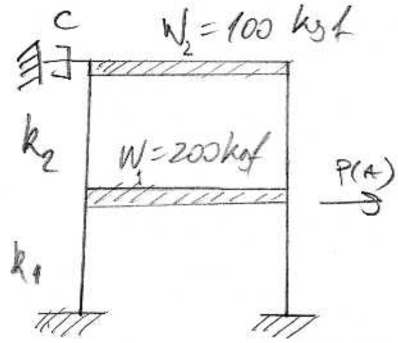
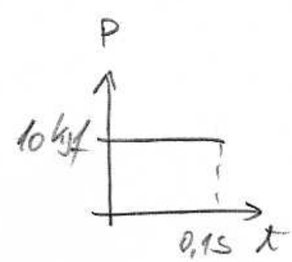
P1 |



Q_{max}?



$g = 9.8 \ m/s^2$

$k_1 = 3000 \ kgf/m$

$k_2 = 5000 \ kgf/m$

$c = 2 \ \frac{kgf \cdot s}{m}$

Sol

Edificio de corte:

$$\Rightarrow [M] = \begin{bmatrix} \frac{W_2}{g} & 0 \\ 0 & \frac{W_1}{g} \end{bmatrix} = \begin{bmatrix} 10.204 & 0 \\ 0 & 20.408 \end{bmatrix} \frac{kgf \cdot s^2}{m}$$

$$[K] = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_1 + k_2 \end{bmatrix} = \begin{bmatrix} 5000 & -5000 \\ -5000 & 8000 \end{bmatrix} kgf/m$$

$$\Rightarrow [\phi] = \begin{bmatrix} 0.852 & 0.775 \\ -0.523 & 0.631 \end{bmatrix} \qquad \omega = \begin{pmatrix} 28.123 \\ 9.543 \end{pmatrix} \frac{rad}{s} \Rightarrow T = \frac{2\pi}{\omega} = \begin{pmatrix} 0.223 \\ 0.658 \end{pmatrix} s$$

Normalizadas x $[M_m]$, i.e $[\tilde{M}_m] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$[M_m] = [\phi]^T [M][\phi] = \begin{bmatrix} 12.999 & 0 \\ 0 & 14.272 \end{bmatrix}$$

$$\{\phi_i\}^T [M] \{\phi_i\} = M_{m_i}$$

$$\frac{1}{\alpha} \{\phi_i\}^T [M] \{\phi_i\} \frac{1}{\alpha} = 1$$

$$\frac{M_{m_i}}{\alpha^2} = 1 \Rightarrow \alpha = \sqrt{M_{m_i}}$$

luego normalizamos: $\{\tilde{\phi}_i\} = \frac{\{\phi_i\}}{\sqrt{M_{m_i}}} \Rightarrow [\tilde{\phi}] = \begin{bmatrix} 0.236 & 0.205 \\ -0.145 & 0.167 \end{bmatrix}$

$$[C] = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \frac{kgf \cdot s}{m} \qquad [\tilde{\phi}]^T [C][\tilde{\phi}] = \begin{bmatrix} 0.112 & 0.097 \\ 0.097 & 0.084 \end{bmatrix} = [C_m]$$

$$[C_m] = \begin{bmatrix} \ddots & & \\ & 2M_{m_i} w_i \beta_i & \\ & & \ddots \end{bmatrix} \Rightarrow \begin{aligned} 2M_{m_2} W_2 \beta_2 &= 0.112 \\ 2M_{m_1} W_1 \beta_1 &= 0.084 \end{aligned} \Rightarrow \begin{aligned} \beta_1 &= 0.0044 \\ \beta_2 &= 0.002 \end{aligned}$$

Analizamos td/T

$$\frac{td}{T} = \begin{pmatrix} 0.152 \\ 0.448 \end{pmatrix} \longrightarrow \begin{aligned} &< 0.25 \rightarrow \text{Impacto corta duración} \\ &> 0.25 \rightarrow \text{'' larga ''} \end{aligned}$$

Por simplicidad, resolver para corta duración (dicho en control)

$$\Rightarrow y_n(t) = \frac{e^{-\beta_n w_n \cdot t}}{M_{m_n} W_{mD}} P_n(t) \cdot td \cdot \operatorname{sen}(W_D \cdot t) \qquad W_D \approx W$$

$$P_n = \tilde{\phi}^T P = \begin{Bmatrix} -0.145 \; P(t) \\ 0.167 \; P(t) \end{Bmatrix} \begin{aligned} &\rightarrow \text{Modo 2} \\ &\rightarrow \text{Modo 1} \end{aligned}$$

$$\Rightarrow y_1(t) = 0.0175 \cdot e^{-0.042t} \cdot \operatorname{sen}(9.543t)$$

$$y_2(t) = -0.0052 \cdot e^{-0.056t} \operatorname{sen}(28.123t)$$

$$\Rightarrow v_{inferior}(t) = \sum \phi_{i,2} \cdot y_i(t) = \phi_{12} \cdot y_1 + \phi_{22} \cdot y_2$$

$$Q_{Base}(t) = k_1 \cdot v_{inferior}(t)$$

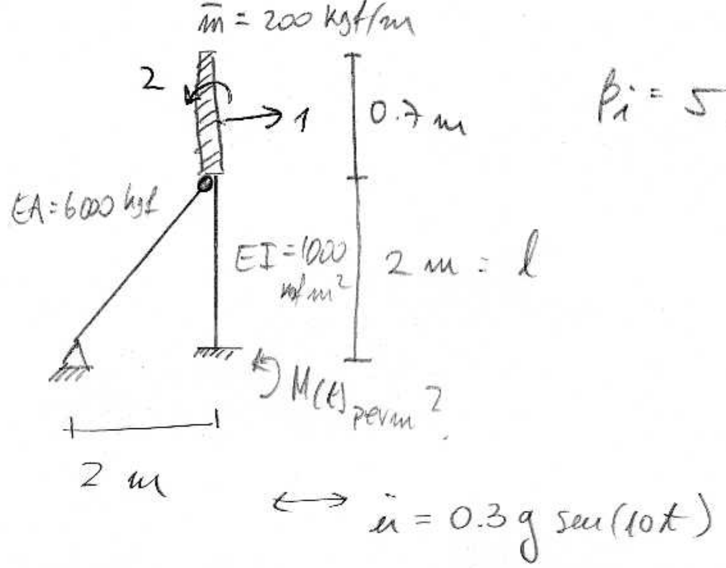
$$\Rightarrow Q_{Base}(t) = 8.767 e^{-0.042t} \operatorname{sen}(9.543 \cdot t) + 2.243 e^{-0.056t} \operatorname{sen}(28.123 t)$$

¿ Q_{max} ?

Opción a) Graficar en TI $\Rightarrow$ max $\approx$ 8.9 kgf

Opción b) Suponer que controla el primer modo

P2



$\beta_i = 5\% \ \forall i$

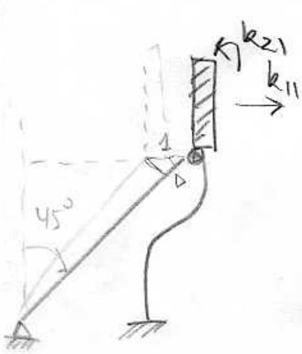
Sol

- Matriz de Masa:

$$[M] = \begin{bmatrix} \frac{\bar{m}}{g} \cdot l_b & 0 \\ 0 & \frac{1}{12}\left(\frac{\bar{m}}{g} l_b\right) l_b^2 \end{bmatrix} = \begin{bmatrix} 14.286 & 0 \\ 0 & 0.583 \end{bmatrix}$$

- Matriz de rigidez

$u_1 = 1 \quad u_2 = 0$



$\Delta = \cos 45°$

$F_{B_x} = \frac{AE}{l_B} \cdot \Delta \cdot \cos 45° \cdot X$

$\Rightarrow k_{B_x} = \frac{AE}{l_B} \cos^2 45° = \frac{AE}{2\sqrt{2}\, l}$

$$k_{11} = \frac{12EI}{l^3} + \frac{EA}{2\sqrt{2}\, l}$$

$$k_{12} = \frac{6EI}{l^2} + k_{11} \cdot \frac{l_b}{2}$$

$$= \frac{6EI}{l^2} + \frac{l_b}{2}\left(\frac{12EI}{l^3} + \frac{EA}{2\sqrt{2}\, l}\right)$$

$u_1 = 0 \quad u_2 = 1$

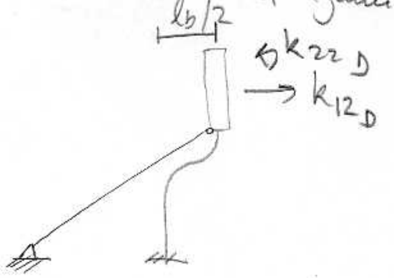
$$\frac{\ }{\ell^2} + \frac{\ }{2}\left(\frac{\ }{\ell^3} + \frac{\ }{2\sqrt{2}\ell}\right)$$

$u_1 = 0 \quad u_2 = 1$

Efecto giro:

$$k_{12g} = \frac{6EI}{\ell^2} \qquad k_{22g} = \frac{4EI}{\ell} + k_{12g}\cdot\frac{\ell_b}{2}$$

Efecto desplazamiento



$$k_{12D} = \frac{12EI}{\ell^3}\,\frac{\ell_b}{2} + \frac{AE}{2\sqrt{2}\ell}\cdot\frac{\ell_b}{2}$$

$$k_{22D} = \frac{6EI}{\ell^2}\cdot\frac{\ell_b}{2} + k_{12D}\cdot\frac{\ell_b}{2}$$

$k_{ij} = k_{ijg} + k_{ijD}$

$$\Rightarrow k_{22D} = \frac{4EI}{\ell} + \frac{6EI}{\ell^2}\cdot\frac{\ell_b}{2} + \frac{6EI\,\ell_b}{\ell^2\,2} + \left(\frac{12EI}{\ell^3}\,\frac{\ell_b}{2} + \frac{AE}{2\sqrt{2}\ell}\,\frac{\ell_b}{2}\right)\frac{\ell_b}{2}$$

Evaluando:

$$[K] = \begin{bmatrix} 2561 & 2396 \\ 2396 & 3364 \end{bmatrix} \Rightarrow \phi = \begin{bmatrix} -0.812 & -0.029 \\ 0.584 & -1 \end{bmatrix}$$

Normalizamos:

$$\tilde{\phi}_i = \frac{\phi_i}{\sqrt{M_{m_i}}} \qquad [M_m] = \phi^T M \phi = \begin{bmatrix} 9.610 & 0 \\ 0 & 0.595 \end{bmatrix}$$

$$\Rightarrow [\tilde{\phi}] = \begin{bmatrix} -0.262 & -0.038 \\ 0.188 & -1.296 \end{bmatrix} ; \quad \omega = \begin{pmatrix} 7.65 \\ 76.727 \end{pmatrix} \frac{rad}{s} \quad T = \begin{pmatrix} 0.821 \\ 0.082 \end{pmatrix} s$$

Para un sismo horizontal, $\{r\} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \Rightarrow \alpha = [\phi]^T [M] \{r\} = \begin{Bmatrix} -3.740 \\ -0.544 \end{Bmatrix}$

recordar:

$$M_m \ddot{y} + c_m \dot{y} + k_m y = -\{\alpha\} \ddot{u}_g = -\alpha \ddot{u}_{go} \operatorname{sen}(\bar{\omega} t)$$

Como normalizamos por masa modal:

$$\ddot{y}_i + 2\beta_i \omega_i \dot{y}_i + \omega_i^2 y_i = -\alpha_i \ddot{u}_{go} \operatorname{sen}(\bar{\omega} t)$$

Sol. en regimen permanente:

$$y_i(t) = \frac{P_{oi}}{k_i} \cdot D_i \operatorname{sen}(\bar{\omega} t - \theta_i) \qquad \theta_i = \operatorname{arctg}\left(\frac{2\beta_i \gamma_i}{1-\gamma_i^2}\right) = \begin{pmatrix} -0.182 \\ 0.013 \end{pmatrix} rad$$

$$\gamma_i = \frac{\bar{\omega}}{\omega_i} = \begin{pmatrix} 1.307 \\ 0.130 \end{pmatrix}$$

$\Rightarrow D$ ... 1

$$\gamma_i = \frac{\omega}{\omega_i} = \begin{pmatrix} 1.307 \\ 0.130 \end{pmatrix}$$

$$\Rightarrow D_i = \frac{1}{\sqrt{(1-\gamma_i)^2 + (2\beta_i\gamma_i)^2}} = \begin{pmatrix} 1.388 \\ 1.017 \end{pmatrix}$$

De esta forma

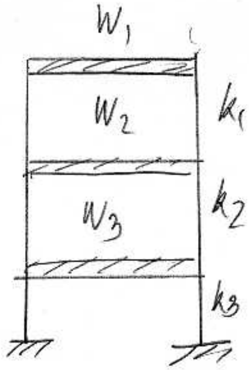
$$y_1(t) = 0.2886 \,\text{sen}(\bar{\omega}t + 0.182)$$
$$y_2(t) = 0.0029 \,\text{sen}(\bar{\omega}t - 0.013)$$

además

$$v(t) = [\phi]\{y\} = \begin{Bmatrix} v_1(t) \\ v_2(t) \end{Bmatrix}$$

$$\Rightarrow M_{Base}(t) = \frac{6EI}{l^2} v_1(t) + \left(\frac{6EI}{l^2}\cdot\frac{b}{2} + \frac{2EI}{l}\right) v_2(t)$$

P3



$$S_a = 0.5g \, \frac{1+4.5\left(\frac{T}{0.3}\right)^{1.5}}{1+\left(\frac{T}{0.3}\right)^{3}}$$

$\ddot{v}_{max}$ //
Δv_{max} = ?
Q_{max} //
M_{cols} //

Sol

Edificio de corte

$$M = \begin{bmatrix} W_1/g & & \\ & W_2/g & \\ & & W_3/g \end{bmatrix} \qquad K = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_3 \end{bmatrix}$$

$$\Rightarrow M = \begin{bmatrix} 7.143 & & \\ & 20.408 & \\ & & 20.408 \end{bmatrix} \qquad K = \begin{bmatrix} 500 & -500 & 0 \\ -500 & 2000 & -1500 \\ 0 & -1500 & 3500 \end{bmatrix}$$

$$\Rightarrow \omega = \begin{pmatrix} 4.904 \\ 9.758 \\ 14.841 \end{pmatrix} \; ; \; T = \begin{pmatrix} 1.281 \\ 0.644 \\ 0.423 \end{pmatrix} \qquad M_n = \begin{bmatrix} 11.784 & & \\ & 9.797 & \\ & & 19.583 \end{bmatrix}$$

$$\Rightarrow \tilde{\phi} = \frac{\phi_i}{\sqrt{M_n}} = \begin{bmatrix} 0.235 & -0.286 & 0.056 \\ 0.154 & 0.103 & -0.121 \\ 0.077 & 0.099 & 0.182 \end{bmatrix} \qquad \{r\} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

Factor de part:

Factor de part.:

$$\{\alpha\} = [\phi][M]\{r\} = \begin{Bmatrix} 6.394 \\ 2.083 \\ 1.656 \end{Bmatrix} \frac{kgf \cdot s^2}{m}$$

Evaluamos Sa:

$$Sa = \begin{pmatrix} 2.528 \\ 6.818 \\ 10.987 \end{pmatrix} m/s^2$$

$$\Rightarrow \ddot{v}_{max_i} = \{\phi_i\} \frac{\alpha_m}{M_m} Sa = \begin{bmatrix} 3.797 & -4.058 & 1.025 \\ 2.493 & 1.461 & -2.201 \\ 1.243 & 1.408 & 3.318 \end{bmatrix} m/s^2$$

Combinando:

$$\ddot{v}_{max} = \begin{Bmatrix} 5.651 \\ 3.632 \\ 3.813 \end{Bmatrix}$$

$$v_{max_i} = \frac{\ddot{v}_{max_i}}{\omega_i^2} = \begin{bmatrix} 0.1579 & -0.0426 & 0.0047 \\ 0.1037 & 0.0153 & -0.0100 \\ 0.0517 & 0.0148 & 0.0151 \end{bmatrix} m$$

$$\Delta v_{max_i} = \begin{bmatrix} -0.0542 & 0.0580 & -0.0146 \\ -0.0520 & -0.0006 & 0.0251 \\ 0.0517 & 0.0148 & 0.0151 \end{bmatrix} m$$

Comb:

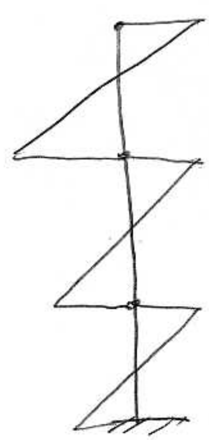
$$\Rightarrow \Delta v_{max} = \begin{pmatrix} 0.081 \\ 0.058 \\ 0.056 \end{pmatrix} m \qquad \underline{SRSS}$$

<u>Qmax</u>

$$Q_{max_i} = \frac{L_m^2}{M_m} \cdot Sa = \begin{pmatrix} 103.357 \\ 29.576 \\ 30.133 \end{pmatrix} kgf$$

Combinando $Q_{max} = 111.649$ kgf

Diagrama Momento:



$$\pm \frac{6EI}{l^2} \cdot \Delta v_{ij}$$

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