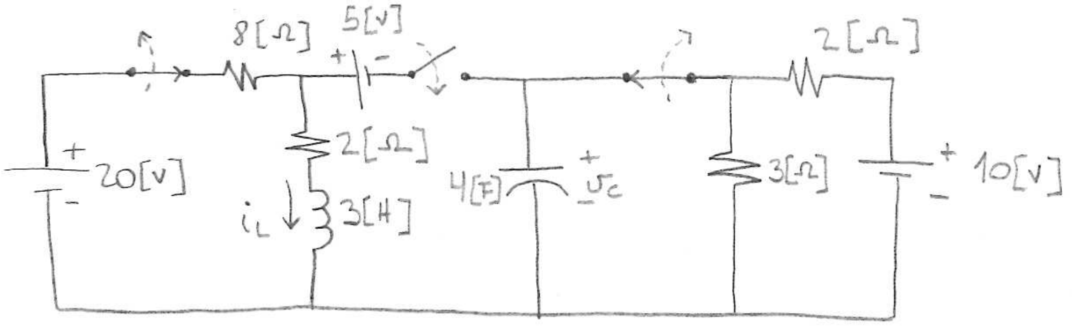
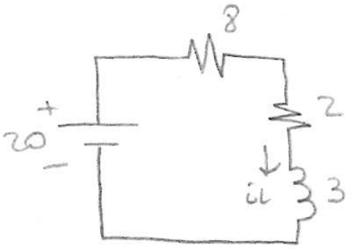
Redes de 2° Orden.

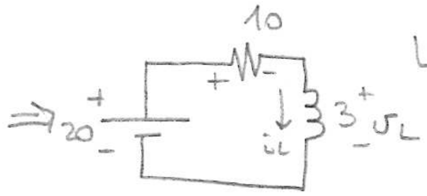
P1 Obtener la respuesta completa para v_C e i_L, considerando que en $t=0$ se mueven los interruptores, y que para $t<0$ el sistema se encontraba en régimen permanente.



SOL) en $t=0^-$:



$\Rightarrow$



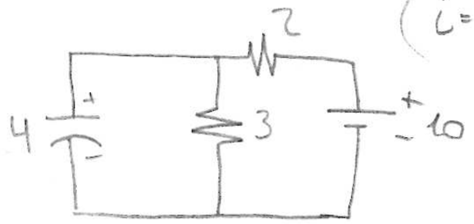
LVK: $20 = 10 i_L + v_L$

en régimen permanente, $v_L \approx 0$ pues inductancia ≈ cortocircuito.

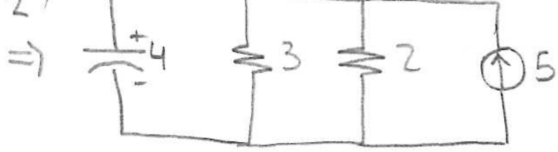
$\Rightarrow \boxed{i_L(0^-) = 2\,[A]}$ y $v_L = L\dfrac{\partial i_L(t)}{\partial t}$ $\Rightarrow$ como $v_L \approx 0$

$\Rightarrow \boxed{\dfrac{\partial i_L}{\partial t}(0^-) = 0}$

el condensador:

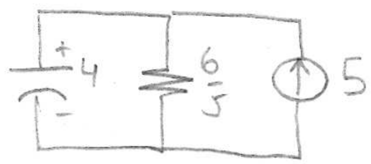


$\left(i = \dfrac{10}{2}\right)$ $\Rightarrow$

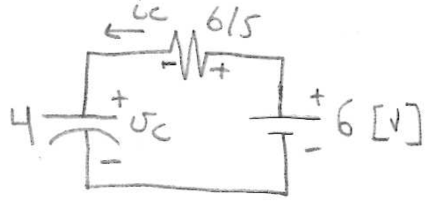


$R_{//} = \dfrac{2\cdot 3}{2+3} = \dfrac{6}{5}$

$\Rightarrow$



$\left(V = 5\cdot\dfrac{6}{5}\right)$ $\Rightarrow$



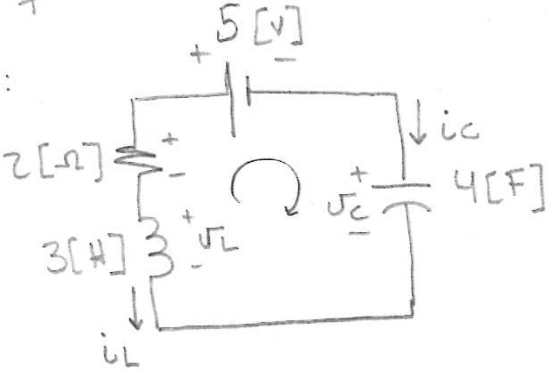
LVK: $6 = v_C + \dfrac{6}{5} i_C$ Pero en régimen permanente, $i_C \approx 0$ pues condensador ≈ circuito abierto.

$\Rightarrow \boxed{v_c(0^-) = 6\,[V]}$ y $i_c = C \dfrac{\partial v_c(t)}{\partial t}$ $\Rightarrow$ como $i_c = 0$

$$\Rightarrow \boxed{\frac{\partial v_c(0^-)}{\partial t} = 0}$$

Ahora que obtuvimos las C.I., analizamos para $t > 0$.

$\underline{t>0:}$



$\underline{LVK:}$ $\boxed{-v_L - 2i_L + 5 + v_c = 0}$ (1)

$\boxed{v_c(0^-) = 6\,[V]}$ $\boxed{\dfrac{\partial v_c}{\partial t}(0^-) = 0}$

$\boxed{i_L(0^-) = 2\,[A]}$ $\boxed{\dfrac{\partial i_L}{\partial t}(0^-) = 0}$

$\underline{LCK:}$ $i_c + i_L = 0 \Rightarrow i_L = -i_c$ $\left(y \;\; i_c = C\dfrac{\partial v_c}{\partial t} \right)$

$$\Rightarrow \boxed{i_L = -4\frac{\partial v_c}{\partial t}} \;(2) \Rightarrow \frac{i_L(0^+)}{-4} = \boxed{\frac{\partial v_c}{\partial t}(0^+) = \frac{-1}{2}} \;(2.1)$$

además, sabemos que $v_L = L\dfrac{\partial i_L}{\partial t} = \dfrac{3\partial i_L}{\partial t} \underset{\substack{\uparrow \\ (2)}}{=} 3 \cdot \dfrac{\partial}{\partial t}\left(-4\dfrac{\partial v_c}{\partial t}\right) = -12\dfrac{\partial^2 v_c}{\partial t^2}$

$$\Rightarrow \boxed{v_L = -12\frac{\partial^2 v_c}{\partial t^2}} \;(3)$$

$\Rightarrow$ Reemplazo (2) y (3) en (1):

$$-\left(-12\frac{\partial^2 v_c}{\partial t^2}\right) - 2\cdot\left(-4\frac{\partial v_c}{\partial t}\right) + 5 + v_c = 0$$

$$\Rightarrow 12\frac{\partial^2 v_c}{\partial t^2} + 8\frac{\partial v_c}{\partial t} + v_c = -5 \qquad / \cdot \frac{1}{12} \quad \left(\begin{array}{c}\text{normalizo la} \\ \text{2ª derivada}\end{array}\right)$$

$$\Rightarrow \boxed{\frac{\partial^2 v_c}{\partial t^2} + \frac{2}{3}\frac{\partial v_c}{\partial t} + \frac{1}{12}v_c = \frac{-5}{12}} \;(4)$$

$\Rightarrow$ Ahora aplico RESC y RENC para obtener la respuesta completa para v_c.

RENC: ⇒ "entrada = 0, C.I. ≠ 0."

$$\Rightarrow (4): \boxed{\frac{\partial^2 v_c}{\partial t^2} + \frac{2}{3}\frac{\partial v_c}{\partial t} + \frac{1}{12} v_c = 0} \quad \boxed{v_c(0^+) = 6\,[V]} \quad \boxed{\frac{\partial v_c(0^+)}{\partial t} = \frac{-1}{2}}$$

(2.1)

Y si me acuerdo de la forma $\boxed{\frac{\partial^2 x}{\partial t^2} + 2\alpha \frac{\partial x}{\partial t} + w_0^2 x = 0}$

Reconozco que: $\boxed{\alpha = \frac{1}{3}}$ $\boxed{w_0^2 = \frac{1}{12}}$ $\Rightarrow w_0 = \frac{1}{\sqrt{12}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$

$\alpha^2 = \frac{1}{9}$ y $w_0^2 = \frac{1}{12}$ $\Rightarrow \alpha^2 > w_0^2 \Rightarrow \boxed{\alpha > w_0}$
$\left(\frac{1}{9} > \frac{1}{12}\right)$

Si $\alpha > w_0$: ⇒ caso sobreamortiguado. $(\lambda_1, \lambda_2 < 0)(\lambda_1 \neq \lambda_2)$

⇒ Las raíces de la EDO son: $\lambda^2 + \frac{2}{3}\lambda + \frac{1}{12} = 0$

$$\lambda = \frac{-\frac{2}{3} \pm \sqrt{\frac{4}{9} - \frac{4}{12}}}{2} = \frac{-\frac{2}{3} \pm 2\sqrt{\frac{4-3}{36}}}{2} = -\frac{2}{6} \pm \frac{1}{6} \quad \begin{cases} \lambda_1 = \frac{-1}{6} \\ \lambda_2 = \frac{-1}{2} \end{cases}$$

$$\Rightarrow \boxed{v_c(t) = K_1 e^{-\frac{1}{6}t} + K_2 e^{-\frac{1}{2}t}}$$ (Forma de la sol. del caso sobreamortiguado)

$$\frac{\partial v_c(t)}{\partial t} = -\frac{1}{6}K_1 e^{-\frac{1}{6}t} - \frac{1}{2}K_2 e^{-\frac{1}{2}t}$$

Ahora, obtengo K_1 y K_2 de las C.I.

$$\Rightarrow v_c(0) = 6 = K_1 + K_2 \longrightarrow 6 = K_1 + K_2$$

$$\frac{\partial v_c(0)}{\partial t} = \frac{-1}{2} = -\frac{K_1}{6} - \frac{K_2}{2} \quad |\cdot 6 \Rightarrow 3 = K_1 + 3K_2$$

$$3 = -2K_2 \Rightarrow \boxed{K_2 = \frac{-3}{2}}$$

$$\Rightarrow K_1 = 6 + \frac{3}{2} \Rightarrow \boxed{K_1 = \frac{15}{2}}$$

$$\Rightarrow \boxed{v_c(t)_{RENC} = \frac{15}{2} e^{-\frac{1}{6}t} - \frac{3}{2} e^{-\frac{1}{2}t}}$$

RESC: $\Rightarrow$ "entrada $\neq 0$, C.I. = 0"

$$(4) \Rightarrow \boxed{\frac{\partial^2 v_c}{\partial t^2} + \frac{2}{3}\frac{\partial v_c}{\partial t} + \frac{1}{12} v_c = \frac{-5}{12}.}$$

Homogénea: $\lambda_1 = \frac{-1}{6}$ $\lambda_2 = -\frac{1}{2}$

$$\Rightarrow v_c(t)_{Hom.} = K_1 e^{-\frac{1}{6}t} + K_2 e^{-\frac{1}{2}t}$$

Particular: $\frac{1}{12} v_c = -\frac{5}{12} \Rightarrow \boxed{v_{c\,Part.}(t) = -5}$

$$\Rightarrow v_c(t) = v_{c\,Hom.} + v_{c\,Part.} = K_1 e^{-\frac{1}{6}t} + K_2 e^{-\frac{1}{2}t} - 5$$

$$\frac{\partial v_c}{\partial t}(t) = -\frac{K_1}{6} e^{-\frac{1}{6}t} - \frac{K_2}{2} e^{-\frac{1}{2}t}$$

(Y como es Resc, $\Rightarrow v_c(0) = 0$, $\frac{\partial v_c}{\partial t}(0) = 0$)

$$\Rightarrow v_c(0) = \boxed{0 = K_1 + K_2 - 5} \Rightarrow K_1 = 5 - K_2$$

$$\frac{\partial v_c}{\partial t}(0) = \boxed{0 = -\frac{K_1}{6} - \frac{K_2}{2}} \Rightarrow K_1 = -3K_2 \longrightarrow \boxed{K_2 = -5/2}$$

$$\Rightarrow K_1 = 5 - K_2 = 5 + \frac{5}{2} = \boxed{\frac{15}{2} = K_1}$$

$$\Rightarrow \boxed{v_{c\,resc}(t) = \frac{15}{2} e^{-\frac{1}{6}t} - \frac{5}{2} e^{-\frac{1}{2}t} - 5}$$

⇒ Respuesta Completa = RENC + RESC

$$\Rightarrow v_C(t) = \frac{15}{2} e^{-\frac{1}{6}t} - \frac{3}{2} e^{-\frac{1}{2}t} + \frac{15}{2} e^{-\frac{1}{6}t} - \frac{5}{2} e^{-\frac{1}{2}t} - 5$$

$$\Rightarrow \boxed{v_C(t) = \underbrace{15 e^{-\frac{1}{6}t} - 4 e^{-\frac{1}{2}t}}_{\text{Parte Transitoria}} \underbrace{-5}_{\text{Parte Permanente.}}}$$

(⇒ en $t \to \infty$
$v_C(t) \to -5$ [V])

(Se verifica que $v_C(0) = 6$)

Y para obtener i_L, recuerdo la ecuación (2).

$$\Rightarrow i_L(t) = -4 \frac{\partial}{\partial t}\left(15 e^{-\frac{1}{6}t} - 4 e^{-\frac{1}{2}t} - 5\right)$$

$$= +\cancel{4}^{2} \cdot \cancel{15}^{5} \cdot \frac{+1}{\cancel{6}} e^{-\frac{1}{6}t} + \cancel{4}^{2} \cdot + 4 \cdot \frac{-1}{\cancel{2}} e^{-\frac{1}{2}t}$$

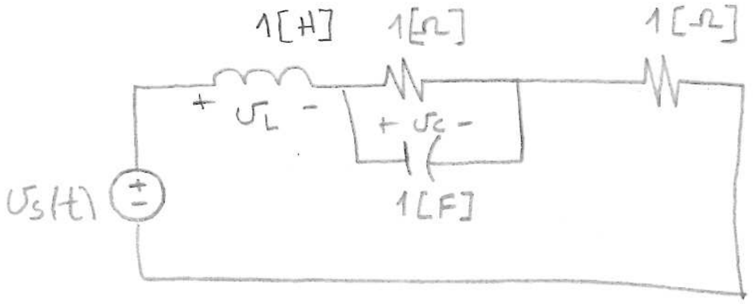
$$\Rightarrow \boxed{i_L(t) = \underbrace{10 e^{-\frac{1}{6}t} - 8 e^{-\frac{1}{2}t}}_{\text{Parte Transitoria}}}$$

(⇒ en $t \to \infty$
$i_L(t) \to 0$.)

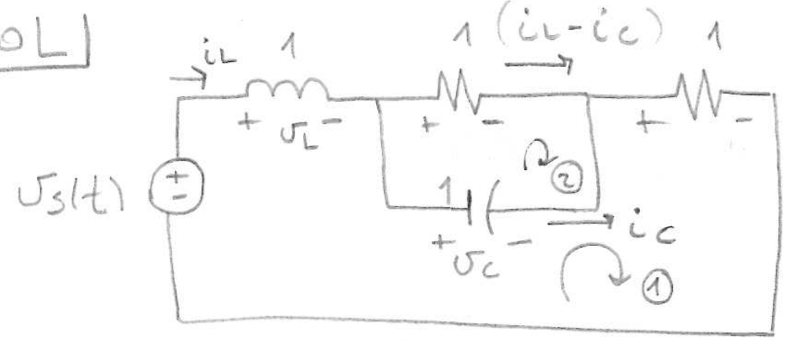
(Parte Permanente = 0)

Se verifica que $i_L(0) = 2$

P2 Determinar $h(t)$ (respuesta al impulso) como una respuesta de estado cero con condiciones iniciales en $t=0^+$ (Determinar $h(t)$ para el voltaje en el condensador).



SOL)



$$\text{LVK}_{①}: \boxed{v_S = v_L + v_C + i_L \cdot 1} \quad (1)$$

$$\text{LVK}_{②}: \boxed{i_L - i_C - v_C = 0} \quad (2)$$

$$v_L = 1\cdot\frac{\partial i_L}{\partial t} \qquad i_C = 1\cdot\frac{\partial v_C}{\partial t}$$

$$\Rightarrow (2): \boxed{i_L = \frac{\partial v_C}{\partial t} + v_C} \rightarrow \text{Reemplazo en } (1).$$

$$\Rightarrow (1): v_S = \frac{\partial}{\partial t}\left(\frac{\partial v_C}{\partial t} + v_C\right) + v_C + \left(\frac{\partial v_C}{\partial t} + v_C\right)$$

$$\Rightarrow \boxed{\frac{\partial^2 v_C}{\partial t^2} + 2\frac{\partial v_C}{\partial t} + 2v_C = v_S} \ (3) \left(\begin{array}{r} \text{y una RESC} \Rightarrow v_C(0^-) = 0 \\ \dfrac{\partial v_C(0^-)}{\partial t} = 0 \end{array}\right)$$

Ahora, reemplazo v_s por un $\delta(t)$, e integro (3) entre $t=0^-$ y $t=0^+$

$$\Rightarrow \underbrace{\int_{0^-}^{0^+} \frac{\partial^2 v_c}{\partial t^2}\,\partial t}_{③} + \underbrace{2\int_{0^-}^{0^+} \frac{\partial v_c}{\partial t}\,\partial t}_{②} + \underbrace{2\int_{0^-}^{0^+} v_c\,\partial t}_{①} = \underbrace{\int_{0^-}^{0^+} \delta(t)\,\partial t}_{=1 \text{ por definición del impulso.}} \qquad (4)$$

La única forma de que la ecuación (3) se pueda balancear, siendo $v_s = \delta(t)$, es que v_c sea contínua, siendo así $\frac{\partial v_c}{\partial t}$ un escalón y $\frac{\partial^2 v_c}{\partial t^2}$ un impulso.

$$\Rightarrow ① = \int_{0^-}^{0^+} v_c\,\partial t = 0$$, se anula por integrar en un intervalo ínfimo sobre una variable contínua.

$$\Rightarrow ② = 2\underbrace{\left(v_c(0^+) - v_c(0^-)\right)}_{=0} \qquad \Rightarrow \boxed{v_c(0^+) = 0}$$

pues $v_c(t)$ es contínua, y $v_c(0^-)=0$

$\Rightarrow$ el único término que sobrevive de (4) es ③

$$\Rightarrow (4):\ \frac{\partial v_c(0^+)}{\partial t} - \cancel{\frac{\partial v_c(0^-)}{\partial t}} = 1 \qquad \Rightarrow \boxed{\frac{\partial v_c(0^+)}{\partial t} = 1}$$

0 por RESC.

$\Rightarrow$ nuestro nuevo problema a resolver para $t>0$ ($\delta(t)=0$ para $t>0$) es:

$$\boxed{\begin{array}{l} \dfrac{\partial^2 v_c}{\partial t^2} + \dfrac{2\,\partial v_c}{\partial t} + 2 v_c = 0 \qquad (5) \\ v_c(0^+) = 0 \\ \dfrac{\partial v_c}{\partial t}(0^+) = 1 \end{array}}$$

← La ecuación (3) para $t>0$.

$\Rightarrow$ de la forma $\boxed{\dfrac{\partial^2 x}{\partial t^2} + 2\alpha \dfrac{\partial x}{\partial t} + w_0^2 x = 0}$ en (5) distingo:

$\alpha = 1 \qquad \alpha^2 = 1 \quad \Rightarrow \quad \alpha^2 < w_0^2 \Rightarrow \boxed{\alpha < w_0}$

$w_0^2 = 2$

$\Rightarrow$ caso subamortiguado.

$\Rightarrow$ sol. de la forma $\left[v_C(t) = K_1 e^{-t} \cos(w_d t + \phi) \right]$

con $w_d = \sqrt{w_0^2 - \alpha^2} = \sqrt{2-1} = 1$

$\Rightarrow \boxed{v_C(t) = K_1 e^{-t} \cos(t + \phi)}$ Ahora obtengo K_1 y ϕ ctes, usando las C.I.

$$\left(\frac{\partial v_C}{\partial t}(t) = -K_1 e^{-t} \cos(t+\phi) - K_1 e^{-t} \operatorname{sen}(t+\phi) \right)$$

$v_C(0^+) = \boxed{0 = K_1 \cos(\phi)}$. $\quad K_1 \neq 0 \Rightarrow \boxed{\phi = \frac{\pi}{2} \vee \phi = -\frac{\pi}{2}}$

$$\frac{\partial v_C}{\partial t}(0^+) = 1 = \underbrace{-K_1 \cos(\phi)}_{=0} - K_1 \operatorname{sen}(\phi)$$

$\Rightarrow \quad 1 = -K_1 \operatorname{sen}(\phi) \rightarrow \boxed{\text{si } \phi = \frac{\pi}{2} \Rightarrow K_1 = -1}$

$\boxed{\text{si } \phi = -\frac{\pi}{2} \Rightarrow K_1 = 1}$ $\rightarrow$ (Ambas sirven)

$\Rightarrow \boxed{h(t) = v_C(t) = e^{-t} \cos\left(t - \frac{\pi}{2}\right) \cdot u(t)}$ respuesta al impulso del circuito.

que es equivalente a

$\boxed{h(t) = v_C(t) = -e^{-t} \cos\left(t + \frac{\pi}{2}\right) \cdot u(t)}$ Por propiedades del coseno.

$(\cos(\theta) = -\cos(\theta + \pi))$.