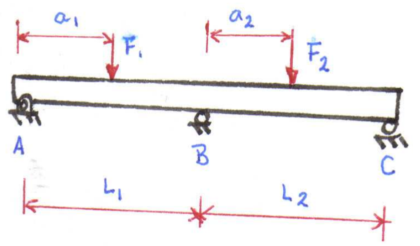
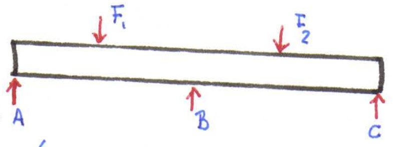
Algunos problemas en flexión

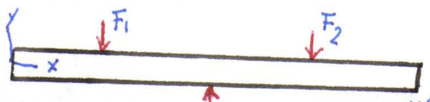
(a)





2 ec.
3 incog

$$\begin{cases} \sum F_y = 0 \Rightarrow A + B + C = F_1 + F_2 \\ \sum_A M_3 = 0 \Rightarrow B L_1 + C_2 (L_1 + L_2) = F_1 a_1 + F_2 (L_1 + a_2) \end{cases}$$



$y(0) = 0$

$\frac{d^2y}{dx^2}(0) = 0$

$y(L_1 + L_2) = 0$

$\frac{d^2y}{dx^2}(L_1 + L_2) = 0$

$\boxed{y(L_1) = 0} \Rightarrow B$ ✓

$$\frac{d^4y}{dx^4} = -\frac{1}{EI}\left\{ F_1\, \delta(x - a_1) - B\, \delta(x - L_1) + F_2\, \delta(x - L_1 - a_2) \right\}$$

$$\Rightarrow \frac{d^3y}{dx^3} = -\frac{1}{EI}\left\{ F_1\, \Pi(x - a_1) - B\, \Pi(x - L_1) + F_2\, \Pi(x - L_1 - a_2) \right\} + \alpha_3$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{EI}\left\{ F_1 (x - a_1)\Pi(x - a_1) - B (x - L_1)\Pi(x - L_1) + F_2 (x - L_1 - a_2)\Pi(x - L_1 - a_2) \right\} + \alpha_3 x + \alpha_2$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{EI}\left\{ \frac{1}{2}F_1 (x - a_1)^2\Pi(x - a_1) - \frac{B}{2}(x - L_1)^2\Pi(x - L_1) + \frac{F_2}{2}(x - L_1 - a_2)^2\Pi(x - L_1 - a_2) \right\} + \frac{\alpha_3}{2}x^2 + \alpha_2 x + \alpha_1$$

$$\Rightarrow y(x) = -\frac{1}{EI}\left\{ \frac{F_1}{6}(x - a_1)^3\Pi(x - a_1) - \frac{B}{6}(x - L_1)^3\Pi(x - L_1) + \frac{F_2}{6}(x - L_1 - a_2)^3\Pi(x - L_1 - a_2) \right\} + \frac{\alpha_3}{6}x^3 + \frac{\alpha_2}{2}x^2 + \alpha_1 x + \alpha_0$$

↑ la solución depende de B que todavía no se conoce

$y(0) = 0 \Rightarrow \alpha_0 = 0$

$\frac{d^2y}{dx^2}(0) = 0 \Rightarrow \alpha_2 = 0$

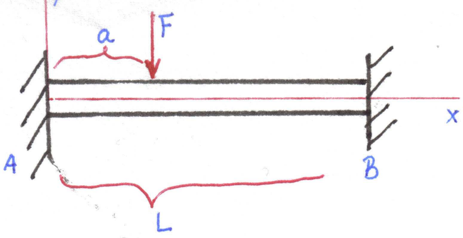
$$\frac{d^2y}{dx^2}(L_1 + L_2) = 0 \Rightarrow -\frac{1}{EI}\left\{ F_1 (L_1 + L_2 - a_1) - B L_2 + F_2 (L_2 - a_2) \right\} + \alpha_3 (L_1 + L_2) = 0$$

$$\Rightarrow \alpha_3 = \frac{1}{EI(L_1 + L_2)}\left[F_1 (L_1 + L_2 - a_1) - B L_2 + F_2 (L_2 - a_2) \right]$$

$y(L_1 + L_2) = 0 \Rightarrow \alpha_1$ depende de B

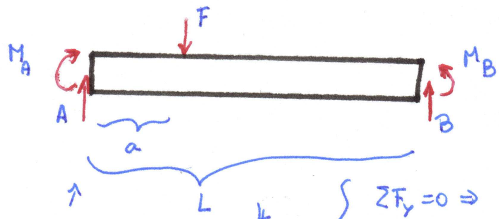
$y(L_1) = 0 \Rightarrow B$ ✓

(b)



(b)

Calcule las reacciones en A y B



$y(L)=0$

$\frac{dy}{dx}(L)=0$

$y(0)=0$

$\frac{dy}{dx}(0)=0$

4 incog.
2 ec.

$\Sigma F_y = 0 \Rightarrow A + B = F$

$\Sigma_A M_z = 0 \Rightarrow BL + M_B = Fa$

$$\frac{d^4y}{dx^4} = -\frac{F}{EI}\,\delta(x-a)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{F}{EI}(x-a)\,\Pi(x-a) + \alpha_3 x + \alpha_2$$

$$\frac{dy}{dx} = -\frac{F}{2EI}(x-a)^2\,\Pi(x-a) + \alpha_3\frac{x^2}{2} + \alpha_2 x + \alpha_1$$

$$y(x) = -\frac{F}{6EI}(x-a)^3\,\Pi(x-a) + \alpha_3\frac{x^3}{6} + \alpha_2\frac{x^2}{2} + \alpha_1 x + \alpha_0$$

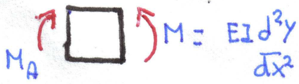
$y(0)=0 \Rightarrow \alpha_0 = 0$ $\quad \frac{dy}{dx}(0)=0 \Rightarrow \alpha_1 = 0$ $\quad \frac{dy}{dx}(L)=0 \Rightarrow -\frac{F}{2EI}(L-a)^2 + \alpha_3\frac{L^2}{2} + \alpha_2 L = 0$

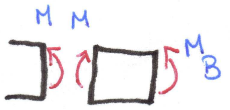
$$\Rightarrow \alpha_2 = \frac{F}{2EI}\frac{(L-a)^2}{L} - \alpha_3\frac{L}{2}$$

$$y(L)=0 \Rightarrow \quad -\frac{F}{6EI}(L-a)^3 + \alpha_3\frac{L^3}{6} + \left[\frac{F}{2EI}\frac{(L-a)^2}{L} - \alpha_3\frac{L}{2}\right]\frac{L^2}{2} = 0$$

$$\alpha_3\left(\underbrace{\frac{L^3}{6} - \frac{L^3}{4}}_{\frac{1}{6}-\frac{1}{4}=\frac{2-3}{12}=\frac{-1}{12}}\right) = \frac{F}{6EI}(L-a)^3 - \frac{F}{4EI}L(L-a)^2$$

$M_A \overset{?}{=}$





$$\Rightarrow M_A = EI\frac{d^2y}{dx^2}(0)$$

$$\Rightarrow M_A = \alpha_2$$

$$M_B = EI\frac{d^2y}{dx^2}(L)$$

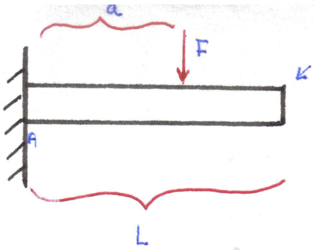
$$\Downarrow$$

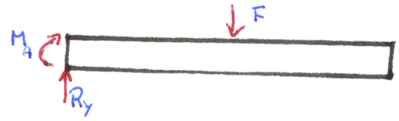
$$M_B = -F(L-a) + EI\alpha_3 L + EI\alpha_2$$

con $M_B \Rightarrow B = \frac{1}{L}(Fa - M_B)$ ✓

$\Rightarrow A = F - B$ ✓

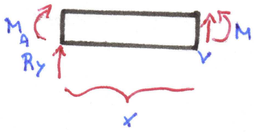
Ⓒ Calcule la deflexión máxima Ⓒ

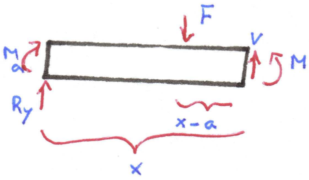




$M_A = -aF$

$R_y = F$

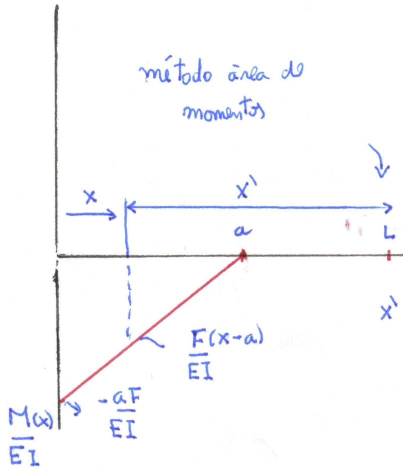




$M = M_A + R_y x$
$= -aF + Fx$
$= F(x-a)$

$M = M_a + R_y x - F(x-a)$
$= F(x-a) - F(x-a)$
$= 0$

método área de momentos



$\Delta = \int_a^L x' \frac{M}{EI}\, dx$

$x' = L - x \Rightarrow \Delta = \int_0^a \underbrace{(L-x)}_{x'} \frac{F(x-a)}{EI}\, dx$ de $a \to L$ $M(x) = 0$

$= \frac{F}{EI} \int_0^a Lx - La - x^2 + xa\, dx$

$= \frac{F}{EI} \int_0^a x(L+a) - x^2 - La\, dx$

$= \frac{F}{EI} \left\{ \frac{x^2}{2}(L+a) - \frac{x^3}{3} - Lax \right\}\Big|_0^a$

$= \frac{F}{EI} \left\{ \underbrace{\frac{a^2}{2}(L+a)}_{\frac{a^2L}{2} + \frac{a^3}{2}} - \frac{a^3}{3} - La^2 \right\}$

$\frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$

$= \frac{F}{EI} \left\{ \frac{a^3}{6} - \frac{a^2L}{2} \right\}$

Ecuación de la elástica

$\frac{d^4y}{dx^4} = -\frac{F}{EI}\delta(x-a) \Rightarrow \frac{d^3y}{dx^3} = -\frac{F}{EI}\Pi(x-a) + \alpha_3 \Rightarrow \frac{d^2y}{dx^2} = -\frac{F}{EI}(x-a)\Pi(x-a) + \alpha_3 x + \alpha_2$

$\frac{dy}{dx} = -\frac{F}{2EI}(x-a)^2\Pi(x-a) + \alpha_3\frac{x^2}{2} + \alpha_2 x + \alpha_1 \Rightarrow y(x) = -\frac{F}{6EI}(x-a)^3\Pi(x-a) + \alpha_3\frac{x^3}{6} + \alpha_2\frac{x^2}{2} + \alpha_1 x + \alpha_0$

C.B. $y(0) = 0 \Rightarrow \alpha_0 = 0$ $\frac{dy}{dx}(0) = 0 \Rightarrow \alpha_1 = 0$ $M(L) = 0 \Rightarrow \frac{d^2y}{dx^2}(L) = 0 \Rightarrow -\frac{F(L-a)}{EI} + \alpha_3 L + \alpha_2 = 0$

$V(L) \overset{?}{=} 0 \Rightarrow \frac{d^3y}{dx^3}(L) = 0 \Rightarrow -\frac{F}{EI} + \alpha_3 = 0 \Rightarrow \alpha_3 = \frac{F}{EI} \Rightarrow \alpha_2 = \frac{F(L-a)}{EI} - \alpha_3 L$

$= \cancel{\frac{FL}{EI}} - \frac{Fa}{EI} - \cancel{\frac{FL}{EI}}$

(d)

$$\Rightarrow y(x) = -\frac{F}{6EI}(x-a)^3\Pi(x-a) + \frac{F}{EI}\frac{x^3}{6} - \frac{Fa}{EI}\frac{x^2}{2}$$

$$y(L) = -\frac{F}{6EI}\underbrace{(L-a)^3}_{(L^2-2La+a^2)(L-a)} + \frac{F}{EI}\frac{L^3}{6} - \frac{Fa}{EI}\frac{L^2}{2}$$

$$\underbrace{(L^2-2La+a^2)(L-a)}_{L^3 - L^2a - 2L^2a + 2La^2 + a^2L - a^3} = L^3 - 3L^2a + 3La^2 - a^3$$

$$= -\cancel{\frac{F}{6EI}L^3} + \cancel{\frac{F}{2EI}L^2a} - \frac{F}{2EI}La^2 + \frac{F}{6EI}a^3 + \cancel{\frac{FL^3}{EI\,6}} - \cancel{\frac{Fa}{EI}\frac{L^2}{2}}$$

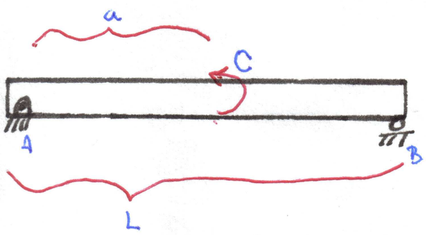
$$= \frac{F}{EI}\left\{\frac{a^3}{6} - \frac{La^2}{2}\right\} \checkmark$$

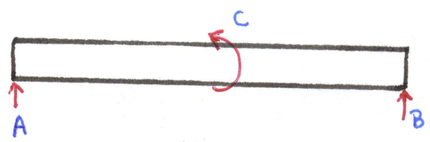
$$M(x) = EI\frac{d^2y}{dx^2} = -F(x-a)\Pi(x-a) + \underbrace{Fx - Fa}_{F(x-a)}$$

$$= \begin{cases} F(x-a) & x < a \\ 0 & x \geq a \end{cases} \checkmark$$

$$V(x) = -EI\frac{d^3y}{dx^3} = F\Pi(x-a) - F = \begin{cases} -F & x < a \\ 0 & x \geq a \end{cases} \checkmark$$

(d)





$A + B = 0$

$\sum_A M_3 = 0$

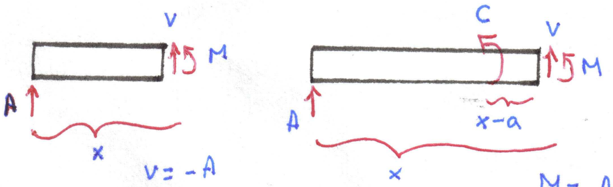
$\Downarrow$

$BL + C = 0$

$\Rightarrow B = -\frac{C}{L}$

$$\Rightarrow \underbrace{A = \frac{C}{L}}_{\text{importa la ubicación de C ?}}$$

ojo V no está definido en $x=a$ $\Swarrow$

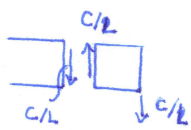


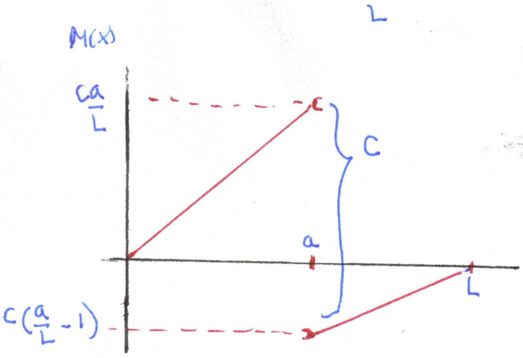
$V = -A$

$M = Ax = \frac{Cx}{L}$

$V = -A$

$M = Ax - C$
$= C\left(\frac{x}{L} - 1\right)$





Ecuación elástica?

$\Leftarrow \frac{d^2y}{dx^2}$ no es continuo

$\frac{d^4y}{dx^4} = 0 \quad x<a$

$\frac{d^4y}{dx^4} = 0 \quad x>a$

e

$$\Rightarrow \quad y(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 \qquad x<a$$

$$y(x) = \beta_0 + \beta_1 (x-L) + \beta_2 (x-L)^2 + \beta_3 (x-L)^3 \qquad x>a$$

$$y(0)=0 \qquad \frac{d^2y}{dx^2}(0)=0 \qquad \frac{d^2y}{dx^2}(a^-) \overset{?}{=} \frac{C}{EI}\frac{a}{L}$$

$$y(L)=0 \qquad \frac{d^2y}{dx^2}(L)=0 \qquad \frac{d^2y}{dx^2}(a^+) \overset{?}{=} \frac{C}{EI}\left(\frac{a}{L}-1\right) = \frac{C}{EI}\frac{1}{L}(a-L)$$

$$\Downarrow$$

$$y(0)=0 \Rightarrow \alpha_0=0 \quad \frac{d^2y}{dx^2}(0)=0 \Rightarrow \alpha_2=0 \quad \Rightarrow y(x) = \alpha_1 x + \alpha_3 x^3 \qquad x<a$$

$$y(L)=0 \Rightarrow \beta_0=0 \quad \frac{d^2y}{dx^2}(L)=0 \Rightarrow \beta_2=0 \quad \Rightarrow y(x) = \beta_1 (x-L) + \beta_3 (x-L)^3 \qquad x>a$$

$$\Rightarrow \frac{dy}{dx} = \alpha_1 + 3\alpha_3 x^2 \qquad \frac{d^2y}{dx^2} = 6\alpha_3 x \qquad x<a$$

$$\frac{dy}{dx} = \beta_1 + 3\beta_3 (x-L)^2 \qquad \frac{d^2y}{dx^2} = 6\beta_3 (x-L) \qquad x>a$$

$$\Rightarrow \frac{d^3y}{dx^3} = \begin{cases} 6\alpha_3 & x<a \\ 6\beta_3 & x>a \end{cases}$$

$$\Downarrow$$

$$6\alpha_3 a = \frac{C}{EI}\frac{a}{L} \Rightarrow \alpha_3 = \frac{C}{6EI}\frac{1}{L}$$

$$6\beta_3(a-L) = \frac{C}{EI}\frac{1}{L}(a-L)$$

$$\Downarrow \quad \beta_3 = \frac{C}{6EI}\frac{1}{L}$$

$$\boxed{\alpha_3 = \beta_3}$$

$$\underbrace{y(a^-) = y(a^+)}_{\Downarrow} \qquad \underbrace{\frac{dy}{dx}(a^-) = \frac{dy}{dx}(a^+)}_{\Downarrow}$$

$$\alpha_1 a + \alpha_3 a^3 = \beta_1 (a-L) + \beta_3 (a-L)^3$$

$$\alpha_1 + 3\cancel{\alpha_3} a^2 = \beta_1 + 3\beta_3 \underbrace{(a-L)^2}_{a^2-2aL+L^2}$$

$$= \beta_1 + 3\cancel{\beta_3} a^2 - 6\beta_3 aL + 3\beta_3 L^2$$

$$a(\beta_1 - 6\beta_3 aL + 3\beta_3 L^2) + \alpha_3 a^3 = \beta_1 (a-L) + \beta_3 (a-L)^3$$

$$\Rightarrow -\beta_1 L = -6\beta_3 a^2 L + 3\beta_3 L^2 a + \beta_3 a^3 - \beta_3 \underbrace{(a-L)^3}$$

$$(a^2-2aL+L^2)(a-L)$$
$$a^3 - a^2L - 2a^2L + 2aL^2 + L^2 a - L^3$$
$$= a^3 - 3a^2L + 3aL^2 - L^3$$

$$\Rightarrow -\beta_1 L = -6\beta_3 a^2 L + \cancel{3\beta_3 L^2 a} + \cancel{\beta_3 a^3} - \cancel{\beta_3 a^3} + 3\beta_3 a^2 L - \cancel{3\beta_3 aL^2} + \beta_3 L^3$$

$$= -3\beta_3 a^2 L + \beta_3 L^3 \Rightarrow \beta_1 = -\beta_3 L^2 + 3\beta_3 a^2$$

$$\beta_1 = \beta_3 (3a^2 - L^2)$$

$$\Rightarrow \alpha_1 = 3\beta_3 a^2 - \beta_3 L^2 - 6\beta_3 aL + 3\beta_3 L^2$$

$$= 2\beta_3 L^2 - 6\beta_3 aL + 3\beta_3 a^2 = \beta_3 (2L^2 - 6aL + 3a^2)$$

$$\Rightarrow \alpha_3 = \frac{C}{6EI}\frac{1}{L} \qquad \beta_3 = \frac{C}{6EI}\frac{1}{L} \qquad \beta_1 = \frac{C}{6EIL}(3a^2 - L^2) \qquad \alpha_1 = \frac{C}{6EIL}(2L^2 - 6aL + 3a^2)$$