Table III: Reliability integrated analysis for the improved models

Model	M	N	T(hours)	A(%)	$P(10^3\$)$	L(hours)	$f(10^3\$/\text{year})$
HPP	25	-	-	93	1072.1	70000	160.8
Renewal process	19	20	3500	92	946.2	70000	103.9
Imperfect repair	7	10	6784	83	856.9	67840	24.40
NHPP	4	-	-	78	680.1	61760	9.105

IV. DISCUSSION

This work has presented an improved model formulation about the optimal negotiation of maintenance service contracts, in which price is bargained. The model integrates the analysis for equipments not aging (HPP model, r=0) and equipments that does age with time (Renewal process, Imperfect repair, and NHPP models). It considers corrective maintenance as minimal and preventive maintenance being perfect (Renewal Process), minimal (NHPP), or normal (Imperfect Repair). In this sense, the paper extends previous works of maintenance service contracts that only have considered constant failure rates.

The optimization was constrained in the sense that the time to replace should be shorter than the useful life of the unit, $L \leq$7000(hours). Note this unequality becomes active for the situations in which the failure rate is the same than initial one at any time $t > 0$ (HPP and Renewal process). Note also that the better the quality of planned overhauls is, the greater the period between replacements should be, which is consequent since age delays as far as the quality of preventive actions is. Finally, it is observed that the equilibrium fee increases as the number of clients increase, as expected according to previous papers.

The model can be extended in different manners. Here we propose a pair of those. Our model deals with the case of a single server. Murthy & Asgharizadeh [14] proposed and developed the optimal number of service channels problem, defining the service facility sizing of a monopolistic contractor repairing equipments with constant failure rates. In addittion, our model assumes that the repair times are exponentially distributed. Mahon & Bailey [15] suggest that distributions with decreasing repair rate are more appropriate for modelling repair times. This approach results interesting specially when a short-run horizon is analyzed.

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