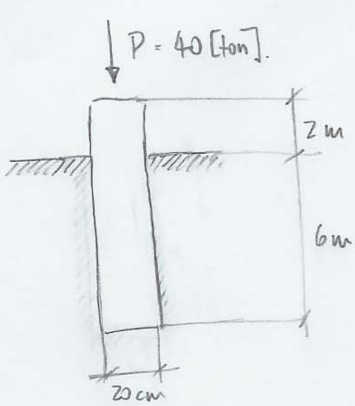
AUXILIAR # 2 26/03/08 ME46A

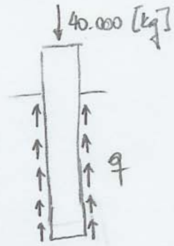
Prof: Roger Bustamante

P1



- Datos: * $E = 150.000 \ [kg/cm^2]$

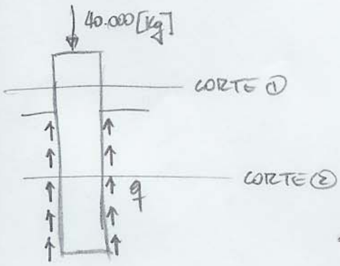
* DCL.



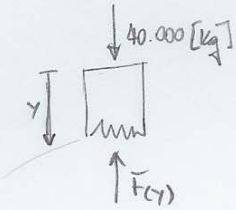
a) Calcular la variación de longitud del pivote.

$$\sum F_y = 0 \Rightarrow q \, (\underbrace{20[cm] \cdot 6[m]}_{600[cm]}) \cdot 4 - 40.000 \, [kg] = 0 \Rightarrow \boxed{q = 0{,}833 \, [kg/cm^2]}$$

CORTES:



CORTE 1:

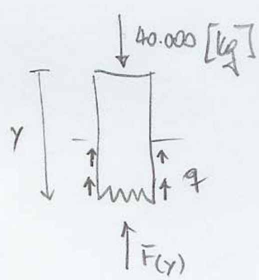


$$\sum F_y = 0 \Rightarrow 40.000 \, [kg] - F(y) = 0$$

$$\Rightarrow \boxed{F(y) = 40.000 \, [kg]}$$

para $y \in [0, 200)$.

CORTE 2:



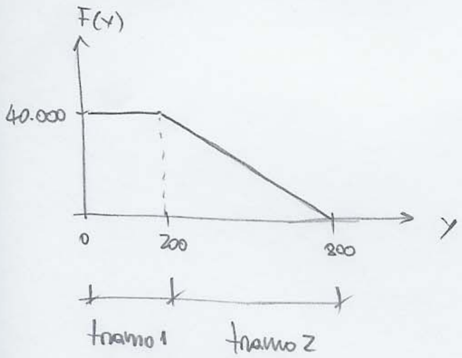
$$\sum F_y = 0 \Rightarrow 40.000 - F(y) - q \, (y - 200) \cdot 20 \cdot 4 = 0$$

$$\Rightarrow \boxed{F(y) = (53328 - 66{,}64 \cdot y) \, [kg]}$$

para $y \in [200, 800)$.

(y en [cm])

DIAGRAMA (NO ES NECESARIO)



- para el tramo 1, se cumple que:

$* \ \sigma = F/A = \varepsilon E = \frac{\Delta L}{L} \cdot E \quad \Rightarrow \quad \boxed{\Delta L = \frac{F L}{E A}}$

$* \ F = F(y) = 40.000 \ [kg]$

$* \ L = L_{1} = 200 \ [cm]$

$$\Rightarrow \Delta L_{1} = \frac{40000 \ [\cancel{kg}] \cdot 200 \ [cm]}{150.000 \left[\cancel{kg/cm^2}\right] \cdot 20^2 \ [\cancel{cm^2}]} \quad \Rightarrow \quad \boxed{\Delta L_{1} = 0,133 \ [cm]}$$

- para el tramo 2 se cumple que:

$* \ \sigma = F/A = \varepsilon E = \frac{dl}{dy} \cdot E \quad$ y como $\quad F = F(y)$

$$\Rightarrow \frac{F(y)}{A} = \frac{dl}{dy} E \quad \Rightarrow \quad dl = \frac{1}{EA} \cdot F(y)\, dy \quad \Big/ \int_{200}^{800}$$

$$\Rightarrow \Delta L_{2} = \frac{1}{EA} \left(53328 \cdot y - 66,64 \frac{y^2}{2}\right)\Bigg|_{200}^{800}$$

$$\Delta L_{②} = \frac{1}{150000\,[\cancel{kg}/\cancel{cm^2}] \cdot 20^2\,[\cancel{cm^2}]} \left[53328(800-200) - \frac{66,64}{2}(800^2 - 200^2) \right] [\cancel{kg} \cdot cm]$$

$$\Rightarrow \boxed{\Delta L_{②} = 0,2\ [cm]} \qquad \therefore \boxed{\Delta L_{TOTAL} = \Delta L_{①} + \Delta L_{②} = 0,333\ [cm]}$$

$$\boxed{L_F = L_i - \Delta L_{TOTAL} = 799,667\ [cm]}$$

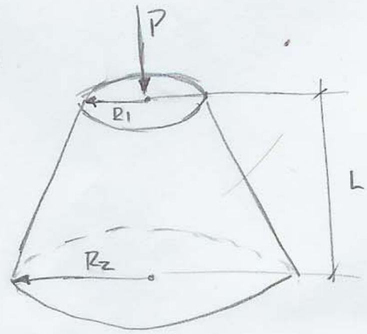
b) Calcule el esfuerzo máximo.

- tramo ①: $\sigma_1 = \dfrac{40.000\ [kg]}{20^2\ [cm^2]} \Rightarrow \boxed{\sigma_1 = 100\ [kg/cm^2]}$

- tramo ②: $\sigma_2 = \dfrac{(53328 - 66,64 \cdot 200)\ [kg]}{20^2\ [cm^2]} \Rightarrow \boxed{\sigma_2 = 100\ [kg/cm^2]}$

$\therefore \boxed{\sigma_{máx} = 100\,[kg/cm^2]}$ //

P2.

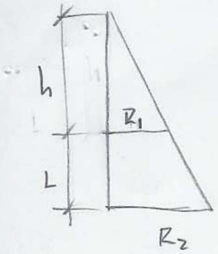


- Determinar cuánto se deforma el sólido producto de la fuerza P y de su peso propio.

- Datos:
  * E [kg/m²]
  * P [kg]
  * L [m]
  * R_1 [m]
  * R_2 [m]
  * γ [kg/m³]

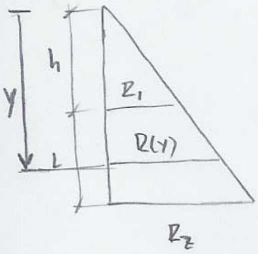
- $\sigma = F/A$, tanto F como A dependen de y $\Rightarrow$ $F = F(y)$, $A = A(y)$.

- para obtener $A(y)$ necesitamos conocer $r(y)$.



$$\Rightarrow \frac{h}{R_1} = \frac{h+L}{R_2} \Rightarrow h\,\frac{R_2}{R_1} = h+L$$

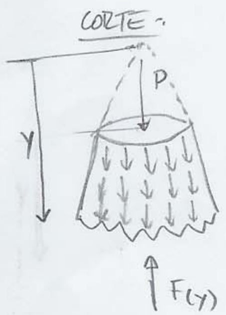
$$\Rightarrow h = \frac{L}{R_2/R_1 - 1} \Rightarrow \boxed{h = \frac{R_1 \cdot L}{R_2 - R_1}}$$



$$\Rightarrow \frac{y}{R(y)} = \frac{h}{R_1} \Rightarrow R(y) = \frac{R_1 \cdot y}{h} = \frac{\cancel{R_1} \cdot y}{\frac{\cancel{R_1} \cdot L}{(R_2 - R_1)}}$$

$$\Rightarrow \boxed{R(y) = \frac{(R_2 - R_1) \cdot y}{L}}$$

- para conocer $F = F(y)$ hacemos un corte:



$$\Sigma F_y = 0 \Rightarrow P - F(y) + \gamma \cdot V(y) = 0$$

$$\boxed{F(y) = P + \gamma V(y)}$$

pero $V(y) = \frac{1}{3}(\pi R^2(y) \cdot y) - \frac{1}{3}(\pi \cdot R_1^2) \cdot h$

$$\boxed{V(y) = \frac{\pi}{3}\left[\frac{R_1^2 \cdot y^3}{h^2} - R_1^2 h\right]}$$

- ahora obtenemos la deformación:

$$\sigma(y) = \frac{F(y)}{A(y)} = \varepsilon E = \frac{dl}{dy} \cdot E \Rightarrow \frac{1}{E}\,\frac{F(y)}{A(y)} \cdot dy = dl \quad // \int_h^{h+L}$$

$$\Rightarrow \frac{1}{E}\int_h^{h+L} \frac{P + \gamma \frac{\pi}{3}\left[\frac{R_1^2 y^3}{h^2} - R_1^2 h\right]}{\pi \cdot \frac{R_1^2 y^2}{h^2}}\, dy = \frac{1}{E}\int_h^{h+L} \frac{P + \gamma \frac{\pi}{3} \frac{R_1^2}{h^2}\left(y^3 - h^3\right)}{\frac{\pi R_1^2 y^2}{h^2}}\, dy = \Delta L$$

$$\Rightarrow \Delta L = \frac{1}{E}\left[\frac{Ph^2}{\pi R_1^2} \int_h^{h+L} \frac{dy}{y^2} + \frac{\cancel{\gamma\pi/3\, R_1^2/h^2}}{\cancel{\pi R_1^2/h^2}} \left(\int_h^{h+L} y\,dy - h^3 \int_h^{h+L} dy/y^2 \right) \right]$$

$$\Delta L = \frac{1}{E}\left[\frac{Ph^2}{\pi R_1^2} \left(\frac{1}{h} - \frac{1}{h+L} \right) + \gamma/3 \left(\frac{(h+L)^2 - h^2}{2} - h^3 \left(\frac{1}{h} - \frac{1}{h+L} \right) \right) \right]$$

$$\Delta L = \frac{1}{E}\left[\frac{Ph^2}{\pi R_1^2} \frac{L}{h(h+L)} + \gamma/3 \left(\frac{2hL + L^2}{2} - h^3 \cdot \frac{L}{h(h+L)} \right) \right]$$

$$\Delta L = \frac{1}{E}\left[\frac{PhL}{\pi R_1^2 (h+L)} + \gamma/3 \left(\frac{(2hL + L^2)h(h+L) - 2h^3 L}{2h(h+L)} \right) \right]$$

$$\Rightarrow \boxed{\Delta L = \frac{1}{E}\left[\underbrace{\frac{PhL}{\pi R_1^2 (h+L)}}_{\text{dif. debido a la fza. P}} + \underbrace{\gamma/6 \; \frac{L^2(3h+L)}{(h+L)}}_{\text{dif. debido al peso propio.}} \right]}$$