AUXILIAR # 3

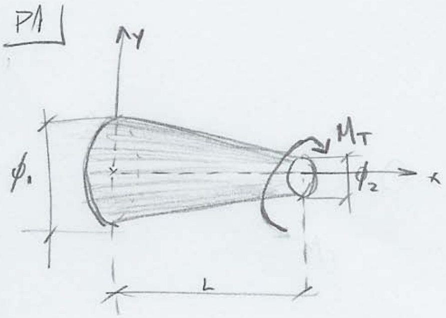
09/04/08

Prof: Rogen Bustamante

ME46A

I

P1



- CALCULAR EL ESFUERZO DE CORTE MÁXIMO Y LA ROTACIÓN, DEL EXTREMO LIBRE.

- DATOS:

$\phi_1 = 15\, cm$

$\phi_2 = 5\, cm$

$L = 50\, cm$

$M_T = 27000\, [kg\, cm]$

$G = 8{,}4 \times 10^5\, [kg/cm^2]$

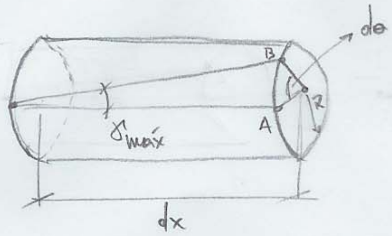
- ESFUERZO MÁXIMO EN EL EXTREMO LIBRE

$$\tau = \frac{M_T\, r}{J} \quad ; \quad J = \frac{\pi \phi^4}{32}$$

$\tau_{máx} \Rightarrow$ evaluar τ en $r = \phi_2/2$

$$\tau_{máx} = \frac{27000\,[kg\,cm]\cdot \frac{5}{2}\,[cm]}{\frac{\pi \cdot 5^4}{32}\,[cm^4]} \Rightarrow \boxed{\tau_{máx} = 1100\, [kg/cm^2]}$$

- ROTACIÓN DEL EXTREMO LIBRE



$$\widehat{AB} = R\,d\theta = dx\,\gamma_{máx}$$

$$\Rightarrow d\theta = \frac{\gamma_{máx}}{R}\,dx \quad (1)$$

pero $\gamma_{máx} = \dfrac{\tau_{máx}}{G}$ (2)

(2) en (1)

$$\Rightarrow d\theta = \frac{\tau_{máx}}{GR}\,dx \quad (3)$$

pero: $\tau_{máx} = \dfrac{M_T \cdot (\phi/2)}{J}$ $(\phi/2) \rightarrow R$ ④

① en ③ $\Rightarrow d\theta = \dfrac{M_T \cdot \cancel{(\phi/2)}}{J} \cdot \dfrac{1}{G\cancel{R}} dx \Rightarrow \boxed{d\theta = \dfrac{M_T}{JG} dx}$

CASOS POSIBLES:

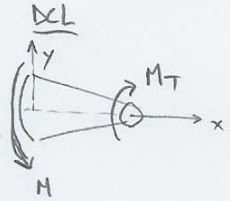
$\rightarrow M_T = M_T(x)$

$\rightarrow J = J(x)$

$\rightarrow G = cte$ (siempre)

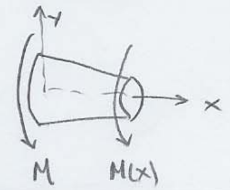
- en este caso $M_T = M_T(x) \rightarrow$ NO (se comprueba haciendo cortes y graficando $M(x)$)

DCL



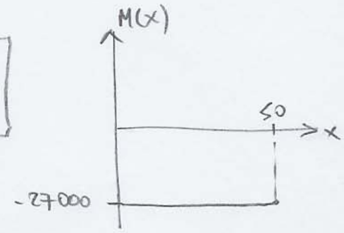
$\Sigma M_x = 0 \Rightarrow M - M_T = 0 \Rightarrow \boxed{M = 27000 \text{ [kg cm]}}$

CORTE ①



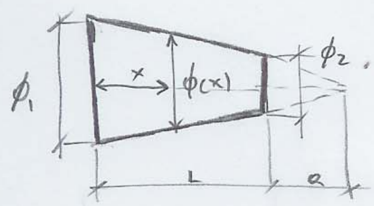
$\Sigma M_x = 0 \Rightarrow M + M(x) = 0 \Rightarrow \boxed{M(x) = -27000 \text{ [kg cm]}}$

$\hookrightarrow x \in [0, 50)$



$\Rightarrow \underline{M(x) = cte}$

- en este caso $J = J(x) \rightarrow$ si (ya que se trata de un cono, por lo que varía su radio a medida que se avanza en x, osea $\phi = \phi(x)$ y como $J = J(\phi) \Rightarrow J = J(x)$)



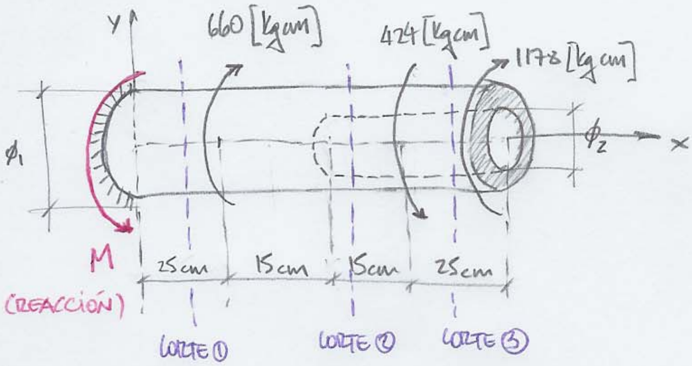
$\dfrac{\phi_1/2}{L+a} = \dfrac{\phi_2/2}{a} \Rightarrow a = \dfrac{\phi_2 L}{\phi_1 - \phi_2} = \dfrac{5 \cdot 50}{15 - 5} \Rightarrow \boxed{a = 25 \text{ [cm]}}$

$\dfrac{\phi(x)/2}{L+a-x} = \dfrac{\phi_2/2}{a} \Rightarrow \phi(x) = \dfrac{(L+a-x)}{a}\phi_2 \Rightarrow \boxed{\phi(x) = \dfrac{75 - x}{5}}$

$$\Rightarrow d\theta = \frac{M_T}{JG} dx = \frac{-27000 \text{ [kg cm]}}{\dfrac{\pi (75-x)^4}{32 \cdot 5^4} \cdot [cm^4] \cdot 8{,}4 \times 10^5 \left[\text{kg/cm}^2\right]} dx \text{ [cm]} \quad \Big/ \int_0^{50}$$

$\Rightarrow \boxed{\begin{array}{l} \theta = 0{,}004204 \text{ [rad]} \\ \theta = 0{,}241° \end{array}}$

P2) - DETERMINAR EL ESFUERZO DE CORTE MÁXIMO
- DETERMINAR EL ÁNGULO QUE GIRA LA SECCIÓN LIBRE.

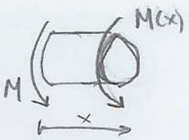


Datos: $G = 0{,}84 \times 10^6$ [kg/cm²]
$\phi_1 = 5$ [cm]
$\phi_2 = 2{,}5$ [cm]

$\sum M_x = 0 \Rightarrow M - 660 + 424 - 1178 = 0 \Rightarrow \boxed{M = 1414\ [kg \cdot cm]}$

CORTES:

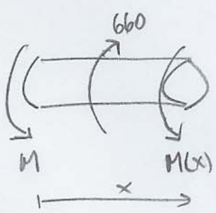
- CORTE ①:



$\sum M_x = 0 \Rightarrow M + M(x) = 0 \Rightarrow \boxed{M(x) = -1414\ [kg\,cm]}$

↳ $x \in [0, 25)$

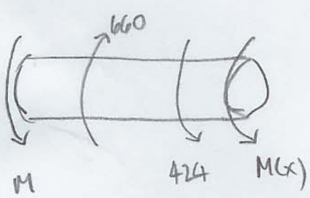
- CORTE ②:



$\sum M_x = 0 \Rightarrow M - 660 + M(x) = 0 \Rightarrow \boxed{M(x) = -754\ [kg \cdot cm]}$

↳ $x \in [25, 55)$

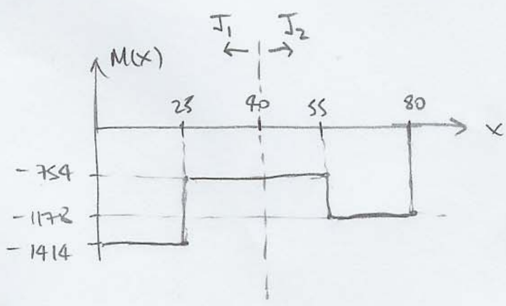
- CORTE ③:



$\sum M_x = 0 \Rightarrow M - 660 + 424 + M(x) = 0$

$\Rightarrow \boxed{M(x) = -1178\ [kg\,cm]}$

↳ $x \in [55, 80)$



IV

$$\tau_{máx} = \frac{M_T \cdot \phi/2}{J}$$

$$J_1 = \frac{\pi \phi_1^4}{32} \Rightarrow J_1 = \frac{\pi 5^4}{32} \Rightarrow \boxed{J_1 = 61,3592\ [cm^4]}$$

$$J_2 = \frac{\pi}{32}\left[\phi_1^4 - \phi_2^4\right] \Rightarrow J_2 = \frac{\pi}{32}\left[5^4 - 2,5^4\right] \Rightarrow \boxed{J_2 = 57,5243\ [cm^4]}$$

- para $x \in [0, 25)$

$$\tau_{máx} = \frac{-1414 \cdot 5/2}{61,3592} \Rightarrow \boxed{\tau_{máx} = -57,61\ [kg/cm^2]}$$

- para $x \in [25, 40)$

$$\tau_{máx} = \frac{-754 \cdot 5/2}{61,3592} \Rightarrow \boxed{\tau_{máx} = -30,72\ [kg/cm^2]}$$

- para $x \in [40, 55)$

$$\tau_{máx} = \frac{-754 \cdot 5/2}{57,5243} \Rightarrow \boxed{\tau_{máx} = -32,77\ [kg/cm^2]}$$

- para $x \in [55, 80)$

$$\tau_{máx} = \frac{-1178 \cdot 5/2}{57,5243} \Rightarrow \boxed{\tau_{máx} = -51,20\ [kg/cm^2]}$$

$$\Rightarrow \boxed{\tau_{máx} = -57,61\ [kg/cm^2] \text{ para } x \in [0, 25)}$$

- el signo $\ominus$ indica la dirección del esfuerzo, luego el esfuerzo máximo será aquel esfuerzo cuyo valor absoluto sea mayor.

- ÁNGULO QUE GIRA EL EXTREMO LIBRE

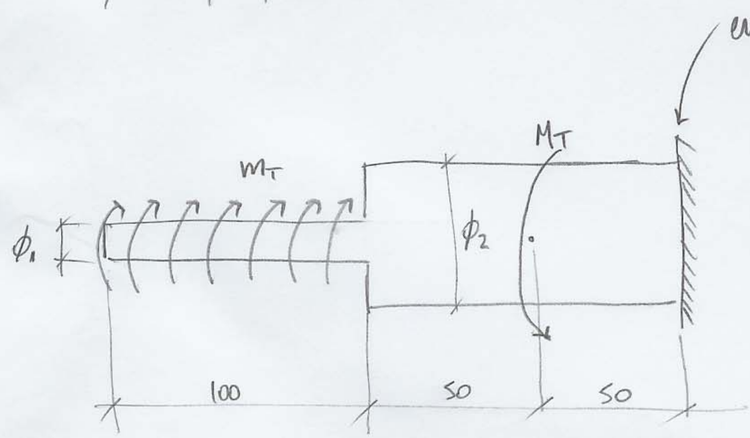
$$d\theta = \frac{M_T}{I_G}\,dx \quad \Big/ \int_0^{80}, \quad G = cte$$

$$\Rightarrow \theta = \frac{1}{G}\cdot\left[\int_0^{25}\frac{-1414}{61{,}3592}dx + \int_{25}^{40}\frac{-754}{61{,}3592}dx + \int_{40}^{55}\frac{-754}{57{,}5243}dx + \int_{55}^{80}\frac{-1178}{57{,}5243}dx\right]$$

$G \rightarrow (0{,}84\times10^6\ [kg/cm^2])$

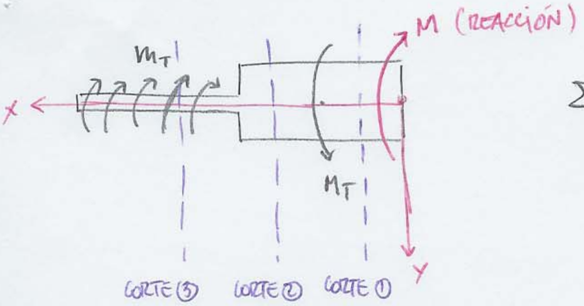
$$\Rightarrow \boxed{\theta = -0{,}001749\ [rad]}$$
$$\boxed{\theta = -0{,}1°}$$

P3| En el tramo de menor diámetro del eje circular de la figura se aplica un momento de torsión uniformemente distribuido $m_T = 500$ [kg cm/cm]. En el tramo siguiente se aplica el momento de torsión indicado, de sentido contrario $M_T = 200.000$ [kg cm]. Calcule los diámetro ϕ_1 y ϕ_2 para que el máximo esfuerzo de corte en el material no supere 1200 [kg/cm²]. No considere el peso propio.



- cotas en [cm].
- $m_T = 500$ [kg·cm/cm]
- $M_T = 200.000$ [kg cm]

DCL:

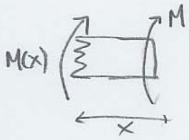


$\Sigma M_x = 0 \Rightarrow M - M_T + m_T \cdot 100 = 0$

$\Rightarrow \boxed{M = 150.000 \, [kg\,cm]}$

CORTES:

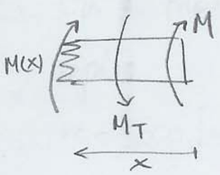
- CORTE ①:



$\Sigma M_x = 0 \Rightarrow M + M(x) = 0 \Rightarrow \boxed{M(x) = -150.000 \, [kg\,cm]}$

↳ $x \in [0, 50)$

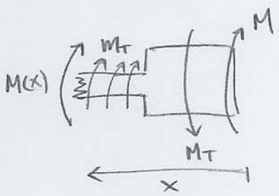
- CORTE ②:



$\Sigma M_x = 0 \Rightarrow M - M_T + M(x) = 0 \Rightarrow \boxed{M(x) = 50.000 \, [kg\,cm]}$

↳ $x \in [50, 100)$

- CORTE ③:

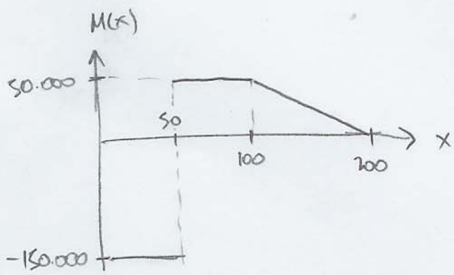


$\Sigma M_x = 0 \Rightarrow M - M_T + m_T(x - 100) + M(x) = 0$

$\Rightarrow \boxed{M(x) = 500(200 - x) \, [kg\,cm]}$

↳ $x \in [100, 200)$

$\Rightarrow$



→ El momento a lo largo del eje no es continuo ya que existen momentos puntuales (M_T)

→ para $x \in [0, 50)$: $$|\tau_{máx}| = \left| \frac{-150.000 \cdot \cancel{\phi_2}/2}{\frac{\pi\ \phi_2^{\cancel{4}3}}{32}} \right| \le 1200\ [kg/cm^2]$$

$$\Rightarrow \phi_2 \geq \left[\frac{150000 \cdot \frac{1}{2} \cdot 32}{\pi \cdot 1200} \right]^{1/3} \Rightarrow \boxed{\phi_2 \geq 8{,}603\ [cm]}$$

→ para $x \in [50, 100)$: $$|\tau_{máx}| = \left| \frac{50.000\ \phi_2/2}{\frac{\pi\ \phi_2^4}{32}} \right| \le 1200\ [kg/cm^2]$$

$$\Rightarrow \phi_2 \geq \left[\frac{50000 \cdot \frac{1}{2} \cdot 32}{\pi \cdot 1200} \right]^{1/3} \Rightarrow \boxed{\phi_2 \geq 5{,}965\ [cm]}$$

→ para $x \in [100, 200)$: $$|\tau_{máx}| = \left| \frac{50.000 \cdot \phi_1/2}{\frac{\pi \phi_1^4}{32}} \right| \le 1200\ [kg/cm^2]$$

$$\Rightarrow \phi_1 \geq \left[\frac{50000 \cdot \frac{1}{2} \cdot 32}{\pi \cdot 1200} \right]^{1/3} \Rightarrow \boxed{\phi_1 \geq 5{,}965\ [cm]}$$

$$\Rightarrow \boxed{\begin{array}{c} \phi_1 = 5{,}965\ [cm] \\ \hline \phi_2 = 8{,}603\ [cm] \end{array}}$$

- con estos valores para los diámetros del eje, el esfuerzo máximo de corte en el material no supera los 1200 [kg/cm²].