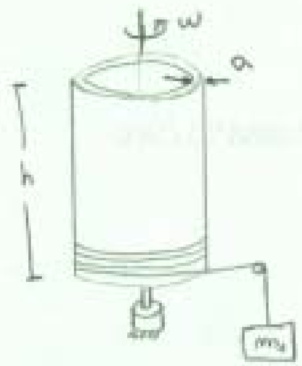
PAUTA P2 GUÍA



ECUACIONES BÁSICAS:

$$\tau = \mu \frac{du}{dy}$$

$$\sum F = m \cdot a$$

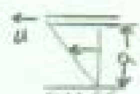
$$\sum M = I \cdot \alpha$$

SUPOSICIONES:

FLUIDO NEWTONIANO

PERFIL DE VELOCIDAD LINEAL

EN EL CLARO:



$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{a} = \frac{\mu R \omega}{a}$$

$$T = \tau \cdot A \cdot R = \frac{\mu R \omega}{a}(2\pi R h) \cdot R \rightarrow \boxed{T = \frac{2\pi R^3 \mu h}{a} \cdot \omega}$$

(τ ES UN ESFUERZO, i.e. TIENE UNIDADES DE $\vec{F}/A$, Y T ES TORQUE, POR LO QUE SUS UNIDADES SON $F \cdot r = \frac{F}{A} \cdot A \cdot r$)

SI LLAMAMOS F_c A LA TENSIÓN EN LA CUERDA:

CILINDRO: $$\sum M = F_c R - T = I \cdot \alpha = m_2 \cdot R^2 \frac{d\omega}{dt} \quad (1)$$

MASA: $$\sum F_y = m_1 g - F_c = m_1 \cdot a = m_1 \frac{dV}{dt} = m_1 R \frac{d\omega}{dt} \quad (2)$$

$$\therefore F_c = m_1 g - m_1 R \frac{d\omega}{dt}$$

SUSTITUYENDO F_c EN (1):

$$\left(m_1 g - m_1 R \frac{d\omega}{dt}\right) \cdot R - T = m_2 R^2 \frac{d\omega}{dt} \Rightarrow m_1 g \cancel{R} - m_1 R^{\cancel{2}} \frac{d\omega}{dt} - \frac{2\pi R^{\cancel{3}\,2} \mu h}{a} \cdot \omega = m_2 R^{\cancel{2}} \frac{d\omega}{dt}$$

$$\Rightarrow m_1 g - m_1 R \frac{d\omega}{dt} - m_2 R \frac{d\omega}{dt} = \frac{2\pi R^2 \mu h}{a} \omega$$

$$\Rightarrow -R(m_1 + m_2) \frac{d\omega}{dt} + m_1 g = \frac{2\pi R^2 \mu h}{a} \omega \rightarrow R(m_1 + m_2) \frac{d\omega}{dt} = m_1 g - \frac{2\pi R^2 \mu h}{a} \omega$$

PARA DISMINUIR NOTACIÓN: $R(m_1 + m_2) = b$

$m_1 g = c$

$\frac{2\pi R^2 \mu h}{a} = e$

$$\Rightarrow b \cdot \frac{d\omega}{dt} = c - e\omega \Rightarrow \frac{b}{c - e\omega} \frac{d\omega}{dt} = 1 \Rightarrow \frac{b}{c - e\omega} d\omega = dt \rightarrow b \int_0^\omega \frac{d\omega}{c - e\omega} = \int_0^t dt$$

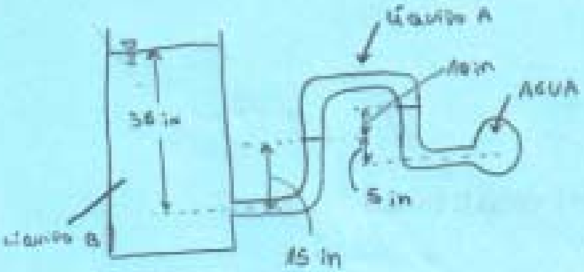
$$\rightarrow -\frac{b}{e} \cdot \ln(c - e\omega) = t \rightarrow \ln(c - e\omega) = \frac{-e \cdot t}{b} \rightarrow c - e\omega = \mathrm{EXP}\left(\frac{-et}{b}\right) \rightarrow -e\omega = \mathrm{EXP}\left(\frac{-et}{b}\right) - c$$

$$\Rightarrow \omega = \left(-\mathrm{EXP}\left(\frac{-et}{b}\right) + c\right) \frac{1}{e} \Rightarrow \omega = \frac{a}{2\pi R^2 \mu h}\left(m_1 g - \mathrm{EXP}\left(\frac{-2\pi R^{\cancel{2}} \mu h}{a \cdot \cancel{R}(m_1 + m_2)} \cdot t\right)\right) = \boxed{\frac{a}{2\pi R^2 \mu h}\left(m_1 g - \mathrm{EXP}\left(\frac{-2\pi R \mu h}{a\,(m_1 + m_2)} \cdot t\right)\right) = \omega(t)}$$

ω_{MAX} OCURRE CUANDO $t \rightarrow \infty$

$$\Rightarrow \omega_{MAX} = \frac{a}{2\pi R^2 \mu h}(m_1 g - 0) \Rightarrow \boxed{\omega_{MAX} = \frac{m_1 \cdot g \cdot a}{2\pi R^2 \mu h}}$$

PAUTA P4 GUÍA



ECUACIÓN BÁSICA: $\frac{dP}{dz} = -\rho \cdot g \cdot dz$

SUPOSICIONES: FLUIDO ESTÁTICO

- γ = cte.
- LA ÚNICA FUERZA PRESENTE ES LA GRAVEDAD

$dP = -\gamma dz$

como $\gamma = cte \Rightarrow \Delta P = -\gamma \Delta z \rightarrow P_j - P_i = -\gamma (z_j - z_i)$

$P_2 - P_1 = -\gamma_B (z_2 - z_1)$

$P_3 - P_2 = -\gamma_B (z_3 - z_2)$

$P_4 - P_3 = -\gamma_A (z_4 - z_3)$

$P_5 - P_4 = -\gamma_{H_2O} (z_5 - z_4)$

Si: $P_5 = P_a \wedge P_1 = P_{ATM} \Rightarrow P_a - P_{ATM} = -\gamma_B (z_3 - z_1) - \gamma_A (z_4 - z_3) - \gamma_{H_2O} (z_5 - z_4)$

$$= 1{,}2 \cdot 62{,}4 \frac{lb}{ft^3} \cdot 32{,}2 \frac{ft}{s^2} \cdot 21\, \cancel{in} \cdot \frac{ft}{12\, \cancel{in}} - 0{,}95 \cdot 62{,}4 \frac{lb}{ft^3} \cdot 32{,}2 \frac{ft}{s^2} \cdot \frac{10}{12} ft$$

$$+ 62{,}4 \frac{lb}{ft^3} \cdot 32{,}2 \frac{ft}{s^2} \cdot \frac{15}{12} ft$$

$\Rightarrow P_a - P_{ATM} = 5140{,}408 \frac{lb}{ft\, s^2}$

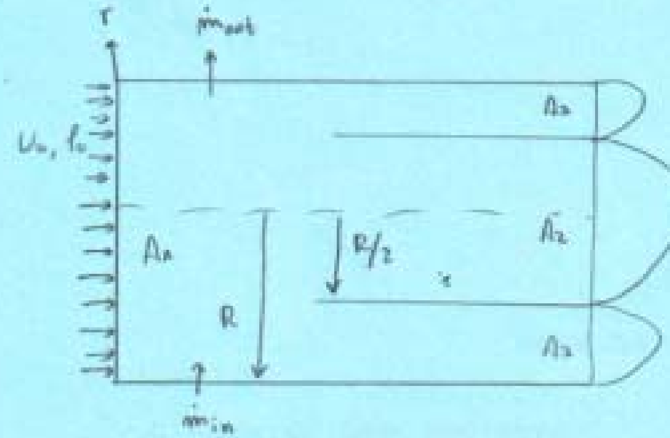
$P_a - P_{ATM} = 7649{,}7698\ Pa$

$P_a = (7649{,}7698 + 10^5)\ Pa$

$\Rightarrow \boxed{P_a = 107649{,}77\ [Pa]}$

$\boxed{P_a = 15{,}613\ [Psi]}$

PAUTA P6



ECUACIÓN BÁSICA

$$\frac{\partial}{\partial t}\int_{VC} \rho \, dV + \int_{SC} \rho V \cdot dA = 0$$

COMO EL FLUJO ES ESTACIONARIO, LA DERIVADA TEMPORAL ES NULA

$$\Rightarrow \int_{SC} \rho V \cdot dA = 0 \Leftrightarrow \int_{A_1} \rho V dA + \int_{A_2} \rho V dA + \int_{A_3} \rho V dA + \dot{m}_{in} - \dot{m}_{out} = 0$$

COMO NO NOS INTERESA $\dot{m}_{in}$ NI $\dot{m}_{out}$ EN PARTICULAR, SINO SU SUMA: $\dot{m}_{in} - \dot{m}_{out} = \dot{m}_{in/out}$

$$\dot{m}_{A_1} = \int_{A_1} \rho V dA = \int_0^R \int_0^{2\pi} \rho_0 \cdot U_0 \cdot r \, d\theta \, dr = \int_0^R 2\pi \rho_0 U_0 r \, dr = \cancel{2}\pi \rho_0 U_0 \frac{R^2}{\cancel{2}} \rightarrow \boxed{\dot{m}_{A_1} = \pi \rho_0 U_0 R^2} \text{ (ENTRA)}$$

$$\dot{m}_{A_2} = \int_{A_2} \rho V dA = \int_0^{R/2} \int_0^{2\pi} \frac{\rho_0}{2} \cdot U_{max} \left(1 - \frac{2r}{R}\right)^2 r \, d\theta \, dr = \frac{\rho_0 U_{max} \cdot 2\pi}{2} \int_0^{R/2} r\left(1 - \frac{2r}{R}\right)^2 dr$$

$$\Rightarrow \dot{m}_{A_2} = \rho_0 U_{max} \cdot \pi \int_0^{R/2} r\left(1 - \frac{4r}{R} + \frac{4r^2}{R^2}\right) dr = \pi \rho_0 U_{max} \int_0^{R/2} \left(r - \frac{4r^2}{R} + \frac{4r^3}{R^2}\right) dr$$

$$\rightarrow \dot{m}_{A_2} = \pi \rho_0 U_{max} \left(\frac{(R/2)^2}{2} - \frac{4}{R}\frac{(R/2)^3}{3} + \frac{\cancel{4}}{R^2}\frac{(R/2)^4}{\cancel{4}}\right) = \pi \rho_0 U_{max}\left(\frac{R^2}{8} - \frac{\cancel{4}}{3}\frac{R^{\cancel{3}}}{\cancel{R}\cdot 8} + \frac{R^{\cancel{4}}}{16 \cancel{R^2}}\right)$$

$$\Rightarrow \dot{m}_{A_2} = \pi \rho_0 U_{max}\left(\frac{R^2}{8} - \frac{R^2}{6} + \frac{R^2}{16}\right) = \pi \rho_0 U_{max} R^2 \left(\frac{1}{8} - \frac{1}{6} + \frac{1}{16}\right) = \pi \rho_0 U_{max} R^2 \left(\frac{6 - 8 + 3}{48}\right)$$

$$\Rightarrow \boxed{\dot{m}_{A_2} = \frac{\pi \rho_0 U_{max} R^2}{48}} \text{ (SALE)}$$

$$\dot{m}_{A_3} = \int_{A_3} \rho V dA = \int_{R/2}^R \int_0^{2\pi} \rho_0 \cdot \frac{U_0}{2}\left[1 - \left(\frac{2r}{R}\right)^2 + \frac{3}{\ln(2)} \cdot \ln\left(\frac{2r}{R}\right)\right] r \, d\theta \, dr = \frac{\rho U_0}{\cancel{2}} \cancel{2}\pi \int_{R/2}^R \left[r - \frac{4r^3}{R^2} + \frac{3r}{\ln(2)} \cdot \ln\left(\frac{2r}{R}\right)\right] dr$$

$$\rightarrow \dot{m}_{A_3} = \pi \rho_0 U_0 \left(\frac{r^2}{2}\Big|_{R/2}^R - \frac{\cancel{4}}{R^2} \cdot \frac{r^4}{\cancel{4}}\Big|_{R/2}^R + \frac{3}{\ln(2)} \int_{R/2}^R r \ln\left(\frac{2r}{R}\right) dr\right) = \pi \rho_0 U_0 \left(\frac{1}{2}\left(R^2 - \left(\frac{R}{2}\right)^2\right) - \frac{1}{R^2}\left(R^4 - \left(\frac{R}{2}\right)^4\right) + \frac{3}{\ln(2)} \int_{R/2}^R r \ln\left(\frac{2r}{R}\right) dr\right)$$

$$\rightarrow \dot{m}_{A_3} = \pi \rho_0 U_0 \left(\frac{1}{2}\left(\frac{3R^2}{4}\right) - \frac{1}{R^2} \cdot \frac{15R^4}{16} + \frac{3}{\ln(2)}\left(\ln\left(\frac{2r}{R}\right) \cdot \frac{r^2}{2}\Big|_{R/2}^R - \int_{R/2}^R \frac{r^2}{\cancel{2}} \cdot \frac{\cancel{R}}{2r} \cdot \frac{\cancel{2}}{\cancel{R}} dr\right)\right) = \pi \rho_0 U_0 \left(\frac{3R^2}{8} - \frac{15R^2}{16} + \frac{3}{\ln(2)}\left(\frac{1}{2} r^2 \ln\left(\frac{2r}{R}\right)\Big|_{R/2}^R - \int_{R/2}^R \frac{r}{2} dr\right)\right)$$

$$\rightarrow \dot{m}_{A_3} = \pi \rho_0 U_0 \left(\frac{3R^2}{8} - \frac{15R^2}{16} + \frac{3}{\ln(2)}\left(\frac{1}{2}\left(R^2 \ln\left(\frac{2R}{R}\right) - \left(\frac{R}{2}\right)^2 \cancel{\ln\left(\frac{R}{R}\right)}^{0}\right) - \frac{1}{2}\frac{r^2}{2}\Big|_{R/2}^R\right)\right) = \pi \rho_0 U_0 \left(\frac{3R^2}{8} - \frac{15R^2}{16} + \frac{3}{\ln(2)}\left(\frac{R^2}{2}\ln(2) - \frac{1}{4}\left(R^2 - \left(\frac{R}{2}\right)^2\right)\right)\right)$$

$$\rightarrow \dot{m}_{A_3} = \pi \rho_0 U_0 \left(\frac{3R^2}{8} - \frac{15R^2}{16} + \frac{3R^2}{2} - \frac{3}{4\ln(2)} \cdot \frac{3R^2}{4}\right) = \pi \rho_0 U_0 \left(\frac{3R^2}{8} - \frac{15R^2}{16} + \frac{3R^2}{2} - \frac{9R^2}{16\ln(2)}\right)$$

$$\rightarrow \boxed{\dot{m}_{A_3} = \pi \rho_0 U_0 R^2 \left(\frac{15}{16} - \frac{9}{16\ln(2)}\right)} \text{ (SALE)}$$

SIGUE →

PARA CALCULAR $\dot{m}_{in/out}$:

$$\dot{m}_{A_1} - \dot{m}_{A_2} - \dot{m}_{A_3} + \dot{m}_{in/out} = 0$$

- Si $\dot{m}_{in/out} > 0 \Rightarrow$ ENTRA
- " " $< 0 \Rightarrow$ SALE

$$\Rightarrow \dot{m}_{in/out} = \dot{m}_{A_2} + \dot{m}_{A_3} - \dot{m}_{A_1} = \frac{\pi \cdot \rho_0 \, U_{max} \cdot R^2}{48} + \pi \cdot \rho_0 U_0 R^2 \left(\frac{15}{16} - \frac{9}{16 \ln(2)} \right) - \pi \rho_0 U_0 R^2$$

Como $U_{max} = 2{,}5\, U_0$:

$$\dot{m}_{in/out} = \pi \rho_0 U_0 R^2 \left(\frac{2{,}5}{48} + \left(\frac{15}{16} - \frac{9}{16 \ln(2)} \right) - 1 \right)$$

$$\Rightarrow \boxed{\dot{m}_{in/out} = -2{,}58\, \rho_0 U_0 R^2}$$

$$\rightarrow \boxed{\dot{m}_{in/out} = 2{,}58\, \rho_0 U_0 R^2}$$ (SALE)