**EJERCICIO Nº 6**

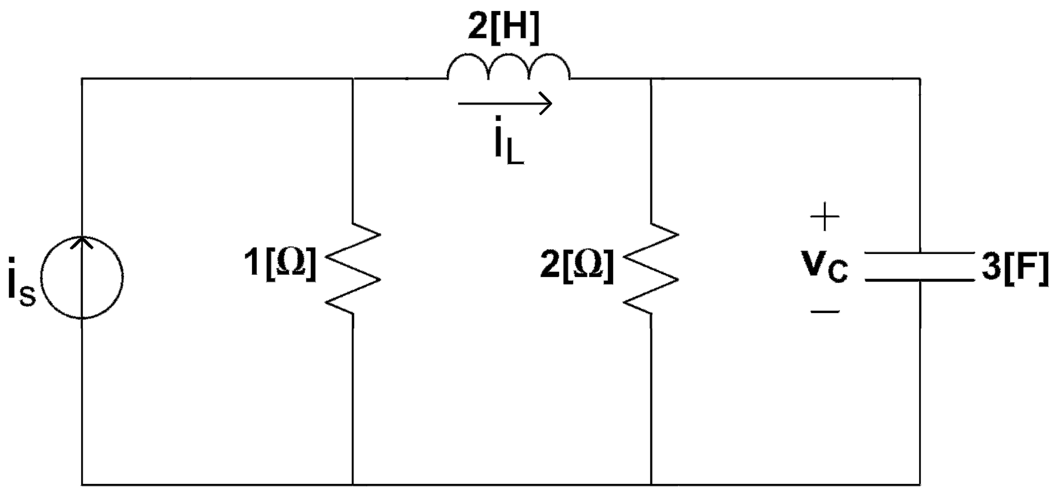
**EL 31-A ANALISIS DE REDES I**

**Prof : Santiago Bradford V.** **07 de octubre de 2008**
**Prof. Aux : Heinz Gerdin H.**

1. Para la red lineal e invariante de la figura determine:

i) La ecuación diferencial de la variable $v_c(t)$, ocupando el método de las mallas (circuitos regionales) y el operador D (no considere las condiciones iniciales como fuentes)

ii) Las condiciones iniciales para $v_c(t)$ y su derivada, que permiten que la solución sea única, mediante el planteamiento del circuito en $t=0$.

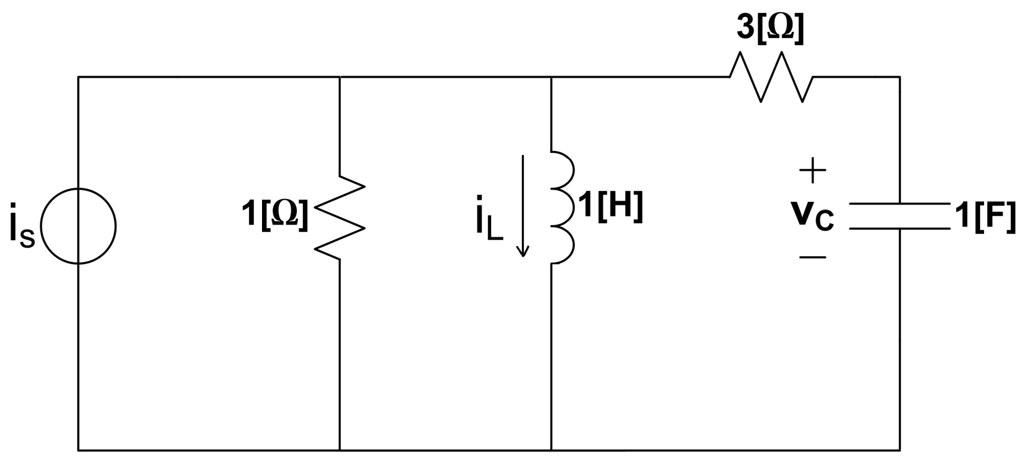



$v_C(0) = 1[V] \quad i_L(0) = 2[A]$

2. Para la red lineal e invariante de la figura, con $\boldsymbol{i_S(t) = e^{-t}u(t)}$ determine:

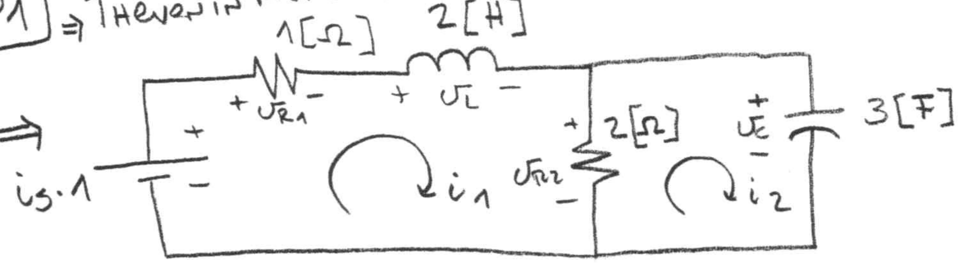
i) La ecuación diferencial de la variable $v_c(t)$, ocupando el método de los nodos y el operador D (considere condiciones iniciales nulas).

ii) La respuesta de Estado Cero para la variable $v_c(t)$.



$\boxed{P1}$ $\Rightarrow$ Thevenin Norton.

$\boxed{i}$ $\Rightarrow$




$\boxed{v_{R1} = i_1 \cdot 1}$ $\qquad i_c = 3Dv_c \Rightarrow \boxed{v_c = \frac{i_2}{3D}}$ (1)

$\boxed{v_L = 2Di_1}$

$\boxed{v_{R2} = (i_1 - i_2)\cdot 2}$

$\Rightarrow$ LVK's: $v_{R1} + v_L + v_{R2} = i_s$

$\Rightarrow i_1 + 2Di_1 + (i_1 - i_2)\cdot 2 = i_s$

$\Rightarrow \boxed{i_1(3+2D) + i_2(-2) = i_s}$

$-v_{R2} + v_c = 0 \Rightarrow i_2 \cdot 2 - i_1 \cdot 2 + \frac{i_2}{3D} = 0 \Rightarrow \boxed{i_1(-2) + i_2\left(2 + \frac{1}{3D}\right) = 0}$

$\Rightarrow$ matricialmente:

$$\begin{bmatrix} 3+2D & -2 \\ -2 & 2+\frac{1}{3D} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \end{bmatrix} \leftarrow (2)$$

$\boxed{i_2 = \frac{\Delta_2}{\Delta}}$ (3) $\quad \Delta_2 = \begin{vmatrix} 3+2D & i_s \\ -2 & 0 \end{vmatrix} = \boxed{2i_s} = \Delta_2$

$\Delta = \begin{vmatrix} 3+2D & -2 \\ -2 & 2+\frac{1}{3D} \end{vmatrix} \Rightarrow \boxed{\Delta = 4D + \frac{8}{3} + \frac{1}{D}}$

además, de (1): $\underline{i_2 = 3Dv_c}$

en (3):

$\Rightarrow v_c \cdot 3D \cdot \left(4D + \frac{8}{3} + \frac{1}{D}\right) = 2i_s$

$\Rightarrow v_c(12D^2 + 8D + 3) = 2i_s$

$$\Rightarrow \boxed{12\frac{\partial^2 v_c}{\partial t^2} + \frac{8\partial v_c}{\partial t} + 3v_c = 2i_s}$$ Ec. diferencial para $v_c(t)$.

ii $v_c(0) = 1\,[V]$.

$$\frac{\partial v_c}{\partial t}(0) = ?$$

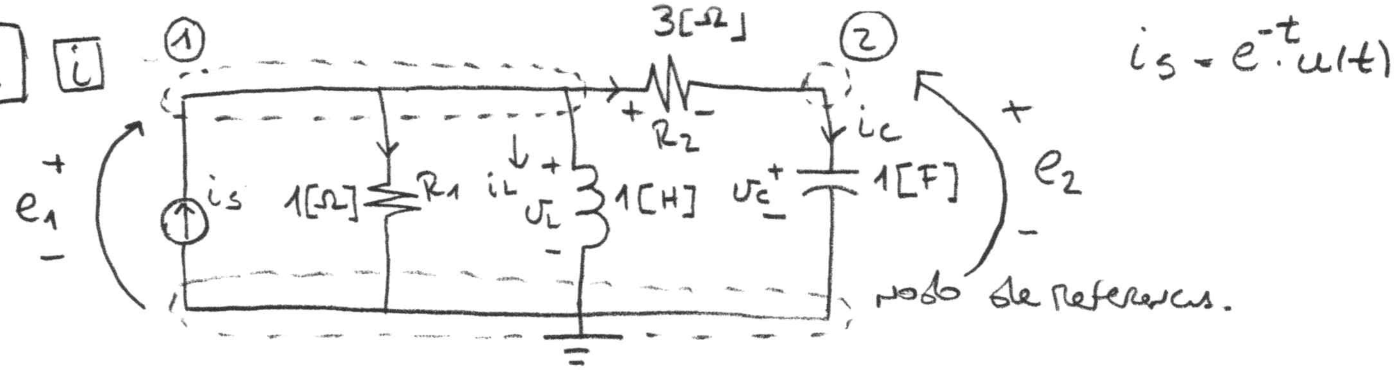
en (2): $-2i_1 + i_2\left(2 + \frac{1}{3D}\right) = 0$ pero de (1): $i_2 = 3Dv_c$

$$\Rightarrow \cancel{3D}v_c\left(\frac{6D+1}{\cancel{3D}}\right) = 2i_1 \qquad \text{y } i_1 = i_L$$

$$\Rightarrow 6\frac{\partial v_c}{\partial t} + v_c = 2i_L \quad \text{en } t=0 \ldots$$

$$\Rightarrow \frac{\partial v_c(0)}{\partial t} = \frac{2\overbrace{i_L(0)}^{2} - \overbrace{v_c(0)}^{1}}{6} \Rightarrow \boxed{\frac{\partial v_c}{\partial t}(0) = \frac{1}{2}}$$ C.I. para $\frac{\partial v_c}{\partial t}$ en $t=0$.

P2 i

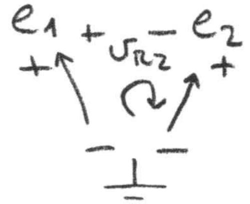



$v_{R1} = R_1 \cdot i_{R1}$ pero $v_{R1} = e_1 \Rightarrow \boxed{i_{R1}} = \frac{e_1}{1} = \boxed{e_1}$

$v_L = L D i_L \quad v_L = e_1, L = 1 \Rightarrow \boxed{i_L = \frac{e_1}{D}}$

$i_C = C D v_C \quad v_C = e_2 \; C = 1 \Rightarrow \boxed{i_C = D e_2}$

$v_{R2} = e_1 - e_2$, (pues $e_1 \overset{+}{\ } v_{R2} \overset{-}{\ } e_2$ LVK $\Rightarrow v_{R2} = e_1 - e_2$)



$v_{R2} = R_2 \cdot i_{R2} \quad R_2 = 3, \Rightarrow \boxed{i_{R2} = \frac{e_1 - e_2}{3}}$

$\Rightarrow$ LCK①: $i_{R1} + i_L + i_{R2} = i_s \Rightarrow e_1 + \frac{e_1}{D} + \frac{e_1 - e_2}{3} = i_s$

$$\Rightarrow \boxed{e_1\left(\frac{4}{3} + \frac{1}{D}\right) + e_2\left(\frac{-1}{3}\right) = i_s}$$

$\Rightarrow$ LCK②: $-i_{R2} + i_C = 0 \Rightarrow \frac{e_2 - e_1}{3} + D e_2 = 0 \Rightarrow \boxed{e_1\left(\frac{-1}{3}\right) + e_2\left(\frac{1}{3} + D\right) = 0}$

$\Rightarrow$ matricialmente:

$$\begin{bmatrix} \frac{4}{3} + \frac{1}{D} & \frac{-1}{3} \\ -\frac{1}{3} & \frac{1}{3} + D \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \end{bmatrix}$$

Pero $v_C = e_2 \Rightarrow$ obtengo $e_2$.

$$e_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_2 = \begin{vmatrix} 4 + \frac{1}{D} & i_s \\ -\frac{1}{3} & 0 \end{vmatrix} = \boxed{\frac{i_s}{3} = \Delta_2}$$

$$\Delta = \left(\frac{4}{3} + \frac{1}{D}\right)\left(\frac{1}{3} + D\right) - \left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)$$

$$\boxed{\Delta = \frac{4D}{3} + \frac{4}{3} + \frac{1}{3D}}$$

$$\Rightarrow e_2 = v_C = \frac{i_s/3}{\frac{4D}{3} + \frac{4}{3} + \frac{1}{3D}} \Rightarrow v_C = \frac{i_s}{4D + 4 + \frac{1}{D}}$$

$$\Rightarrow v_c\left(4D+4+\frac{1}{D}\right)=i_s \qquad /\cdot D$$

$$\Rightarrow 4\frac{\partial^2 v_c}{\partial t^2}+4\frac{\partial v_c}{\partial t}+v_c=\frac{\partial i_s}{\partial t} \qquad i_s=e^{-t}\ (\text{en } t>0)$$

$$\Rightarrow \boxed{4\frac{\partial^2 v_c}{\partial t^2}+4\frac{\partial v_c}{\partial t}+v_c=-e^{-t}}$$ Ecuación diferencial para $v_c(t)$

ii) RESC $\Rightarrow v_c(0)=0$, $i_L(0)=0$.

Homogénea: $4\frac{\partial^2 v_c}{\partial t^2}+4\frac{\partial v_c}{\partial t}+v_c=0 \Rightarrow 4\lambda^2+4\lambda+1=0$

$\Rightarrow (2\lambda+1)^2=0 \Rightarrow \boxed{\lambda=-\frac{1}{2} \text{ frecuencia doble.}}$

$$\Rightarrow v_{cH}=K_1e^{-\frac{1}{2}t}+K_2te^{-\frac{1}{2}t}$$

Particular: $v_{cp}$ tiene la forma de la entrada, $\Rightarrow v_{cp}=Ce^{-t}$

$\Rightarrow$ reemplazo en la Ec. diferencial.

$$\Rightarrow \cancel{4Ce^{-t}}-\cancel{4Ce^{-t}}+Ce^{-t}=-e^{-t} \Rightarrow \boxed{C=-1}$$

$$\Rightarrow v_{cp}=-e^{-t}$$

$$\Rightarrow v_c(t)=K_1e^{-\frac{1}{2}t}+K_2te^{-\frac{1}{2}t}-e^{-t}$$

$$\frac{\partial v_c}{\partial t}(t)=-\frac{1}{2}K_1e^{-\frac{1}{2}t}+K_2e^{-\frac{1}{2}t}-\frac{1}{2}K_2te^{-\frac{1}{2}t}+e^{-t}$$

$$\Rightarrow v_c(0)=0=K_1-1 \Rightarrow \boxed{K_1=1}$$

del LCK ①: $e^{-t}=i_L+i_C+i_{R1}$ Pero $i_{R1}=e_1=v_L=\frac{\partial i_L}{\partial t}$, $i_C=\frac{\partial v_c}{\partial t}$

$$\Rightarrow \boxed{i_L+\frac{\partial v_c}{\partial t}+\frac{\partial i_L}{\partial t}=e^{-t}} \quad (1)$$

del LCK ②: $-\frac{v_L}{3}+\frac{v_c}{3}+\frac{\partial v_c}{\partial t}=0 \Rightarrow \boxed{\frac{\partial v_c}{\partial t}-\frac{1}{3}\frac{\partial i_L}{\partial t}=-\frac{v_c}{3}} \quad (2)$

(1) − (2) evaluado en $t=0$

$$\Rightarrow \cancel{i_L(0)}^{\,0}+\frac{4}{3}\frac{\partial i_L(0)}{\partial t}=\cancel{e^{-0}}^{\,1}+\cancel{\frac{v_c(0)}{3}}^{\,0} \Rightarrow \boxed{\frac{\partial i_L}{\partial t}(0)=\frac{3}{4}}$$ en (2): $\boxed{\frac{\partial v_c}{\partial t}(0)=\frac{1}{4}}$

$$\Rightarrow \frac{\partial v_c}{\partial t}(0) = \frac{1}{4} = -\frac{1}{2}K_1 + K_2 + 1 \qquad (K_1 = 1)$$

$$\Rightarrow K_2 = \frac{1}{4} - 1 + \frac{1}{2} \Rightarrow \boxed{K_2 = \frac{-1}{4}}$$

$$\Rightarrow \boxed{v_{c\,resc}(t) = \left(e^{-\frac{1}{2}t} - \frac{t}{4}e^{-\frac{1}{2}t} - e^{-t}\right)u(t)}$$