AUXILIAR # 1

19/03/2008

Profesor: ROGER BUSTAMANTE

$NF = 0.7 \cdot NC_{DE} + 0.3 \cdot NT$

$NT = \left[\sum_{i=1}^{n} NT_i\right] \frac{1}{n}$, n = # tareas

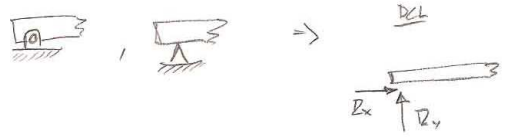
$NC_{DE} = \frac{C_1 + C_2 + C_3 + Ex'}{4}$; (DE: después del examen, el examen reemplaza la peor nota).

$NC_{AE} = \frac{C_1 + C_2 + C_3}{3}$, (AE = antes del examen)

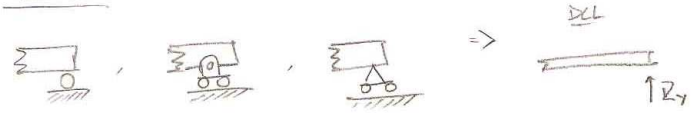
EXIMICIÓN	APROBACIÓN	FECHA CONTROLES
$NC_{AE} \geq 5.5$	$NF \geq 4.0$	C_1 - 02 Abril
$NT \geq 4.0$	(confirmar con el profe.)	C_2 - 07 Mayo
		C_3 - 11 Junio

APOYOS Y REACCIONES

- pasador



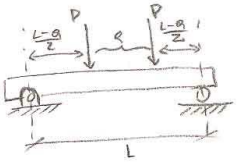
- rodillo



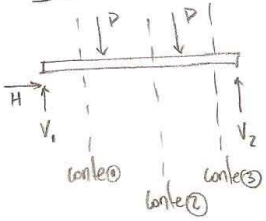
- empotramiento

EJ # 1

Determinar las reacciones y los diagramas $M(x)$, $V(x)$.



DCL



$\Sigma F_x = 0 \Rightarrow \boxed{H = 0}$

$\Sigma F_y = 0 \Rightarrow \boxed{V_1 + V_2 = 2P}$

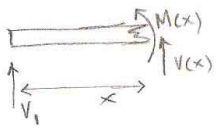
$\Sigma M_2 = 0 \Rightarrow -\left(\frac{L-a}{2}\right)P - \left(\frac{L-a}{2} + a\right)P + LV_2 = 0$

$\Rightarrow V_2 = \frac{P}{L}\left[\frac{L-a}{2} + \frac{L-a}{2} + a\right] \Rightarrow \boxed{V_2 = P}$

$\boxed{V_1 = P}$

CORTES:

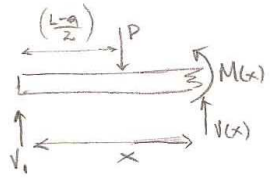
- corte ①:



$\Sigma F_y = 0 \Rightarrow V(x) + V_1 = 0 \Rightarrow \boxed{V(x) = -P}$

$\Sigma M_2 = 0 \Rightarrow -xV_1 + M(x) = 0 \Rightarrow \boxed{M(x) = P \cdot x}$ $\quad x \in \left[0, \frac{L-a}{2}\right)$

- corte ②:

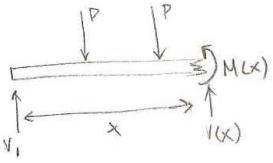


$\Sigma F_y = 0 \Rightarrow V_1 + V(x) - P = 0 \Rightarrow \boxed{V(x) = 0}$

$\Sigma M_2 = 0 \Rightarrow -xV_1 + \left[x - \left(\frac{L-a}{2}\right)\right]P + M(x) = 0$ $\quad x \in \left[\frac{L-a}{2}, \frac{L+a}{2}\right)$

$\Rightarrow M(x) = Px - P\left[x - \left(\frac{L-a}{2}\right)\right] \Rightarrow \boxed{M(x) = P\left(\frac{L-a}{2}\right)}$

- corte ③.



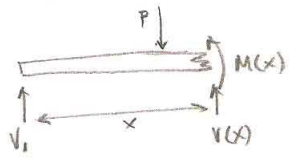
$\Sigma F_y = 0 \Rightarrow V_1 + V(x) - 2P = 0 \Rightarrow \boxed{V(x) = P}$

$\Sigma M_2 = 0 \Rightarrow -xV_1 + \left[x - \left(\frac{L-a}{2}\right)\right]P + \left[x - \left(\frac{L+a}{2}\right)\right]P + M(x) = 0$ $\quad x \in \left[\frac{L+a}{2}, L\right)$

$\Rightarrow M(x) = Px - P\left[x - \left(\frac{L-a}{2}\right) + x - \left(\frac{L+a}{2}\right)\right]$

$\Rightarrow \boxed{M(x) = P(L-x)}$

- corte ②



$\Sigma F_y = 0 \Rightarrow V_1 - P + V(x) = 0 \Rightarrow V(x) = P - \frac{a}{L}P \Rightarrow \boxed{V(x) = P\left(\frac{L-a}{L}\right)}$

$\Sigma M_z = 0 \Rightarrow -xV_1 + \left[x - \left(\frac{L-a}{2}\right)\right]P + M(x) = 0$

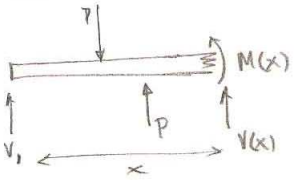
$\Rightarrow M(x) = \frac{a}{L}P \cdot x - P\left[x - \left(\frac{L-a}{2}\right)\right]$

$M(x) = \frac{a}{L}Px - Px + P\left(\frac{L-a}{2}\right)$

$\boxed{M(x) = P\left[x\left(\frac{a}{L} - 1\right) + \left(\frac{L-a}{2}\right)\right]}$

$x \in \left[\frac{L-a}{2}, \frac{L+a}{2}\right)$

- corte ③:



$\Sigma F_y = 0 \Rightarrow V_1 + P - P + V(x) = 0 \Rightarrow \boxed{V(x) = -\frac{a}{L}P}$

$\Sigma M_z = 0 \Rightarrow -xV_1 + \left[x - \left(\frac{L-a}{2}\right)\right]P - \left[x - \left(\frac{L+a}{2}\right)\right]P + M(x) = 0$

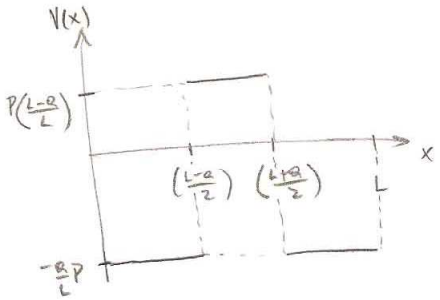
$\Rightarrow M(x) = \frac{a}{L}P \cdot x + P\left[x - \left(\frac{L+a}{2}\right) - x + \left(\frac{L-a}{2}\right)\right]$

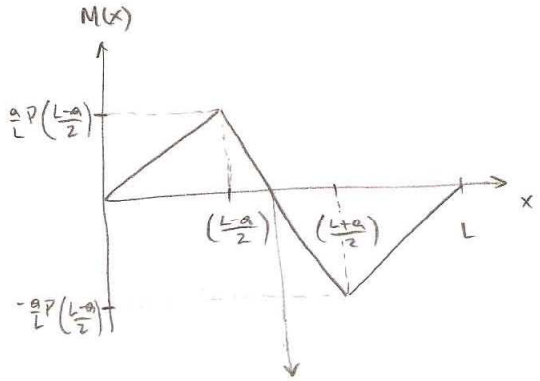
$M(x) = \frac{a}{L}P \cdot x - aP$

$\Rightarrow \boxed{M(x) = aP\left(\frac{x}{L} - 1\right)}$

$x \in \left[\frac{L+a}{2}, L\right)$

DIAGRAMAS:

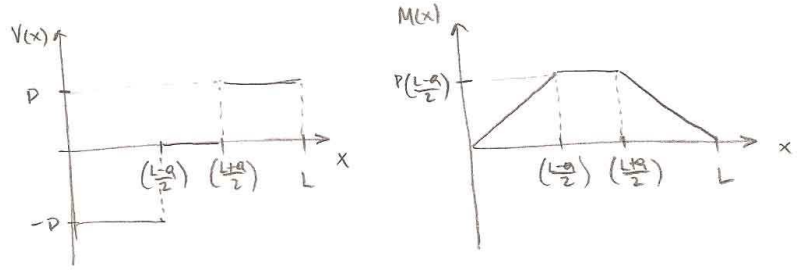




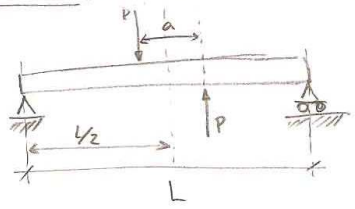
$M(x) = 0 = P\left[x'\left(\frac{a}{L} - 1\right) + \left(\frac{L-a}{2}\right)\right]$

$\Rightarrow x' = \frac{\left(\frac{L-a}{2}\right)}{\left(1 - \frac{a}{L}\right)} = \frac{L-a}{2} \cdot \frac{L}{L-a} \Rightarrow \boxed{x' = L/2}$

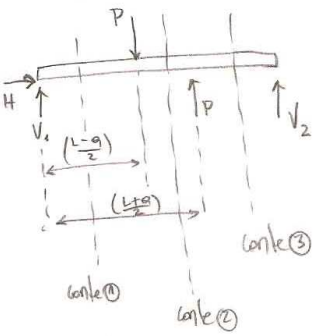
DIAGRAMAS:



EJ # 2 (reacciones, V(x), M(x)).



DCL:



$\Sigma F_x = 0 \Rightarrow \boxed{H = 0}$

$\Sigma F_y = 0 \Rightarrow V_1 + V_2 + P - P = 0 \Rightarrow \boxed{V_1 = -V_2}$

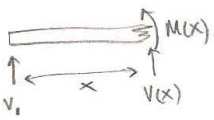
$\Sigma M_2 = 0 \Rightarrow -\left(\frac{L-a}{2}\right)P + \left(\frac{L+a}{2}\right)P + LV_2 = 0$

$\Rightarrow V_2 = \frac{P}{L}\left[\left(\frac{L-a}{2}\right) - \left(\frac{L+a}{2}\right)\right] \Rightarrow \boxed{V_2 = -\frac{a}{L}P}$

$\boxed{V_1 = \frac{a}{L}P}$

CORTES:

- corte ①:

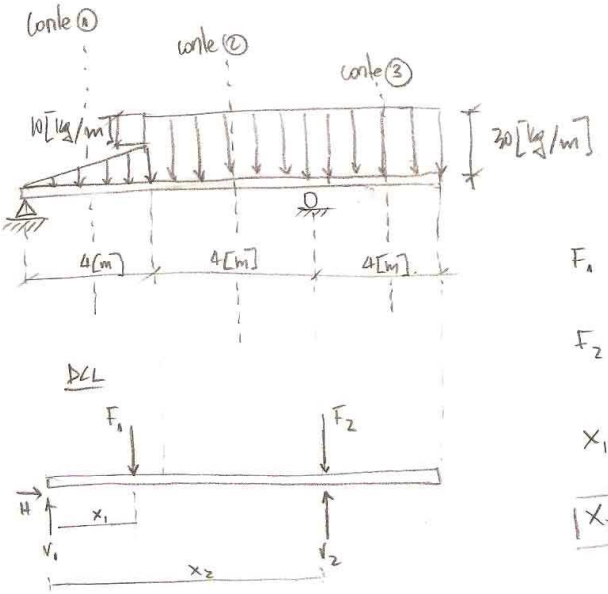


$\Sigma F_y = 0 \Rightarrow V(x) + V_1 = 0 \Rightarrow \boxed{V(x) = -a/L\,P}$

$\Sigma M_2 = 0 \Rightarrow -xV_1 + M(x) = 0 \Rightarrow \boxed{M(x) = a/L \cdot P \cdot x}$

$x \in \left[0, \frac{L-a}{2}\right)$

EJ#3 (reacciones, M(x), V(x))



DCL

$F_1 = 20[kg/m] \cdot 4[m] \cdot \frac{1}{2} \Rightarrow \boxed{F_1 = 40[kg]}$

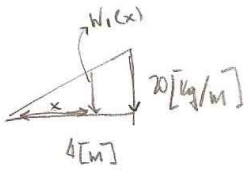
$F_2 = 30[kg/m] \cdot 8[m] \Rightarrow \boxed{F_2 = 240[kg]}$

$X_1 = \frac{2}{3} \cdot 4[m] \Rightarrow \boxed{X_1 = 8/3[m]}$

$\boxed{X_2 = 8[m]}$

otra forma:

$F_1 = \int_0^4 W_1(x)\,dx$;



$\frac{W_1(x)}{x} = \frac{20}{4} \Rightarrow \boxed{W_1(x) = 5 \cdot x\ [kg/m]}$

$F_1 = \int_0^4 5 \cdot x\,dx = 5 \cdot \frac{x^2}{2}\Big|_0^4 \Rightarrow \boxed{F_1 = 40[kg]}$

$\boxed{W_2(x) = 30[kg/m]}$

$F_2 = \int_4^{12} W_2(x)\,dx = \int_4^{12} 30 \cdot dx = 30 \cdot x\Big|_4^{12} \Rightarrow \boxed{F_2 = 240[kg]}$

$\Sigma F_y = 0 \Rightarrow V_1 - F_1 - F_2 + V_2 = 0 \quad \Rightarrow \boxed{V_1 + V_2 = 280\,[kg]}$

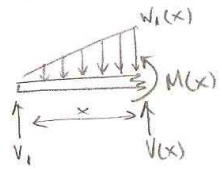
$\Sigma M_2 = 0 \Rightarrow -x_1 F_1 + x_2 (V_2 - F_2) = 0$

$\Rightarrow V_2 = x_1/x_2 \, F_1 + F_2 = \frac{8/3}{8} \cdot 40 + 240 \quad \Rightarrow \boxed{V_2 = 253{,}3\,[kg]}$

$\boxed{V_1 = 26{,}7\,[kg]}$

CORTES:

- corte ①:

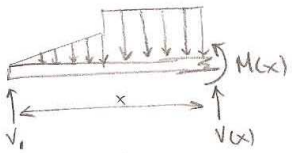


$\Sigma F_y = 0 \Rightarrow V_1 - \frac{5x^2}{2} + V(x) = 0 \quad \Rightarrow \boxed{V(x) = \left(\frac{5x^2}{2} - 26{,}7\right)[kg]}$

$\Sigma M_z = 0 \Rightarrow -xV_1 + \left(\frac{5x^2}{2}\right) \cdot \frac{x}{3} + M(x) = 0$

$\Rightarrow \boxed{M(x) = \left(26{,}7 \cdot x - \frac{5x^3}{6}\right)[kg \cdot m]} \quad x \in [0,4)$

- corte ②:



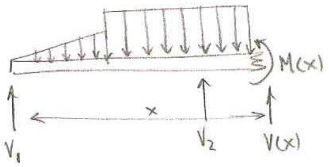
$\Sigma F_y = 0 \Rightarrow V_1 - \frac{20 \cdot 4}{2} - 30(x-4) + V(x) = 0$

$\Rightarrow \boxed{V(x) = (30x - 106{,}7)\,[kg]}$

$\Sigma M_z = 0 \Rightarrow -xV_1 + \left(\frac{20 \cdot 4}{2}\right)\left(x - \frac{2}{3} \cdot 4\right) + 30(x-4)\frac{(x-4)}{2} + M(x) = 0$

$\Rightarrow \boxed{M(x) = \left(-15x^2 + 106{,}7 \cdot x - 133{,}3\right)[kg \cdot m]} \quad x \in [4,8)$

- corte ③:



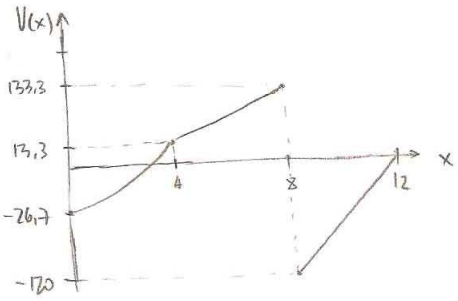
$\Sigma F_y = 0 \Rightarrow V_1 - \frac{20 \cdot 4}{2} - 30(x-4) + V_2 + V(x) = 0$

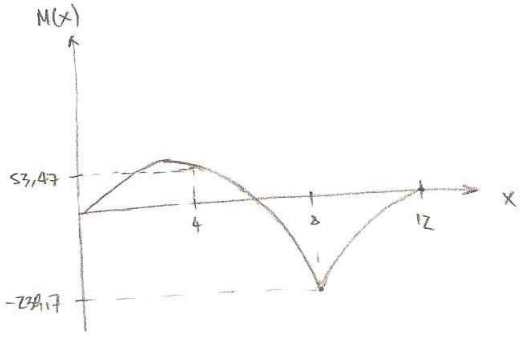
$\Rightarrow \boxed{V(x) = (30 \cdot x - 360)\,[kg]} \rightarrow x \in [8,12)$

$$\Sigma M_z = 0 \Rightarrow -xV_1 + \left(\frac{20 \cdot 4}{2}\right)\left(x - \tfrac{2}{3}\cdot 4\right) + 30(x-4)\frac{(x-4)}{2} - V_2(x-8) + M(x) = 0$$

$$\Rightarrow \boxed{M(x) = (-15x^2 + 360x - 2159.7)\ [kg \cdot m]} \rightarrow x \in [8, 12)$$

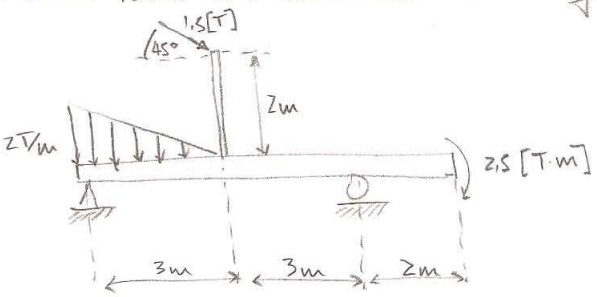
DIAGRAMAS:

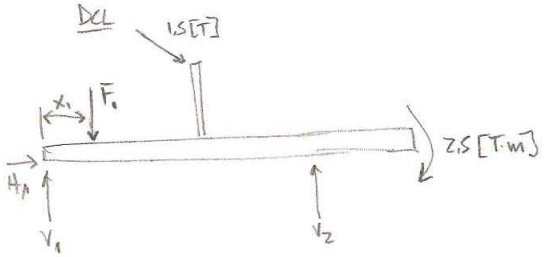




EJ #4

Calculen todas las reacciones de la sgte. figura





$$F_1 = 2[T/m] \cdot 3[m] \cdot \frac{1}{2} \Rightarrow \boxed{F_1 = 3[T]}$$

$$x_1 = \frac{1}{3} \cdot 3[m] \Rightarrow \boxed{x_1 = 1[m]}$$

$$\Sigma F_x = 0 \Rightarrow H_1 + 1{,}5[T] \cdot \cos 45° = 0 \Rightarrow \boxed{H_1 = -1{,}06\ [T]}$$

$$\Sigma F_y = 0 \Rightarrow V_1 - F_1 - 1{,}5[T] \sin 45° + V_2 = 0$$

$$\Rightarrow \boxed{V_1 + V_2 = 4{,}06\ [T]}$$

$\Sigma M_z = 0 \Rightarrow -x_1 F_1 - 2 \cdot 1{,}5 \cos 45° - 3 \cdot 1{,}5 \cdot \operatorname{sen} 45° + 6 V_2 - 2{,}5 = 0$

$$\rightarrow V_2 = \frac{1 \cdot 3 + 2 \cdot 1{,}5 \cdot \cos 45° + 3 \cdot 1{,}5 \cdot \operatorname{sen} 45° + 2{,}5}{6} \Rightarrow \boxed{V_2 = 1{,}8\ [T]}$$

$$\boxed{V_1 = 2{,}26\ [T]}$$

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→ U-CURSOS