ME-46A RESISTENCIA DE MATERIALES

07/11/2003 **CONTROL N°3** **Prof.: M. Elgueta**

Problema 1.

En relación con la figura 1, determine el valor de a de manera que la deflexión en C sea nula.. Resuelva este problema utilizando el método de área de momentos y mediante el método de Castigliano. Considere EI constante y sólo flexión.

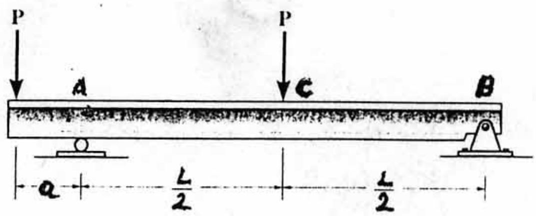


Figura 1

Problema 2.

Para el estado de esfuerzos indicado en la figura 2a,

a) Calcule los esfuerzos principales y dibuje el círculo de Mohr.
b) Utilizando este círculo determine los esfuerzos en un elemento orientado a 30° en sentido antihorario, como se muestra en la figura 2b.
c) Nuevamente utilizando el círculo de Mohr, determine en qué plano(s) el esfuerzo normal es nulo.

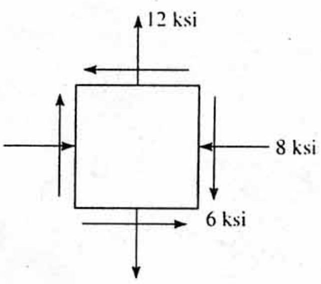


Figura 2a

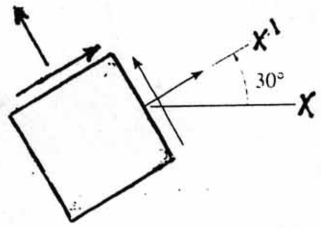


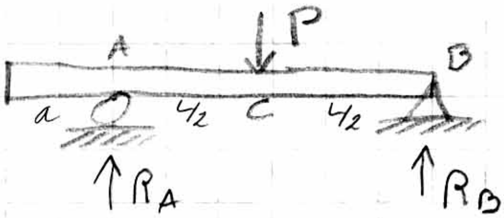
Figura 2b

(1) Control 3 Me46a 7/11/2003

P1

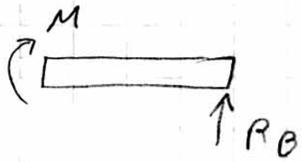
a) Área Momento - Superposición

a.1)



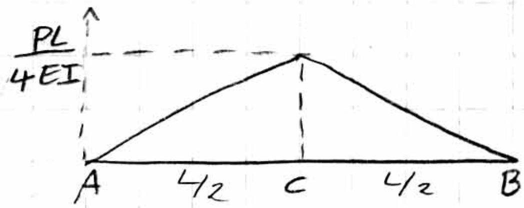
$$R_A = R_B = \frac{P}{2}$$

Entre B y C:



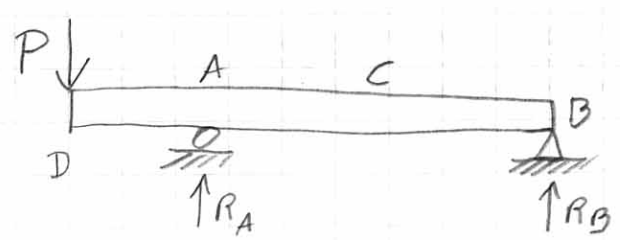
$$\Rightarrow -M + R_B x = 0$$

$$M = \frac{P}{2} x$$



$$t_{A/C} = \frac{2}{3}\frac{L}{2}\frac{L}{2}\frac{PL}{4EI}\frac{1}{2} = \boxed{\frac{PL^3}{48EI} = \delta_{C_1}}$$

a.2)



$$\Sigma F \Rightarrow R_A + R_B - P = 0 \quad (1)$$

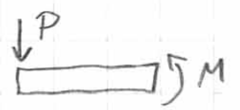
$$\Sigma M_B = 0 \Rightarrow P(L+a) - L R_A = 0$$

$$R_A = \frac{P(L+a)}{L} \qquad \text{en } (1)$$

$$\Rightarrow \frac{P(L+a)}{L} + R_B - P = 0 \Rightarrow R_B = P - \frac{P(L+a)}{L}$$

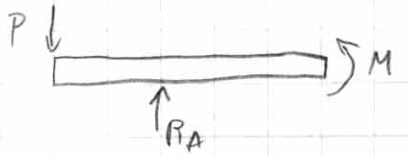
$$R_B = P\left(1 - \frac{L+a}{L}\right) = P\left(\frac{\cancel{L}-\cancel{L}-a}{L}\right) = -\frac{Pa}{L}$$

Entre D y A:

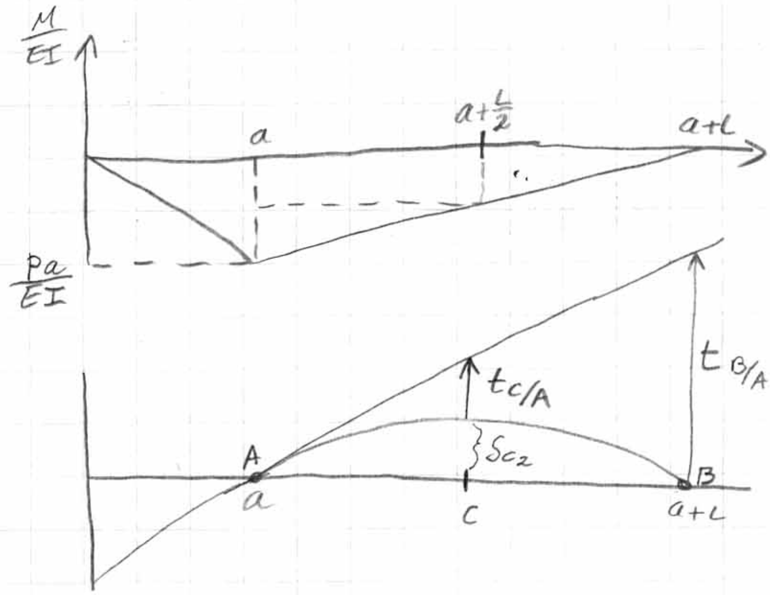


$$M(x) = -Px$$

Entre A y B:



$$M(x) = -Px + (x-a)\frac{P(L+a)}{L}$$



3

$$t_{B/A} = \frac{\cancel{2}}{3} L\, L\, \frac{Pa}{EI} \frac{1}{\cancel{2}} = \frac{PaL^2}{3EI}$$

$$t_{C/A} = \frac{1}{2} \frac{L}{2} \frac{Pa}{2EI} \frac{L}{2} + \frac{\cancel{2}}{3} \frac{L}{\cancel{2}} \frac{L}{2} \frac{Pa}{2EI} \frac{1}{2}$$

$$= \frac{PaL^2}{16EI} + \frac{PaL^2}{24EI} = \frac{5PaL^2}{48EI}$$

$$\frac{\delta_{C_2} + t_{C/A}}{\cancel{L}/2} = \frac{t_{B/A}}{\cancel{L}} \Rightarrow \delta_{C_2} = \frac{t_{B/A}}{2} - t_{C/A}$$

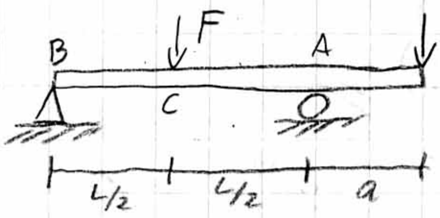
$$\delta_{C_2} = \frac{PaL^2}{6EI} - \frac{5PaL^2}{48EI} = \frac{PaL^2}{16EI} \quad //$$

$$\delta_{C_1} = \delta_{C_2} \Rightarrow \frac{\cancel{P}L^{\cancel{3}}}{48\cancel{EI}} = \frac{\cancel{P}a\cancel{L^2}}{16\cancel{EI}}$$

$$\Rightarrow a = \frac{16L}{48} = \frac{1}{3} L \quad //$$

b) castigliano

En el caso de Castigliano, deben diferenciarse las fuerzas P para no derivar con respecto a ambas.



$$\sum F = 0 \Rightarrow R_A + R_B = P + F \quad (1)$$

$$\sum M_B = 0 \Rightarrow -\frac{FL}{2} + R_A L - P(L+a) = 0$$

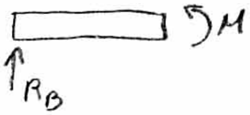
$$\Rightarrow R_A L = \frac{FL}{2} + P(L+a) \Rightarrow R_A = \frac{F}{2} + \frac{P(L+a)}{L} \quad //$$

④ en (1): $\dfrac{F}{2} + \dfrac{P\cdot(L+a)}{L} + R_B = P + F$

$$R_B = P + F - \frac{F}{2} - \frac{P(L+a)}{L} = \frac{F}{2} + P\left(\frac{L}{L} - \frac{(L+a)}{L}\right)$$

$$R_B = \frac{F}{2} - \frac{P\,a}{L}$$

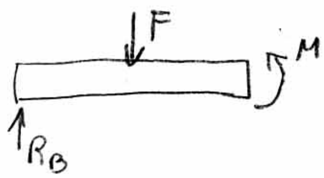
$0 \leq x < L/2$



$$M(x) = x\,R_B$$

$$M_1(x) = x\left(\frac{F}{2} - \frac{Pa}{L}\right) = \frac{Fx}{2} - \frac{Pax}{L}$$

$L/2 \leq x < L$



$$M(x) = x\,R_B - \left(x - \frac{L}{2}\right)F$$

$$M(x) = x\left(\frac{F}{2} - \frac{Pa}{L}\right) - \left(x - \frac{L}{2}\right)F$$

$$M(x) = \frac{Fx}{2} - \frac{Pax}{L} - Fx + \frac{FL}{2}$$

$$M_2(x) = -\frac{Fx}{2} - \frac{Pax}{L} + \frac{FL}{2}$$

$L \leq x < L+a$

$$M(x) = x\,R_B - \left(x - \frac{L}{2}\right)F + (x-L)\,R_A$$

$$M(x) = -\frac{Fx}{2} - \frac{Pax}{L} + \frac{FL}{2} + (x-L)\left(\frac{F}{2} + \frac{P}{L}(L+a)\right)$$

$$M(x) = -\cancel{\frac{Fx}{2}} - \cancel{\frac{Pax}{L}} + \cancel{\frac{FL}{2}} + \cancel{\frac{Fx}{2}} + \frac{Px(L+a)}{L} - \cancel{\frac{FL}{2}} - P(L+a)$$

$$M(x) = Px - P(L+a)$$

$$M_3(x) = P(x - (L+a))$$

$$\frac{\partial M_1}{\partial F} = \frac{x}{2} \qquad \frac{\partial M_2}{\partial F} = -\frac{x}{2} + \frac{L}{2} \qquad \frac{\partial M_3}{\partial F} = 0$$

⑤ $$EI\,\delta_c = \int_0^{L/2} M_1(x)\frac{\partial M_1(x)}{\partial F}\,dx + \int_{L/2}^{L} M_2(x)\frac{\partial M_2(x)}{\partial F}\,dx$$

$$+ \int_L^{L+a} M_3(x)\cancel{\frac{\partial M_3}{\partial F}}^{\,0}\,dx$$

$$\Rightarrow EI\,\delta_c = \int_0^{L/2}\left(\frac{Fx}{2} - \frac{Pax}{L}\right)\left(\frac{x}{2}\right)dx + \int_{L/2}^{L}\left(-\frac{Fx}{2} - \frac{Pax}{L} + \frac{FL}{2}\right)\cdots$$

$$\cdots\left(-\frac{x}{2} + \frac{L}{2}\right)dx \quad \Rightarrow F = P$$

$$\Rightarrow EI\,\delta_c = \int_0^{L/2}\left(\frac{Px}{2} - \frac{Pax}{L}\right)\left(\frac{x}{2}\right)dx + \int_{L/2}^{L}\left(-\frac{Px}{2} - \frac{Pax}{L} + \frac{PL}{2}\right)\cdots$$

$$\cdots\left(-\frac{x}{2} + \frac{L}{2}\right)dx$$

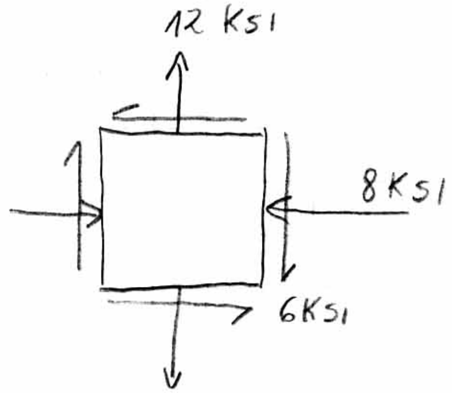
$$EI\,\delta_c = \frac{1}{48}L^3P - \frac{1}{16}L^2P\cdot a = 0$$

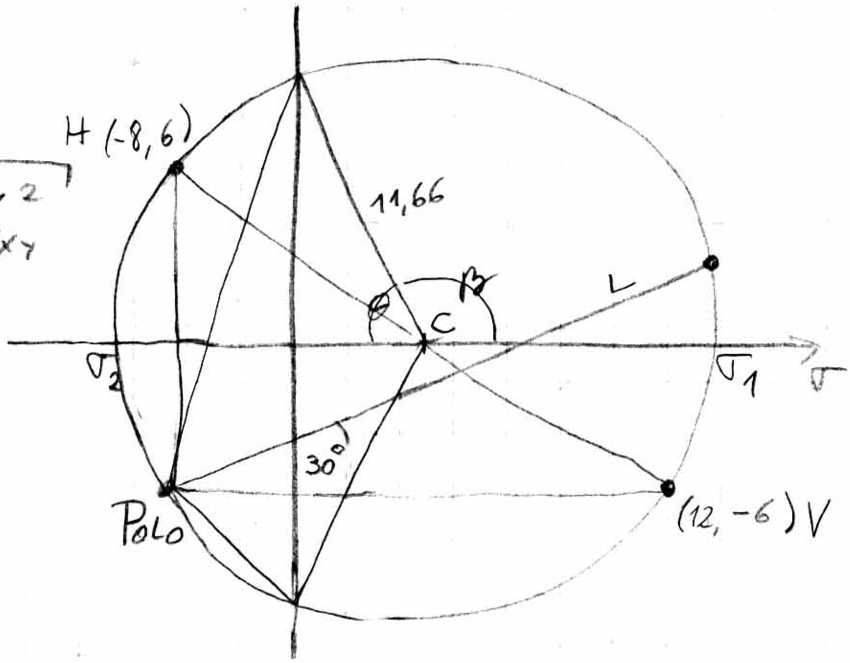
$$\delta_c = 0 \Rightarrow a = \frac{1}{3}L$$

(1) Control 3 Me 46a 7/11/2003

P2

a)





$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{-8+12}{2} \pm \sqrt{\left(\frac{-8-12}{2}\right)^2 + 6^2}$$

$$\sigma_{1,2} = 2 \pm 11,66$$

$$\sigma_1 = 13,66$$
$$\sigma_2 = 9,66$$

$$\tau_{max} = 11,66$$

b) Ec. circulo: $(\sigma_n - 2)^2 + \tau_n^2 = 11,66^2$ (1)

Ec. recta L: $\tau_n = tg(30)\,\sigma_n + b$

El polo pertenece a la recta => $-6 = tg\,30°(-8) + b$

$$b = -6 - tg\,30°(-8) = -1,38$$

$$\Rightarrow \tau_n = 0,577\,\sigma_n - 1,38 \qquad (2)$$

② (2) en (1) $\Rightarrow$ $\sigma_n^2 - 4\sigma_n + 4 + (0{,}577\sigma_n - 1{,}38)^2 = 136$

$\sigma_n^2 - 4\sigma_n + 4 + 0{,}577^2\sigma_n^2 - 2\cdot 0{,}577\sigma_n\cdot 1{,}38 + 1{,}38^2 = 136$

$\sigma_n^2 - 4\sigma_n + 4 + 0{,}33\sigma_n^2 - 1{,}59\sigma_n + 1{,}9 = 136$

$1{,}33\sigma_n^2 - 5{,}59\sigma_n - 130{,}1 = 0$

$\sigma_n = 12{,}21$ // en (2) $\Rightarrow$ $\tau_n = 5{,}66$ //

Falta el otro eje.

c) Ec. círculo: $(\sigma_n - 2)^2 + \tau_n^2 = 136$ $\sigma_n = 0$

$\Rightarrow 4 + \tau_n^2 = 136$

$\tau_n = \sqrt{132} = 11{,}49$ //

$11{,}66 \cos\theta = 2 \Rightarrow \theta = \operatorname{acos}\left(\frac{2}{11{,}66}\right) = 80{,}12°$

$\Rightarrow \beta = 180 - 80{,}12 = 99{,}88°$ //

Falta el segundo plano.