ANALYTICAL MODEL FOR PREDICTING SHEAR STRENGTH OF SQUAT WALLS

By Shyh-Jiann Hwang,[1] Wen-Hung Fang,[2] Hung-Jen Lee,[3] and Hsin-Wan Yu[4]

ABSTRACT: A softened strut-and-tie model for determining the shear strength of squat walls is proposed in this paper. The proposed model originates from the strut-and-tie concept and satisfies equilibrium, compatibility, and constitutive laws of cracked reinforced concrete. The shear capacities of 62 squat walls were calculated and compared with the available experimental results, and reasonable agreement was obtained. Based on the collected experimental data in this paper, the proposed physical model was used to study the effects of boundary elements, cyclic loading, and vertical loads on the wall shear strength. The softened strut-and-tie model can be further developed to improve the current shear wall design procedures by incorporating the actual shear resisting mechanisms in predicting shear strength of walls.

INTRODUCTION

Buildings containing structural walls have exhibited extremely good earthquake performance (Fintel 1991). Structural walls have demonstrated great ability to protect both life and property from an earthquake at the least cost. An adequate design of a structural wall requires that wall shear failure will not curtail ductile response of the structure under seismic excitation. Unfortunately, the design and proportioning of structural walls cannot achieve the same level of confidence presently available for seismic beams and columns. More information on the shear strength of walls is urgently needed.

Reinforced concrete squat walls with a height-to-width ratio <2 find wide application in seismic resistance for low-rise buildings. For squat walls the predominant action is shear, and the ACI 318-95 method for squat walls is based on empirical expressions derived originally for beams by using test results usually exhibiting a broad scatter. Current ACI 318-95 provisions for the design of reinforced concrete squat walls are in disagreement with the observed structural behavior. While current provisions [American Concrete Institute (ACI) 1995] estimate the wall resistance using the tensile strength of the concrete, Lefas et al. (1990) have shown that the shear force resistance is associated with the concrete compressive strength in the compressive zone at the base of the wall. In contrast to what was adopted by the ACI 318 building code (ACI 1995), experimental evidence (Lefas et al. 1990) also showed that horizontal web reinforcement does not appear to have a significant effect on the shear force capacity of a squat wall. Clearly, design methods based on a complete understanding of wall behavior would be preferable in current design procedures.

To predict the shear strength of squat walls, the softened truss model was developed by Hsu and Mo (1985) and modified by Gupta and Rangan (1998). In the softened truss model, the state of stresses in the central panel is assumed to be uniform and the flow of compressive stresses is idealized by a series of parallel compressive struts. However, the internal stress flow of the squat wall is highly disturbed by the presence of concentrated load on the top and the foundation at the bottom. In this disturbed region, it is inappropriate to assume that the shear stress is uniform. Therefore the strut-and-tie model is believed to be a better choice in modeling the flow of the forces of the squat wall with compressive struts representing the flow of concentrated compressive stresses in the concrete and tension ties representing the reinforcing steel.

The purpose of this paper is to present a softened strut-and-tie model for determining the shear strength of squat walls. A similar model for the seismic resistance of beam-column joints has been proposed by Hwang and Lee (1999, 2000). This model is based on the strut-and-tie concept and derived to satisfy equilibrium, compatibility, and constitutive laws of cracked reinforced concrete. The word ''softened'' emphasizes the importance of the compression softening phenomenon which means that cracked reinforced concrete in compression exhibits lower strength and stiffness than uniaxially compressed concrete (Vecchio and Collins, 1993). It is believed that the shear failure relating to concrete crushing should be governed by the softening effect of concrete.

The proposed model is an extension of the softened strut-and-tie model that specifically predicts the shear strength of squat walls associated with diagonal compression failure. In the following, the model and its theory will be presented and developed first. The validity and accuracy of the proposed model is then tested against available experimental data.

RESEARCH SIGNIFICANCE

A new treatment of the shear strength prediction for squat walls is presented. The proposed softened struct-and-tie model can give important insight into shear strength and behavior of reinforced concrete squat walls, and it is in a ready format for the evaluation of their shear strength and behavior.

SOFTENED STRUT-AND-TIE MODEL

Consider a typical reinforced concrete squat wall loaded horizontally on the top and fixed at the bottom as shown in Fig. 1. By taking into account the distances between couples (Fig. 1), it will be sufficiently accurate to express the following relationship between vertical and horizontal shears:

$$\frac{V_{wv}}{V_{wh}} \approx \frac{H}{\ell} \tag{1}$$

where V_{wv} and V_{wh} = vertical and horizontal wall shear forces, respectively; H = distance from point of application of V_{wh} to base; and ℓ = internal lever arm of the couple at the base of the wall.

Without vertical force N acting on the wall (Fig. 1), the

[1]Prof., Dept. of Constr. Engrg., Nat. Taiwan Univ. of Sci. and Technol., Taipei, Taiwan 10672, ROC.
[2]Former Grad. Res. Asst., Dept. of Constr. Engrg., Nat. Taiwan Univ. of Sci. and Technol., Taipei, Taiwan 10672, ROC.
[3]PhD, Dept. of Constr. Engrg., Nat. Taiwan Univ. of Sci. and Technol., Taipei, Taiwan 10672, ROC.
[4]Assoc. Prof., Dept. of Civ. Engrg., Chung Kuo Inst. of Technol. and Commerce, Taipei, Taiwan 117, ROC.
Note. Associate Editor: David Stevens. Discussion open until June 1, 2001. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on September 28, 1998. This paper is part of the ***Journal of Structural Engineering***, Vol. 127, No. 1, January, 2001. ©ASCE, ISSN 0733-9445/01/0001-0043–0050/$8.00 + $.50 per page. Paper No. 19362.

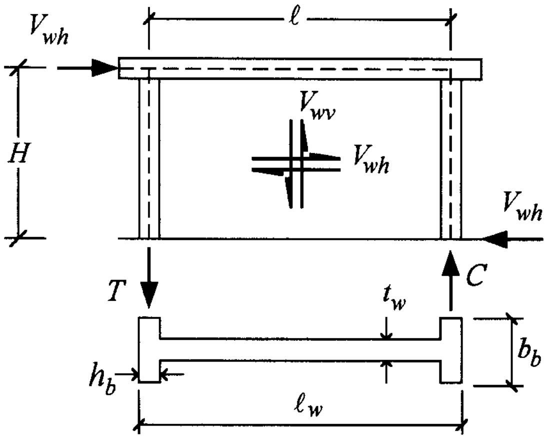

FIG. 1. External Actions and Internal Shears for Squat Wall

vertical shear force V_{wv} is equal to the tensile force T or the compressive force C at the base of the wall. The vertical load in the wall was shown to have a beneficial effect on the shear strength of the wall (Mau and Hsu 1987). In the softened strut-and-tie model, the vertical load may play two roles. One is to enlarge the cross-sectional area of the compression strut and thus increase the shear resistance (Hwang and Lee 2000). The other is to limit the vertical expansion of the wall panel and retard the softening process of the cracked concrete. The vertical loads acting on the boundary elements or directly on the wall panels may produce different effects. This issue will be addressed later in this paper.

Macromodel

After the development of the first cracking pattern in the wall, the steel bars will be subjected to tension and the concrete acts as compressive struts, thus forming a strut-and-tie action. Three strut-and-tie load paths (Hwang and Lee 1999, 2000) are proposed to model the force transfer within the squat wall, and they are the diagonal, horizontal, and vertical mechanisms as depicted in Fig. 2.

The diagonal mechanism [Fig. 2(a)] is a single diagonal compression strut whose angle of inclination θ is defined as

$$\theta = \tan^{-1}\left(\frac{H}{\ell}\right) \tag{2}$$

It is assumed that the direction of the diagonal concrete strut coincides with the direction of the principal compressive stress of the concrete.

The effective area of the diagonal strut A_{str} is defined as

$$A_{str} = a_s \times b_s \tag{3}$$

where a_s = depth of the diagonal strut; and b_s = width of the diagonal strut that can be taken as the width of the wall web t_w.

The depth of the diagonal strut (a_s) depends on its end condition provided by the compression zone at the base of the wall. It is intuitively assumed that

$$a_s = a_w \tag{4}$$

where a_w = depth of the compression zone at the base of the wall.

For general purposes, a_w can be determined by the sectional analysis for the stage when the extreme tensile steel reaches yielding. For simplicity, a_w can be approximated with Paulay and Priestley's (1992) equation for the depth of the flexural compression zone of an elastic column

$$a_w = \left(0.25 + 0.85\frac{N}{A_w f'_c}\right)\ell_w \tag{5}$$

where A_w = net area of the concrete section bounded by the web thickness t_w and the length of the section in the direction of the shear force ℓ_w (Fig. 1); and f'_c = compressive strength of the concrete.

The horizontal mechanism [Fig. 2(b)] includes one horizontal tie and two flat struts. The horizontal tie is made up of the horizontal shear reinforcement. When computing the cross area of the horizontal tie, it is assumed that the horizontal shear reinforcement within the center half of the wall is fully effective, and the other horizontal steel is included as 50% effective (Hwang and Lee 1999, 2000).

The vertical mechanism [Fig. 2(c)] is composed of one vertical tie and two steep struts. The vertical tie includes only the vertical shear reinforcement within the wall web and excludes the vertical reinforcement of the boundary elements. For a wall without boundary elements, the vertical shear reinforcement within the central portion of $0.8\ell_w$ is considered effective to constitute the vertical tie.

For diagonal compression failure, the shear strength of the squat wall is defined as the concrete compressive stress on the nodal zone as the concrete reaches its capacity. The boundary of the nodal zone coincides with the diagonal strut boundary, but the concrete bearing force to be examined is the summa-

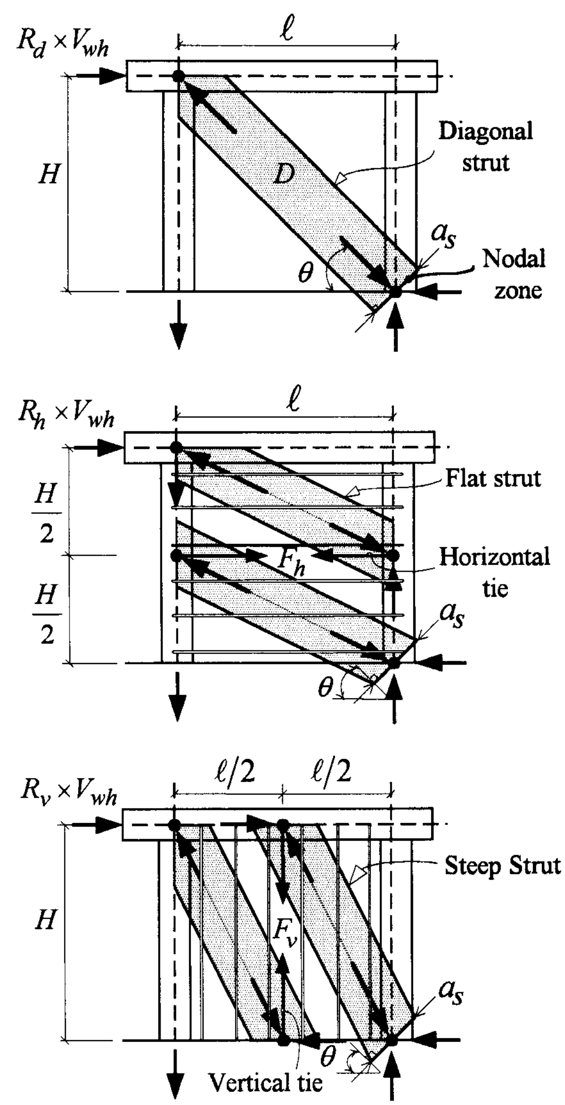

FIG. 2. Wall Shear Resisting Mechanisms: (a) Diagonal; (b) Horizontal; (c) Vertical

tion of compressions from the diagonal, flat, and steep struts as shown in Fig. 2.

Equilibrium Conditions

Fig. 3 shows the proposed strut-and-tie model for a reinforced concrete squat wall. Using the strut-and-tie model, the resistance against the vertical and horizontal wall shears is calculated as

$$V_{wv} = -D \sin \theta + F_h \tan \theta + F_v \tag{6}$$

$$V_{wh} = -D \cos \theta + F_h + F_v \cot \theta \tag{7}$$

where D = compression force in the diagonal strut (positive for tension); F_h and F_v are the tension forces in the horizontal and vertical ties, respectively (positive for tension). Note that the ratio $V_{wv}/V_{wh} = \tan \theta$ is always preserved in the proposed model.

There are three load paths in the wall web, and the shear forces must be apportioned to the resisting mechanisms. It is assumed that the ratios of the horizontal shear V_{wh} assigned among the three mechanisms are defined as (Hwang and Lee 1999, 2000)

$$-D \cos \theta : F_h : F_v \cot \theta = R_d : R_h : R_v \tag{8}$$

where R_d, R_h, and R_v = wall shear ratios resisted by the diagonal, horizontal, and vertical mechanisms, respectively.

The horizontal shear V_{wh} will be distributed among the resisting mechanisms in proportion to their relative stiffnesses. According to Schäfer (1996) and Jennewein and Schäfer (1992), the stiffness ratio between horizontal tie and diagonal strut to transfer the horizontal shear is $\gamma_h/(1 - \gamma_h)$, where γ_h is the fraction of horizontal shear transferred by the horizontal tie in the absence of the vertical tie. The values of γ_h is defined as

$$\gamma_h = \frac{2 \tan \theta - 1}{3} \quad \text{for} \quad 0 \le \gamma_h \le 1 \tag{9}$$

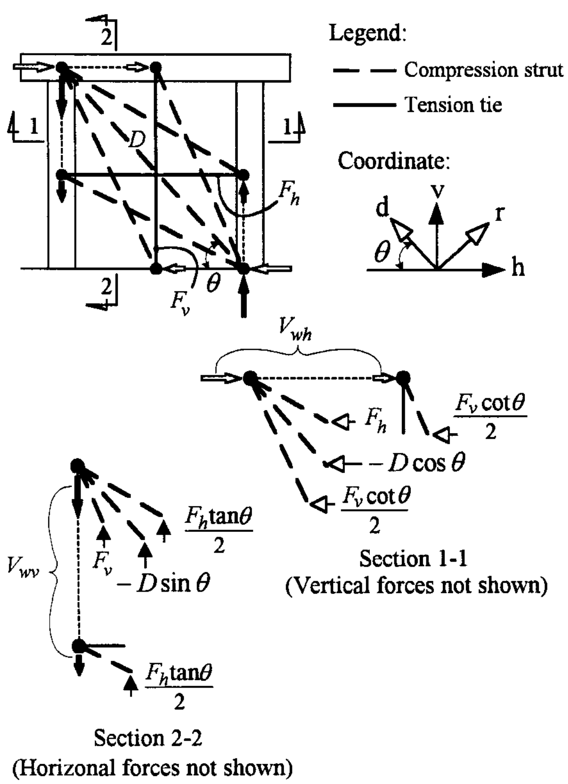

FIG. 3. Strut-and-Tie Model for Squat Wall

Similarly the stiffness ratio between vertical tie and diagonal strut to transfer the vertical shear is $\gamma_v/(1 - \gamma_v)$, where γ_v is the fraction of vertical shear carried by the vertical tie in the absence of the horizontal tie. The value of γ_v is defined as (Jennewein and Schäfer 1992; Schäfer 1996)

$$\gamma_v = \frac{2 \cot \theta - 1}{3} \quad \text{for} \quad 0 \le \gamma_v \le 1 \tag{10}$$

Since the horizontal shear V_{wh} is in proportion to the vertical shear V_{wv}, the stiffness ratio between vertical tie and diagonal strut to transfer the horizontal shear is also $\gamma_v/(1 - \gamma_v)$.

The relative stiffness ratio between horizontal and diagonal mechanisms should be the same with or without the participation of the vertical mechanism, that is

$$\frac{R_h}{R_d} = \frac{\gamma_h}{1 - \gamma_h} \tag{11}$$

A similar relationship exists between vertical and diagonal mechanisms

$$\frac{R_v}{R_d} = \frac{\gamma_v}{1 - \gamma_v} \tag{12}$$

It is useful to scale the sum of R_d, R_h, and R_v as unity

$$R_d + R_h + R_v = 1 \tag{13}$$

By solving (11), (12), and (13), the values of R_d, R_h, and R_v can be obtained as

$$R_d = \frac{(1 - \gamma_h)(1 - \gamma_v)}{1 - \gamma_h \gamma_v} \tag{14}$$

$$R_h = \frac{\gamma_h(1 - \gamma_v)}{1 - \gamma_h \gamma_v} \tag{15}$$

$$R_v = \frac{\gamma_v(1 - \gamma_h)}{1 - \gamma_h \gamma_v} \tag{16}$$

Since the diagonal compression is mainly transferred in the d-direction, the maximum compressive stress $\sigma_{d,\max}$ acting on the nodal zone is assumed to govern the failure. The stress $\sigma_{d,\max}$ resulting from the summation of the compressive forces from the diagonal, flat, and steep struts (Fig. 3) on the nodal zone can be calculated as

$$\sigma_{d,\max} = \frac{1}{A_{str}} \left\{ D - \frac{\cos\left(\theta - \tan^{-1}\left(\frac{H}{2\ell}\right)\right)}{\cos\left(\tan^{-1}\left(\frac{H}{2\ell}\right)\right)} F_h - \frac{\cos\left(\tan^{-1}\left(\frac{2H}{\ell}\right) - \theta\right)}{\sin\left(\tan^{-1}\left(\frac{2H}{\ell}\right)\right)} F_v \right\} \tag{17}$$

Constitutive Laws

In the paper by Mo and Rothert (1997), it can be found that Model A of Vecchio and Collins (1993) and the model of Belarbi and Hsu (Hsu 1993) are the two most accurate softened stress-strain relationships to predict the shear strength of squat walls. Due to its simplicity in mathematical formulation, the softening model of Zhang and Hsu (1998) is chosen in this paper. The ascending branch of the softened stress-strain curve of the cracked concrete (Zhang and Hsu 1998) is represented as follows:

$$\sigma_d = -\zeta f'_c \left[2\left(\frac{-\varepsilon_d}{\zeta\varepsilon_0}\right) - \left(\frac{-\varepsilon_d}{\zeta\varepsilon_0}\right)^2 \right] \quad \text{for} \quad \frac{-\varepsilon_d}{\zeta\varepsilon_0} \le 1 \qquad (18)$$

$$\zeta = \frac{5.8}{\sqrt{f'_c}} \frac{1}{\sqrt{1 + 400\varepsilon_r}} \le \frac{0.9}{\sqrt{1 + 400\varepsilon_r}} \qquad (19)$$

where σ_d = average principal stress of concrete in the d-direction (positive for tension); ζ = softening coefficient; f'_c = compressive strength of a standard concrete cylinder (MPa); ε_d and ε_r = average principal strains in the d- and r-directions, respectively (positive for tension); and ε_0 = concrete cylinder strain corresponding to the cylinder strength f'_c. The value of ε_0 can be defined approximately as (Foster and Gilbert 1996)

$$\varepsilon_0 = 0.002 + 0.001 \left(\frac{f'_c - 20}{80}\right) \quad \text{for} \quad 20 \le f'_c \le 100 \text{ MPa} \qquad (20)$$

If the horizontal and vertical shear reinforcement is assumed to be elastic-perfectly-plastic, the stress-strain relationships are

$$f_s = E_s\varepsilon_s \quad \text{for} \quad \varepsilon_s < \varepsilon_y \qquad (21)$$

$$f_s = f_y \quad \text{for} \quad \varepsilon_s \ge \varepsilon_y \qquad (22)$$

where E_s = elastic modulus of the steel bars; and f_s and ε_s are stress and strain in the mild steel, respectively. Variable f_s becomes f_h or f_v, ε_s becomes ε_h or ε_v, and f_y becomes f_{yh} or f_{yv} when applied to horizontal or vertical shear reinforcement, respectively.

The relationship between forces and strains of the tension ties becomes

$$F_h = A_{th}E_s\varepsilon_h \le F_{yh} \qquad (23)$$

$$F_v = A_{tv}E_s\varepsilon_v \le F_{yv} \qquad (24)$$

where A_{th} and A_{tv} = areas of the horizontal and vertical ties, respectively; and F_{yh} and F_{yv} = yielding forces of the horizontal and vertical ties, respectively.

Compatibility Condition

The compatibility condition employed in this paper is the first strain invariant

$$\varepsilon_r + \varepsilon_d = \varepsilon_h + \varepsilon_v \qquad (25)$$

where ε_h and ε_v = average normal strains in the h- and v-directions, respectively (positive for tension). This equality states that the sum of the normal strains in the perpendicular direction is a constant.

Proposed Solution Procedure

The solution procedure is that of Hwang and Lee (1999, 2000) and starts with a selection of the horizontal shear V_{wh} as shown in the flowchart of Fig. 4. It consists of three major steps as follows:

1. The equilibrium equations are employed to find the $\sigma_{d,\max}$ acting on the nodal zone.
2. By assuming that the strength of the concrete strut is reached, an initial value of the softening coefficient ζ is obtained as $\zeta = -\sigma_{d,\max}/f'_c$. Then, the strains of the struts and ties are calculated with the aid of the corresponding constitutive laws.
3. A new value of ζ is computed by applying the compatibility equation. If the assumed ζ value from Step 2 is close to the new ζ, then V_{wh} selected is the shear strength of the wall, otherwise back to the iteration procedure.

EXPERIMENTAL VERIFICATION

A total of 62 test specimens and their available results in the technical literature are used to verify the proposed model.

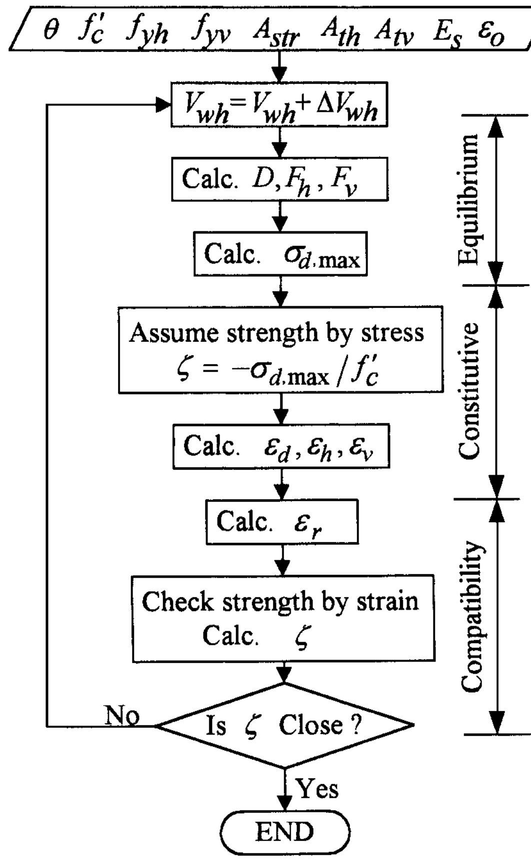

FIG. 4. Flowchart Showing Solution Algorithm

These are the test results by Benjamin and Williams (1957), Hirosawa (1975), Yamada et al. (1974), Barda et al. (1977), Cardenas et al. (1980), and Mo (1993) that are listed in Table 1. The test specimens considered in this paper have three major common features: (1) All walls were reported to have failed in web shear mode; (2) they were one-story isolated walls; and (3) all contained both horizontal and vertical reinforcement uniformly distributed throughout the web.

The shear strength of test walls was calculated using the proposed model and the ACI 318-95 method. The flexural strength of walls was calculated using the conventional analysis of a reinforced concrete section subjected to axial compression and uniaxial bending moment. The smaller of the calculated values is taken as the predicted strength.

The last three columns of Table 1 list the values of $V_{wh,\text{test}}/V_{wh,\text{calc}}$ ratios calculated according to the proposed model (general and simple approaches) and the ACI 318-95 method. In the general approach (Table 1), the depth of the compression zone a_w and the internal lever arm of couple ℓ at the base of the wall are determined by analysis of the fully cracked transformed section with straight-line theory. In the simple approach (Table 1), (5) is used to estimate the value of a_w, and a simplified method of determining ℓ is employed as following. For a wall with boundary elements, ℓ is the distance between the centroids of the boundary elements. For a wall without boundary elements, ℓ is equal to $0.8\ell_w$, where ℓ_w is the length of the entire wall in the direction of the applied shear force.

Table 1 shows satisfactory results. The average strength ratio ($V_{wh,\text{test}}/V_{wh,\text{calc}}$) in the general approach is 1.18 with a coefficient of variation (COV) of 0.17. The average strength ratio in the simple approach is 1.05 with a COV of 0.21.

It has been disputed whether to use a plane sectional analysis for squat walls. However, it is a simple tool to make an approximation. By using the plane sectional analysis, the gen-

TABLE 1. Experimental Verification

Specimen	$H \times \ell_w \times t_w$ (cm)	$b_b \times h_b$ (cm)	f'_c (MPa)	ρ_v (percent)	f_{yv} (MPa)	ρ_h (percent)	f_{yh} (MPa)	$\frac{N}{A_w f'_c}$	$V_{wh,\text{test}}$ (kN)	$V_{wh,\text{test}}/V_{wh,\text{calc}}$		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	General (11)	Simple (12)	ACI (13)
(a) Benjamin and Williams (1957)												
4BII-1	56 × 61 × 5	13 × 10	20.1	0.50	359	0.50	359	0.000	89	1.45	1.28	1.00
3A2-3	56 × 91 × 5	13 × 10	21.5	0.50	359	0.50	359	0.000	155	1.32	1.06	1.15
4BII-3	56 × 122 × 5	13 × 10	19.5	0.50	359	0.50	359	0.000	201	1.17	0.95	1.14
4BII-4	56 × 178 × 5	13 × 10	26.4	0.50	359	0.50	359	0.000	294	0.82	0.63[b]	1.07
(b) Hirosawa (1965) as reported by Hirosawa (1975)												
9-40-WI-1	70 × 60 × 3	10 × 10	25.7	0.23	293	0.21	293	0.271	86	1.48	1.21	2.54
(c) Sugano (1973) as reported by Hirosawa (1975)												
140-1	180 × 396 × 12	36 × 36	20.6	0.66	572	0.66	572	0.120	2,354	1.32	0.90	1.31
141-2	180 × 396 × 12	36 × 36	20.8	0.66	572	0.66	572	0.228	2,942	1.43	0.96	1.64
142-3	180 × 396 × 12	36 × 36	21.3	0.66	572	0.66	572	0.155	3,138	1.62	1.12	1.72
143-4	180 × 396 × 12	36 × 36	19.6	0.33	572	0.33	572	0.097	1,814	1.31	0.91	1.27
144-5	180 × 396 × 12	36 × 36	20.8	0.33	572	0.33	572	0.097	1,912	1.32	0.92	1.32
145-6	180 × 396 × 12	36 × 36	20.5	0.69	284	0.66	284	0.110	2,138	1.31	0.92	1.49
146-7	180 × 396 × 12	36 × 36	19.6	0.69	284	0.66	284	0.106	1,981	1.27	0.89	1.39
147-8	180 × 396 × 12	36 × 36	20.9	0.77	397	0.74	397	0.116	2,305	1.25	0.90	1.28
(d) Yoshizaki (1973) as reported by Hirosawa (1975)												
165-1-56-2[a]	86 × 80 × 6	—	23.5	0.22	433	0.23	433	0.000	102	1.12[b]	1.12[b]	1.12[b]
166-1-56-8[a]	86 × 80 × 6	—	23.5	0.73	433	0.82	433	0.000	147	1.10	1.16	1.08[b]
167-1-88-4[a]	86 × 80 × 6	—	23.5	0.44	433	0.41	433	0.000	135	1.15	1.16	0.94
168-1-88-8[a]	86 × 80 × 6	—	23.5	0.73	433	0.82	433	0.000	159	1.06	1.24	0.88[b]
169-1-88-12[a]	86 × 80 × 6	—	23.5	1.17	433	1.17	433	0.000	175	1.08	1.32	0.90
170-2/3-36-2[a]	86 × 120 × 6	—	24.5	0.24	433	0.23	433	0.000	160	1.04[b]	1.04[b]	1.04[b]
171-2/3-36-8[a]	86 × 120 × 6	—	24.5	0.78	433	0.82	433	0.000	235	1.05	1.05	0.94[b]
172-2/3-52-4[a]	86 × 120 × 6	—	24.5	0.44	433	0.41	433	0.000	220	1.03	1.06	1.01
173-2/3-52-8[a]	86 × 120 × 6	—	24.5	0.78	433	0.82	433	0.000	260	1.05	1.13	0.88
174-2/3-52-12[a]	86 × 120 × 6	—	24.5	1.17	433	1.17	433	0.000	275	1.04	1.16	0.93
175-1/2-27-2[a]	86 × 120 × 6	—	25.5	0.22	433	0.23	433	0.000	199	0.91[b]	0.91[b]	0.91[b]
176-1/2-27-8[a]	86 × 120 × 6	—	25.5	0.80	433	0.82	433	0.000	322	0.92	0.88	0.81[b]
177-1/2-42-4[a]	86 × 120 × 6	—	25.5	0.36	433	0.41	433	0.000	319	0.99	0.95	1.09
178-1/2-42-8[a]	86 × 120 × 6	—	25.5	0.80	433	0.82	433	0.000	382	0.99	1.01	0.95
179-1/2-42-12[a]	86 × 120 × 6	—	25.5	1.17	433	1.17	433	0.000	422	1.09	1.10	1.05
(e) Yamada et al. (1974)												
$\rho_w = 0.003$	60 × 133 × 4	13 × 13	35.6	0.31	286	0.31	286	0.186	373	1.02	0.83	2.95
$\rho_w = 0.006$	60 × 133 × 4	13 × 13	30.4	0.63	286	0.63	286	0.201	370	1.08	0.88	2.19
$\rho_w = 0.012$	60 × 133 × 4	13 × 13	31.5	1.26	286	1.26	286	0.218	438	1.07	0.89	1.78
$t = 30$: $\rho_w = 0.008$	60 × 133 × 3	13 × 13	32.8	0.84	286	0.84	286	0.221	276	0.98	0.76	1.81
$t = 20$: $\rho_w = 0.006$	60 × 133 × 2	13 × 13	30.1	0.63	286	0.63	286	0.305	211	1.32	0.85	2.50
$t = 20$: $\rho_w = 0.012$	60 × 133 × 2	13 × 13	33.7	1.26	286	1.26	286	0.293	213	1.03	0.73	1.67
(f) Barda et al. (1977)												
B1-1	95 × 191 × 10	61 × 10	29.0	0.50	543	0.50	496	0.000	1,276	1.73	1.38	1.75
B2-1	95 × 191 × 10	61 × 10	16.3	0.50	552	0.50	499	0.000	965	1.12	1.69	1.51
B3-2[a]	95 × 191 × 10	61 × 10	27.0	0.50	545	0.50	513	0.000	1,112	1.25	1.28	1.51
B6-4[a]	95 × 191 × 10	61 × 10	21.2	0.25	496	0.50	496	0.000	872	1.31	1.38	1.26
B7-5[a]	48 × 191 × 10	61 × 10	25.7	0.50	531	0.50	501	0.000	1,140	0.90	0.93	1.58
B8-5[a]	191 × 191 × 10	61 × 10	23.4	0.50	527	0.50	496	0.000	889	1.64	1.73	1.26
(g) Cardenas et al. (1980)												
SW-7	206 × 191 × 8	—	43.0	0.94	448	0.27	414	0.000	519	0.93	0.92	1.30
SW-8	206 × 191 × 8	—	42.5	2.93	448	0.27	465	0.000	569	0.95	1.00	1.36
(h) Mo (1993)												
HN4-1[a]	65 × 86 × 7	17 × 8	32.2	0.72	302	0.81	302	0.007	205	1.05	0.99[b]	0.99[b]
HN4-2[a]	65 × 86 × 7	17 × 8	32.2	0.72	302	0.81	302	0.007	247	1.27	1.20[b]	1.20[b]
HN4-3[a]	65 × 86 × 7	17 × 8	32.1	0.72	302	0.81	302	0.007	202	1.04	0.98[b]	0.98[b]
HN6-1[a]	65 × 86 × 7	17 × 8	29.5	0.72	443	0.81	302	0.007	255	1.42	1.08	1.11
HN6-2[a]	65 × 86 × 7	17 × 8	29.5	0.72	443	0.81	302	0.007	204	1.13	0.86	0.89
HN6-3[a]	65 × 86 × 7	17 × 8	31.0	0.72	443	0.81	302	0.007	205	1.10	0.83	0.89
HM4-1[a]	65 × 86 × 7	17 × 8	37.5	0.72	302	0.81	302	0.006	223	1.07[b]	1.07[b]	1.07[b]
HM4-2[a]	65 × 86 × 7	17 × 8	37.5	0.72	302	0.81	302	0.006	231	1.11[b]	1.11[b]	1.11[b]
HM4-3[a]	65 × 86 × 7	17 × 8	39.9	0.72	302	0.81	302	0.005	250	1.20[b]	1.20[b]	1.20[b]
LN4-1[a]	65 × 86 × 7	17 × 8	18.0	0.58	302	0.81	302	0.012	193	1.57	1.27	1.03[b]
LN4-2	65 × 86 × 7	17 × 8	18.0	0.58	302	0.81	302	0.012	217	1.76	1.43	1.15[b]
LN4-3[a]	65 × 86 × 7	17 × 8	29.7	0.58	302	0.81	302	0.007	203	1.14	1.06[b]	1.06[b]
LN6-1[a]	65 × 86 × 7	17 × 8	30.7	0.58	443	0.81	302	0.007	246	1.38	1.03	1.06
LN6-2[a]	65 × 86 × 7	17 × 8	30.2	0.58	443	0.81	302	0.007	200	1.13	0.85	0.87
LN6-3[a]	65 × 86 × 7	17 × 8	30.2	0.58	443	0.81	302	0.007	210	1.19	0.89	0.91
LM6-1[a]	65 × 86 × 7	17 × 8	39.3	0.58	443	0.81	302	0.005	219	1.03	0.80[b]	0.90
LM6-2[a]	65 × 86 × 7	17 × 8	37.0	0.58	443	0.81	302	0.006	205	1.01	0.75[b]	0.86
LM6-3[a]	65 × 86 × 7	17 × 8	34.5	0.58	443	0.81	302	0.006	210	1.08	0.80	0.89
LM4-2[a]	65 × 86 × 7	17 × 8	66.0	0.58	302	0.81	302	0.003	252	1.28[b]	1.28[b]	1.28[b]
LM4-3[a]	65 × 86 × 7	17 × 8	66.0	0.58	302	0.81	302	0.003	227	1.16[b]	1.16[b]	1.16[b]
62									Average	1.18	1.05	1.26
									COV	0.17	0.21	0.34

Note: 1 MPa = 0.1450 ksi; 1 kN = 0.2248 kips.
[a]Cyclic loading.
[b]Predicted as failing in flexure.

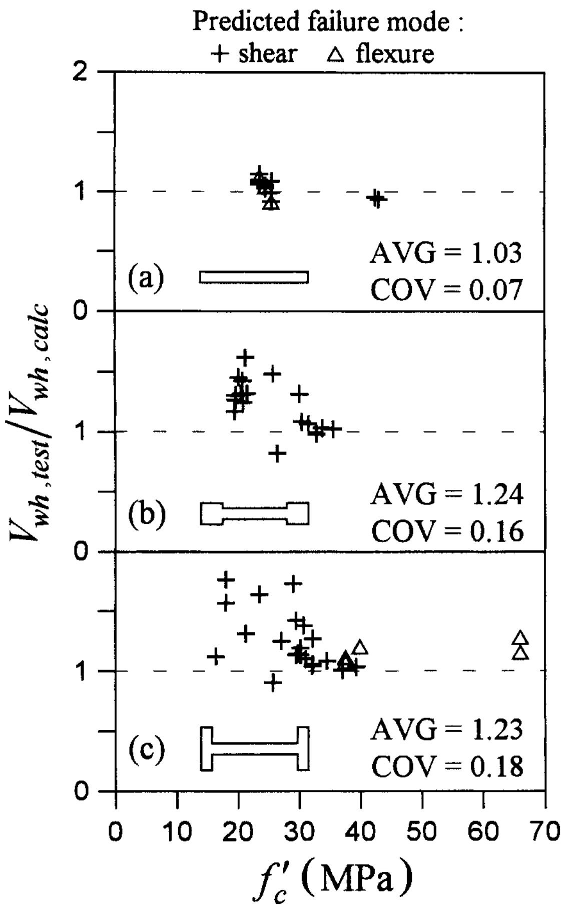

FIG. 5. Effect of Cross-Sectional Shape on Wall Shear Strength

eral approach yields better results than the simple approach (Table 1).

The strength ratio from the general approach is quite sensitive to the cross-sectional shape of the wall. Fig. 5 indicates that the walls with a barbell or flanged section have a strength ratio significantly higher than that of a rectangular section. The higher strength ratio for the wall with boundary elements, about 1.2 times higher, is attributed to the improved end conditions of its diagonal strut provided by the compression boundary element. Web crushing usually occurs in the compressive struts that intersect the compression boundary element at the wall base. Therefore, load carried by crushed struts can be transferred to higher or lower struts depending on the stiffness of the boundary element (Oesterle et al. 1984). As the load is transferred, the depth of strut a_s should be increased accordingly. An adjustment of (4) seems to be needed, and additional research is required to address those effects.

Eq. (5) can artificially increase the depth of the strut of a wall with boundary elements. However, the participation of the boundary elements in the wall shear resistance depends on several parameters such as shape, concrete strength, vertical loading, vertical reinforcement, and confinement reinforcement in the boundary elements. The simple treatment of (5) does not identify the individual influence of parameters described earlier. Therefore, the simple approach using (5) would reduce the strength ratios of walls with boundary elements but would not improve their coefficient of variation for predictions (Table 1). Moreover, it is noted that the simple approach using (5) would lead to results on the unsafe side for walls with boundary elements.

Fig. 6 shows the effect of loading type on wall shear resistance. The loading scheme was not observed to have a significant influence on the shear strength of the specimens (Fig. 6); however, the test-to-calculated strength ratio of walls subjected to cyclic loading was less than the strength ratio of walls subjected to monotonic loading by 10% (1.13/1.25 = 0.90).

In the reported tests (Table 1), the vertical forces were applied to the walls through different paths. The vertical force was applied to one boundary element (Fig. 7) for the specimens tested by Sugano (Hirosawa 1975). In the proposed model, the vertical load (N_1; Fig. 7) provides a beneficial effect

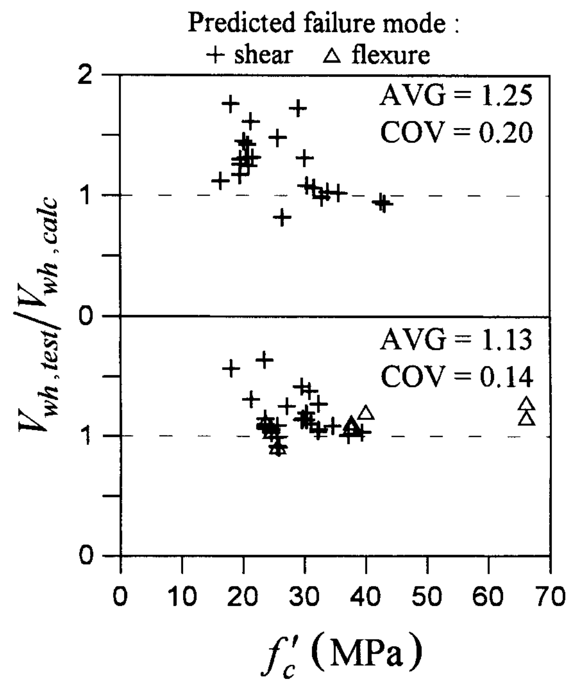

FIG. 6. Effect of Loading Type on Wall Shear Strength: (a) Monotonic; (b) Cyclic

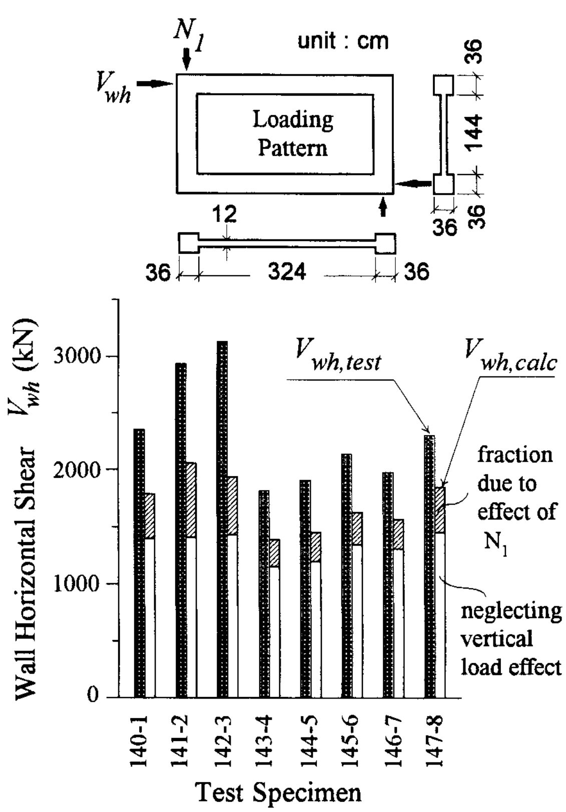

FIG. 7. Wall Specimens Tested by Sugano (Reported by Hirosawa 1975)

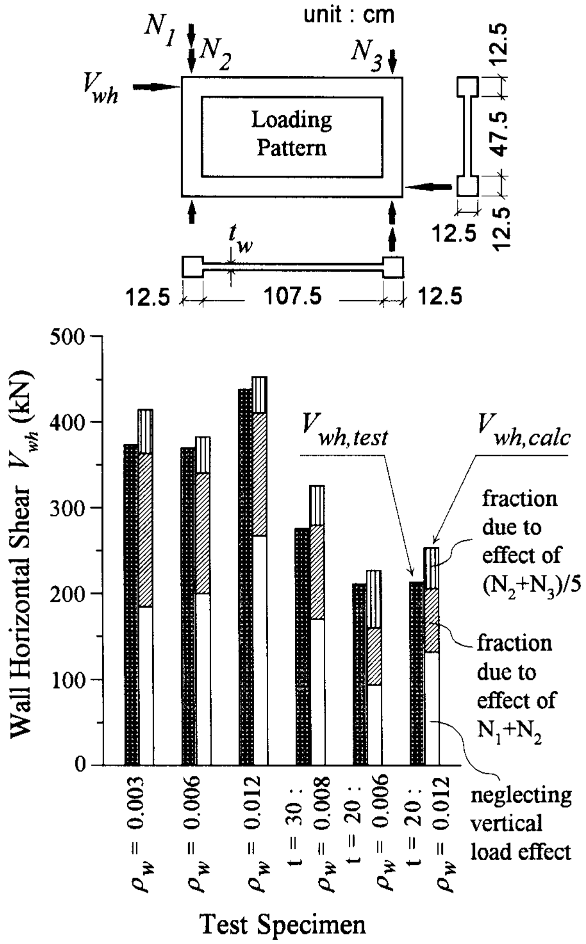

FIG. 8. Wall Specimens Tested by Yamada et al. (1974)

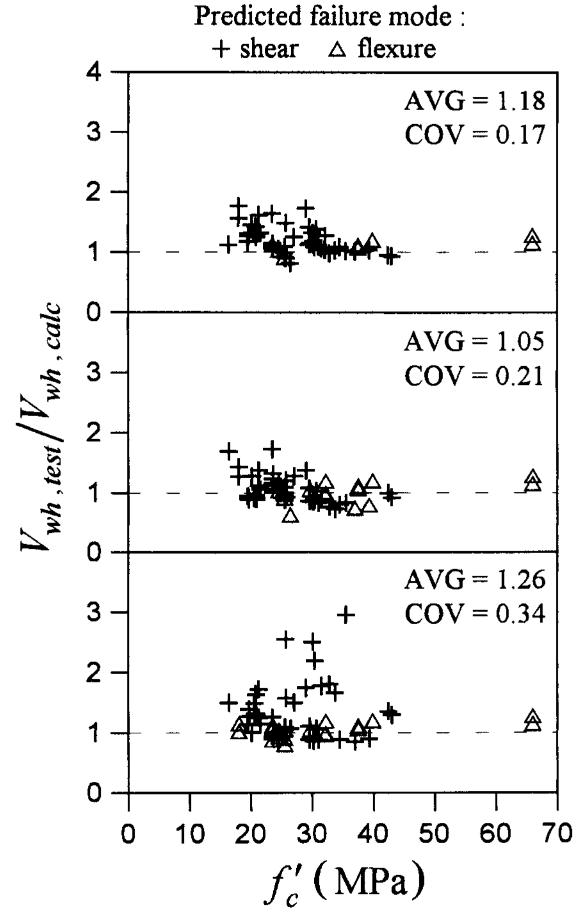

FIG. 9. Correlation of Experimental and Predicted Wall Shear Strength: (a) General Method; (b) Simple Method; (c) ACI 318-95

on wall shear strength because it increases the depth of the strut (4) and (5). Fig. 7 compares the measured wall strengths by Sugano (Hirosawa 1975) with the predicted values from the general approach (Table 1). As shown in Fig. 7, the effect due to the vertical load N_1 accounts for nearly one-fifth of the calculated wall shear strength. The vertical load, which can increase the depth of the strut in the wall, is demonstrated to have a significant effect on the wall strength (Fig. 7).

The specimens tested by Yamada et al. (1974) were subjected to three concentrated loads on the top of two boundary elements (Fig. 8). The first two vertical loads ($N_1 + N_2$; Fig. 8) will certainly increase the depth of the strut, and the fractions including this consideration by the general approach (Table 1) are illustrated in Fig. 8. However, the wall web should receive some vertical compressive stresses from another two loads ($N_2 + N_3$; Fig. 8)—transmitted through the beam action—this is another beneficial effect to be considered.

Mau and Hsu (1987) suggested that the effect of a vertical stress on the shear strength of a wall is equivalent to an additional amount of vertical reinforcement in the wall for underreinforced cases. Similarly, the vertical load on the web can be taken as the additional amount of vertical reinforcement in the softened strut-and-tie model. This vertical reinforcement can develop shear transfer through the vertical mechanism and prevent the vertical expansion of the wall panel.

Since the beam action cannot be fully effective in transmitting the vertical loads ($N_2 + N_3$; Fig. 8) to the wall web, one-fifth of the vertical loads ($N_2 + N_3$) is arbitrarily selected to be calculated as the additional amount of reinforcement for the vertical tie. By including this consideration, the predicted strengths of the specimens of Yamada et al. (1974) will be further increased as shown in Fig. 8. The fractions due to the vertical load effects have the same order of magnitude as the segments caused by omission of the vertical load effects (Fig. 8). Wall shear strengths are indicated from the studies by the softened strut-and-tie model as being quite sensitive to the vertical loads.

In Fig. 9, the experimentally determined wall shear strengths from 62 tests were compared to the shear strengths predicted by the current ACI 318-95 provisions (Sections 21.6.5.3 and 21.6.5.7 of ACI 318-95 building code) and the proposed method. The mean value and the coefficient of variation of the test-to-calculated shear strength ratio by ACI equations were found to be 1.26 and 34%, respectively (Table 1). The predictions of the ACI empirical equations are especially conservative for the walls with low reinforcement ratios and high axial loads (Table 1), and these underestimations are corrected by the proposed model. As seen in Fig. 9, the proposed model (general approach) predicts the failure shears more accurately than the equations of the current ACI 318 building code (ACI 1995).

A key feature of the proposed procedures is that they explicitly consider the influence of shear reinforcement. If a wall contains the shear reinforcement, its shear force will be carried by additional load paths instead of a diagonal strut alone. This will activate more concrete for shear resistance, and in consequence, the shear strength is increased. For example, without considering the effect of shear reinforcement of the specimens in Table 1, the proposed model with a diagonal strut only can yield a prediction about 75% of the value $V_{wh,\mathrm{calc}}$ listed in Table 1.

CONCLUSIONS

A proposal for determining the shear strength of reinforced concrete squat walls has been made. The proposed softened strut-and-tie model is derived from the concept of struts and ties, and this model satisfies equilibrium, compatibility, and constitutive laws of cracked reinforced concrete. Based on the available test results in the literature and their comparison with the proposed model and ACI 318-95 formulas, the following conclusions can be made:

1. Examination of existing experimental data indicated that the proposed model is capable of predicting the shear strengths of reinforced concrete squat walls for diagonal compression failures.
2. The softened strut-and-tie model (general approach in Table 1) predicts the failure shears more accurately than the equations of the current ACI 318-95 building code.
3. In the softened strut-and-tie model, the effects of the vertical stress in the wall can be modeled as enlarging the cross-sectional area of the compression strut or can be taken as an additional amount of reinforcement for the vertical tie. Wall shear strengths are indicated from the studies by the proposed model as being quite sensitive to the vertical loads.
4. The participation of the boundary elements in the wall shear resistance is a complicated phenomenon and additional research is needed.

ACKNOWLEDGMENTS

Much appreciation is due to the Earthquake Engineering Research Center Library, University of California at Berkeley, for providing reference works contributing to this paper through their overseas interlibrary loan service.

APPENDIX. REFERENCES

American Concrete Institute (ACI). (1995). ''Building code requirements for structural concrete.'' *ACI 318-95 and commentary (ACI 318R-95)*, Farmington Hills, Mich.

Barda, F., Hanson, J. M., and Corley, W. G. (1977). ''Shear strength of low-rise walls with boundary elements.'' *Reinforced concrete structures in seismic zones, SP-53*, American Concrete Institute, Farmington Hills, Mich.

Benjamin, J. R., and Williams, H. A. (1957). ''The behavior of one-story reinforced concrete shear walls.'' *J. Struct. Div.*, ASCE, 83(3), 1–49.

Cardenas, A. E., Russell, H. G., and Corley, W. G. (1980). ''Strength of low-rise structural walls.'' *Reinforced concrete structures subject to wind and earthquake forces, SP-63*, American Concrete Institute, Farmington Hills, Mich.

Fintel, M. (1991). ''Shearwalls—an answer for seismic resistance?'' *Concrete Int.*, 13(7), 48–53.

Foster, S. J., and Gilbert, R. I. (1996). ''The design of nonflexural members with normal and high-strength concrete.'' *ACI Struct. J.*, 93(1), 3–10.

Gupta, A., and Rangan, B. V. (1998). ''High-strength concrete (HSC) structural wall.'' *ACI Struct. J.*, 95(2), 194–204.

Hirosawa, M. (1975). ''Past experimental results on reinforced concrete shear walls and analysis on them.'' Kenchiku Kenkyu Shiryo, No. 6, Build. Res. Inst., Ministry of Construction, Tokyo, 277 (in Japanese).

Hsu, T. T. C. (1993). *Unified theory of reinforced concrete*, CRC, Boca Raton, Fla.

Hsu, T. T. C., and Mo, Y. L. (1985). ''Softening of concrete in low-rise shear walls.'' *ACI J.*, 82(6), 883–889.

Hwang, S. J., and Lee, H. J. (1999). ''Analytical model for predicting shear strengths of exterior reinforced concrete beam-column joints for seismic resistance.'' *ACI Struct. J.*, 96(5), 846–857.

Hwang, S. J., and Lee, H. J. (2000). ''Analytical model for predicting shear strengths of interior reinforced concrete beam-column joints for seismic resistance.'' *ACI Struct. J.*, 97(1), 35–44.

Jennewein, M., and Schäfer, K. (1992). ''Standardisierte nachweise von häufigen D-Bereichen.'' *DafStb.*, Heft 430, Beuth-Verlag, Berlin (in German).

Lefas, I. D., Kotsovos, M. D., and Ambraseys, N. N. (1990). ''Behavior of reinforced concrete structural walls: Strength, deformation characteristics, and failure mechanism.'' *ACI Struct. J.*, 87(1), 23–31.

Mau, S. T., and Hsu, T. T. C. (1987). ''Shear behavior of reinforced concrete framed wall panels with vertical loads.'' *ACI Struct. J.*, 84(3), 228–234.

Mo, Y. L. (1993). ''Dynamic tests on reinforced concrete shearwalls.'' *Nat. Sci. Council Proj. Rep. No. 81-0410-E006-521*, Taiwan (in Chinese).

Mo, Y. L., and Rothert, H. (1997). ''Effect of softening models on behavior of reinforced concrete frame shearwalls.'' *ACI Struct. J.*, 94(6), 730–744.

Oesterle, R. G., Aristizabal-Ochoa, J. D., Shiu, K. N., and Corley, W. G. (1984). ''Web crushing of reinforced concrete structural walls.'' *ACI J.*, 81(3), 231–241.

Paulay, T., and Priestley, M. J. N. (1992). *Seismic design of reinforced concrete and masonry buildings*, Wiley, New York.

Schäfer, K. (1996). ''Strut-and-tie models for the design of structural concrete.'' *Notes of workshop*, Dept. of Civ. Engrg., National Cheng Kung University, Tainan, Taiwan.

Vecchio, F. J., and Collins, M. P. (1993). ''Compression response of cracked reinforced concrete.'' *J. Struct. Engrg.*, ASCE, 119(12), 3590–3610.

Yamada, M., Kawamura, H., and Katagihara, K. (1974). ''Reinforced concrete shear walls without openings; Test and analysis.'' *Shear in reinforced concrete, SP-42*, American Concrete Institute, Farmington Hills, Mich.

Zhang, L. X. B., and Hsu, T. T. C. (1998). ''Behavior and analysis of 100 MPa concrete membrane elements.'' *J. Struct. Engrg.*, ASCE, 124(1), 24–34.