7. FLOW NETS FOR ANISOTROPIC MATERIALS

7.1 Introduction

Many soils are formed in horizontal layers as a result of sedimentation through water. Because of seasonal variations such deposits tend to be horizontally layered and this results in different permeabilities in the horizontal and vertical directions.

7.2 Permeability of Layered Deposits

Consider the horizontally layered deposit, shown in Figure 1, which consists of pairs of layers the first of which has a permeability of k_1 and a thickness of d_1 overlaying a second which has permeability k_2 and thickness d_2.

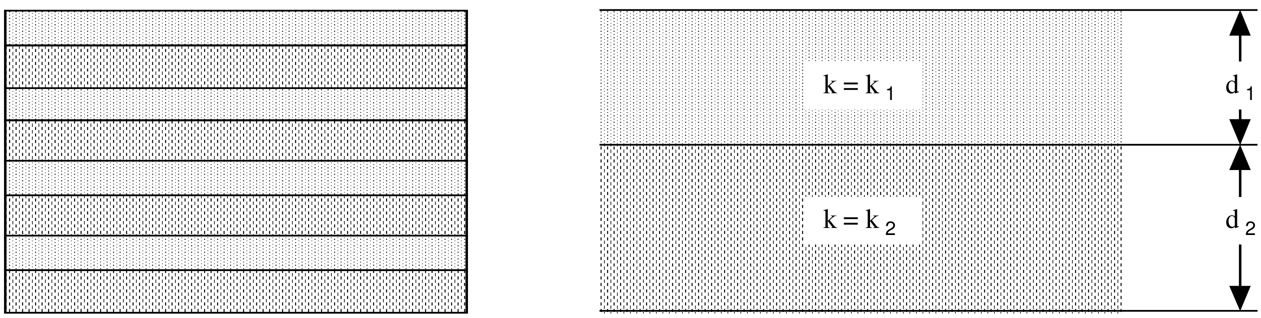

Fig. 1 Layered Soil

First consider horizontal flow in the system and suppose that a head difference of Δh exists between the left and right hand sides as indicated in Fig. 2. It then follows from Darcy's law that:

$$v_1 = k_1 \frac{\Delta h}{L} \quad ; \quad Q_1 = k_1 \frac{\Delta h}{L} d_1 \tag{1a}$$

and

$$v_2 = k_2 \frac{\Delta h}{L} \quad ; \quad Q_2 = k_2 \frac{\Delta h}{L} d_2 \tag{1b}$$

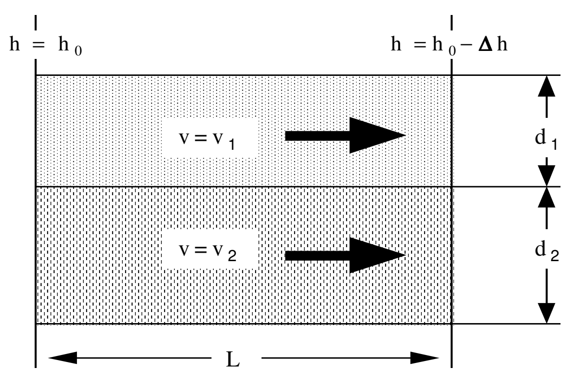

Fig. 2 Horizontal flow through layered soil

It therefore follows:

$$v = \frac{Q_1 + Q_2}{d_1 + d_2} = k_H \frac{\Delta h}{L} \qquad (2a)$$

where

$$k_H = \frac{k_1 d_1 + k_2 d_2}{d_1 + d_2} \qquad (2b)$$

Next consider vertical flow through the system, shown in Fig.3. Suppose that the superficial velocity in each of the layers is v and that the head loss in layer 1 is Δh_1, while the head loss in layer 2 is Δh_2

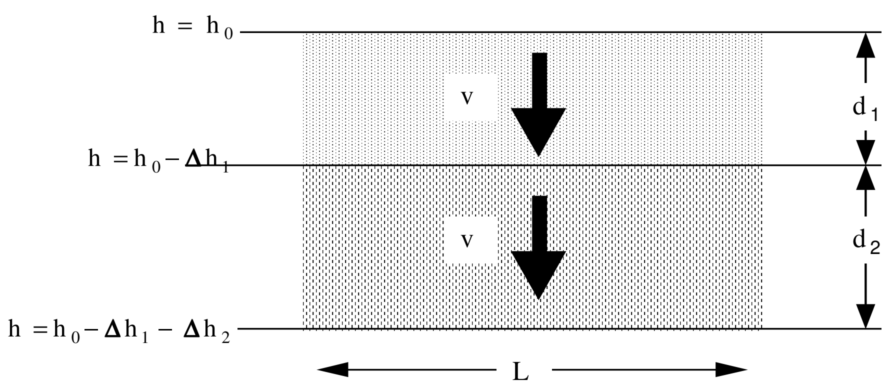

Fig. 3 Vertical flow through layered soil

In layer 1: $$v = k_1 \frac{\Delta h_1}{d_1}$$

so $$\Delta h_1 = \frac{v d_1}{k_1} \qquad (3a)$$

Similarly in layer 2 $v = k_2 \frac{\Delta h_2}{d_2}$ and $\Delta h_2 = \frac{v d_2}{k_2}$ (3b)

The total head loss across the system will be $\Delta h = \Delta h_1 + \Delta h_2$ and the hydraulic gradient will be given by:

$$i = \frac{\Delta h}{d} = \frac{\Delta h_1 + \Delta h_2}{d_1 + d_2} = \frac{\frac{v d_1}{k_1} + \frac{v d_2}{k_2}}{d_1 + d_2} \qquad (3c)$$

For vertical flow Darcy's Law gives

$$v = k_V \frac{\Delta h}{d} \qquad (3d)$$

and hence

$$\frac{d}{k_V} = \frac{d_1}{k_1} + \frac{d_2}{k_2} \qquad (3e)$$

Example

Suppose that that the layers are of equal thickness $d_1 = d_2 = d_0$ and that $k_1 = 10^{-8}$ m/sec and that $k_2 = 10^{-10}$ m/sec, then:

$$k_H = \frac{d_o \times 10^{-8} + d_o \times 10^{-10}}{d_o + d_o} = 5.05 \times 10^{-9}\, m/\sec$$

and

$$k_V = \frac{d_o + d_o}{\dfrac{d_o}{10^{-8}} + \dfrac{d_o}{10^{-10}}} = 1.98 \times 10^{-10}\, m/\sec$$

Showing that, as is generally the case, the vertical permeability is much less than the horizontal.

7.3 Flow nets for soil with anisotropic permeability

Plane flow in an anisotropic material having a horizontal permeability k_H and a vertical permeability k_V is governed by the equation:

$$k_H \frac{\partial^2 h}{\partial x^2} + k_V \frac{\partial^2 h}{\partial z^2} = 0 \tag{4}$$

The solution of this equation can be reduced to that of flow in an isotropic material by the following simple device. Introduce new variables defined as follows:

$$x = \alpha \bar{x} \quad \text{and} \quad z = \bar{z} \tag{5a}$$

the seepage equation then becomes

$$\frac{k_H}{\alpha^2 k_V} \frac{\partial^2 h}{\partial \bar{x}^2} + \frac{\partial^2 h}{\partial \bar{z}^2} = 0 \tag{5b}$$

Thus by choosing:

$$\alpha = \sqrt{\frac{k_H}{k_V}} \tag{5c}$$

It is found that the equation governing flow in an anisotropic soil reduces to that for an isotropic soil, viz.:

$$\frac{\partial^2 h}{\partial \bar{x}^2} + \frac{\partial^2 h}{\partial \bar{z}^2} = 0 \tag{5d}$$

and so the flow in anisotropic soil can be analysed using the same methods (including sketching flow nets) that are used for analysing isotropic soils.

Example - Seepage in an anisotropic soil

Suppose we wish to calculate the flow under the dam shown in Figure 4;

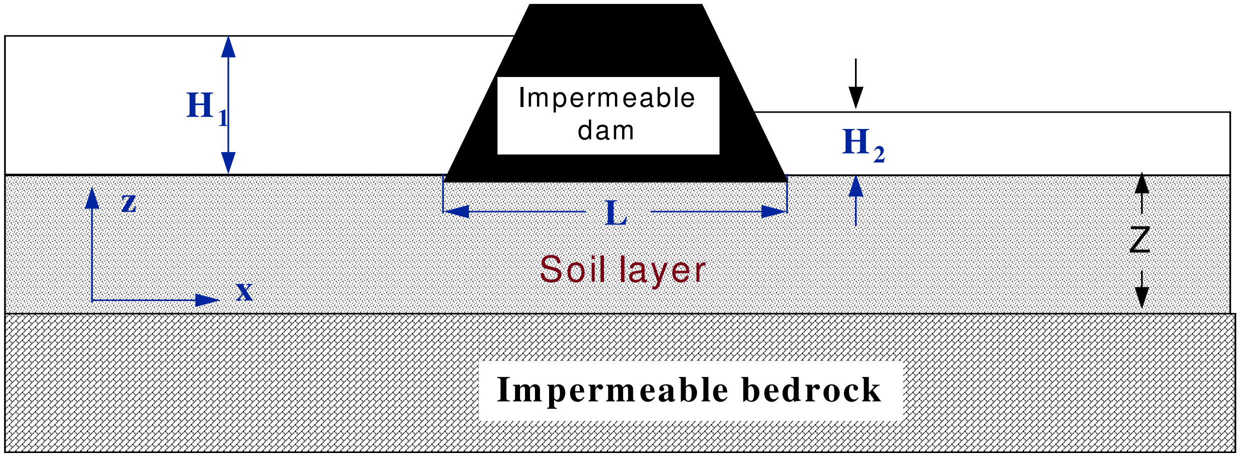

Fig. 4 Dam on a permeable soil layer over impermeable rock (natural scale)

For the soil shown in Fig. (4) it is found that $k_H = 4 \times k_V$ and therefore

$$\alpha = \sqrt{\frac{4 \times k_V}{k_V}} = 2$$

so (6)

$$x = 2\bar{x} \qquad \text{or} \qquad \bar{x} = \frac{x}{2}$$

$$z = \bar{z}$$

In terms of transformed co-ordinates this becomes as shown in Figure 5

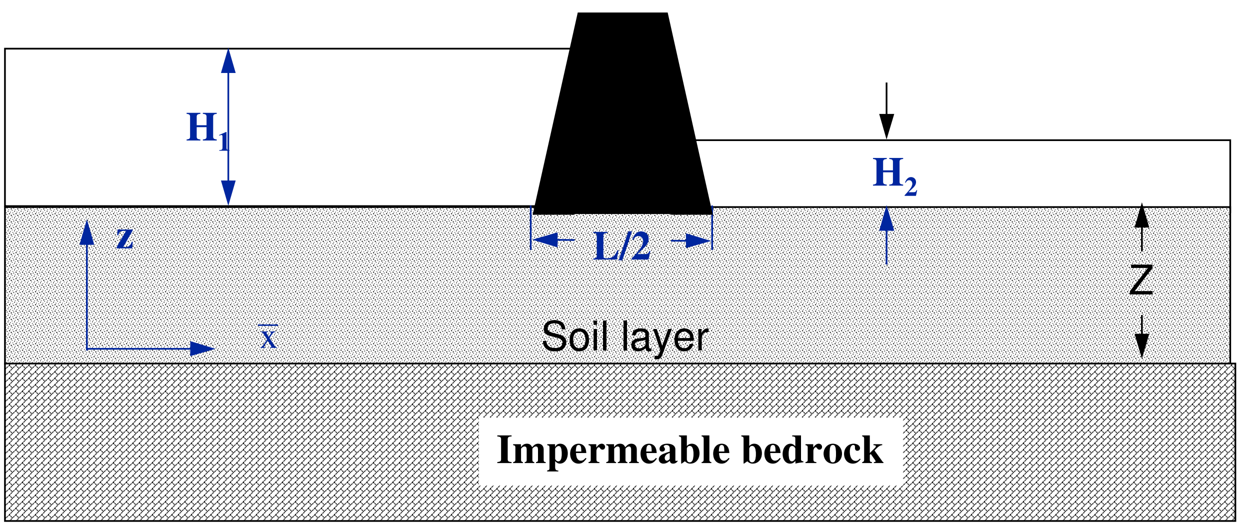

Fig. 5 Dam on a permeable layer over impermeable rock (transformed scale)

The flow net can now be drawn in the transformed co-ordinates and this is shown in Fig.6

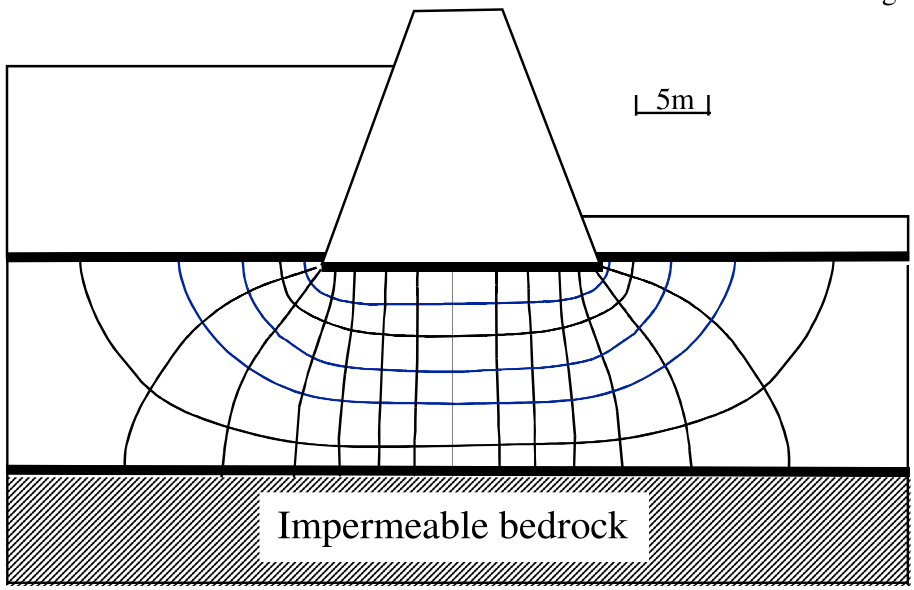

Fig. 6 Flow net for the transformed geometry

It is possible to use the flow net in the transformed space to calculate the flow underneath the dam by introducing an equivalent permeability

$$k_{eq} = \sqrt{k_H k_V} \tag{7}$$

A rigorous proof of this result will not be given here, but it can be demonstrated to work for purely horizontal flow as follows:

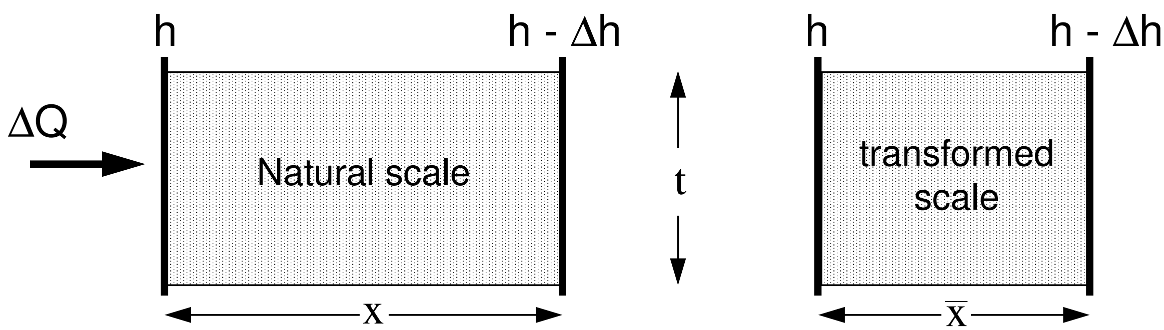

Fig. 7 Horizontal flow through anisotropic soil

For the natural scale $\Delta Q = k_H \, t \, \dfrac{\Delta h}{x}$ (7a)

For the transformed scale $\Delta Q = k_{eq} \, t \, \dfrac{\Delta h}{\bar{x}} = k_{eq} \, t \, \dfrac{\Delta h}{x} \sqrt{\dfrac{k_H}{k_V}}$ (7b)

From Equations 7a and b it can be seen that $k_{eq} = \sqrt{k_H \, k_V}$

Example

Suppose that in Figure 6 H_1 = 13m and H_2 = 2.5m, and that $k_V = 10^{-6}$ m/sec and $k_H = 4 \times 10^{-6}$ m/sec The equivalent permeability is:

$$k_{eq} = \sqrt{(4 \times 10^{-6}) \times (10^{-6})} \quad = \quad 2 \times 10^{-6} \ \text{m / sec} \tag{8a}$$

The total head drop is 10.5 m and there are 14 head drops and thus:

$$\Delta h = \frac{(13 - 2.5)}{14} = 0.75 \text{ m} \tag{8b}$$

The flow through each flow tube, $\Delta Q = k_{eq}\,\Delta h = (2\times10^{-6})\times(0.75) = 1.5 \times 10^{-6}\ m^3/s/m$

There are 6 flow tubes and so the total flow , $Q = 6 \times 1.5 \times 10^{-6}$
$= 9.0\times10^{-6}$ m^3/sec/(m width of dam)

For a dam with a width of 50 m $Q = 450 \times 10^{-6}\ m^3/sec = 41.47\ m^3/day$

7.4 Piping

Many dams on soil foundations have failed because of the sudden formation of a piped shaped discharge channel. As the store water rushes out the channel widens and catastrophic failure results. This results from erosion of fine particles due to water flow. Another situation where flow can cause failure is in producing 'quicksand' conditions. This is also often referred to as piping failure.

In order to analyse this situation consider water flowing upwards through the element shown in Figure 8.

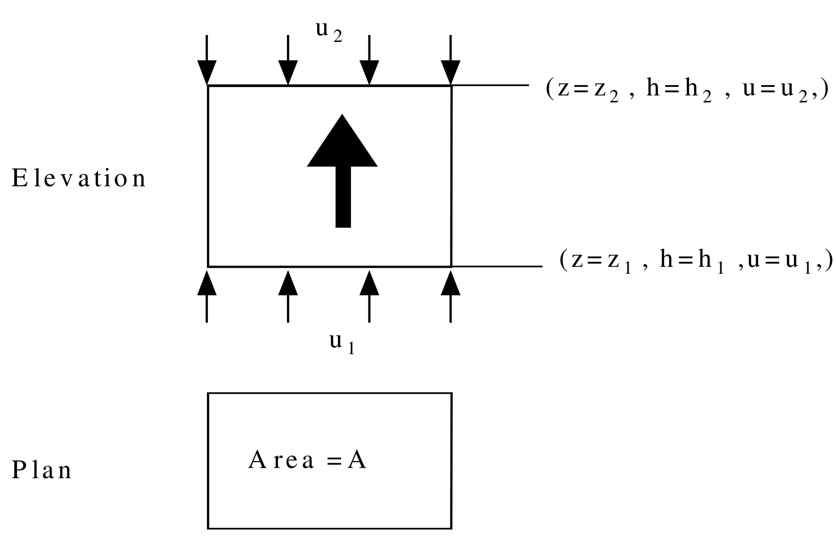

Fig. 8 Analysis of Piping

$$\text{Uplift Force} = A(u_1 - u_2)$$

$$\text{Force due to weight} = A\gamma_{sat}(z_2 - z_1) \qquad (9a)$$

The pore pressure can be calculated from the head and so:

$$u_2 = \gamma_w(h_2 - z_2)$$
and
$$u_1 = \gamma_w(h_1 - z_1) \qquad (9b)$$

For piping to occur the Uplift must be greater than the self-weight of the soil

$$A(u_2 - u_1) > A\gamma_{sat}(z_2 - z_1) \qquad (9c)$$

$$\gamma_w(h_1 - h_2) - \gamma_w(z_1 - z_2) > \gamma_{sat}(z_2 - z_1)$$

$$\gamma_w(h_1 - h_2) > \gamma_{sat}(z_2 - z_1) - \gamma_w(z_2 - z_1)$$

$$\frac{(h_1 - h_2)}{(z_2 - z_1)} > \frac{\gamma_{sat} - \gamma_w}{\gamma_w} \qquad (9d)$$

or alternatively

$$i > i_{crit}$$

where

i = hydraulic gradient $= \dfrac{h_1 - h_2}{z_2 - z_1}$

and

i_{crit} = critical hydraulic gradient $= \dfrac{\gamma_{sat} - \gamma_w}{\gamma_w}$

Example

Suppose the dam shown in Figure 6 is 39 metres wide (this may be determined from the scale drawing), the water levels are the same as in the previous example (H_1 = 13 m, H_2 = 2.5 m), and the saturated unit weight of the soil is 18 kN/m^3. Piping is most likely to occur at the toe of the dam, the hydraulic gradient there can be obtained from the flow net:

$h_1 - h_2 = \Delta h = 0.75$ m (calculated from Fig. 6)

$z_2 - z_1 = 1.125$ m (scaled from Fig. 6)

thus

$$i = \frac{0.75}{1.125} = 0.67$$

Now

$$i_{crit} = \frac{18 - 9.81}{9.81} = 0.83 \tag{10}$$

The safety factor against piping failure is thus $i_{crit}/i = 0.83/0.67 = 1.25$ which is probably not adequate given the potentially disastrous consequences of a piping failure.