EXAMEN

EL 31-A ANALISIS DE REDES I

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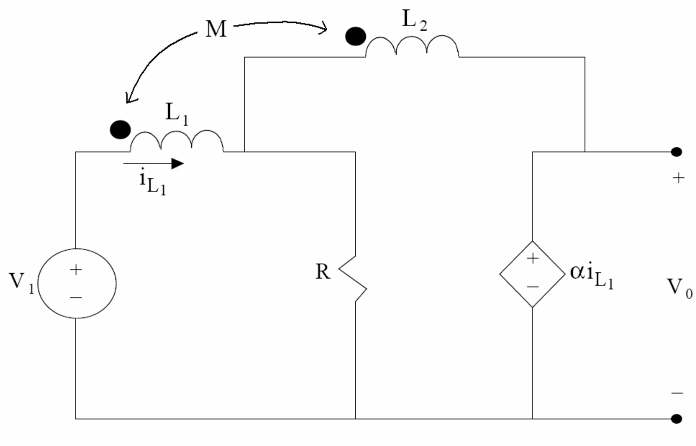
1.- Para la red lineal e invariante de la figura, cuyos elementos poseen condiciones iniciales nulas:

a) Determine la Función de Red $H(s)=\dfrac{V_0(s)}{V_1(s)}$

b) Determine el valor del parámetro α tal que la Función de Red posea 2 polos reales $s_1= -1$ y $s_2=-3$.

c) Encuentre la respuesta al impulso para $V_0(t)$, expresándola en el dominio del tiempo.

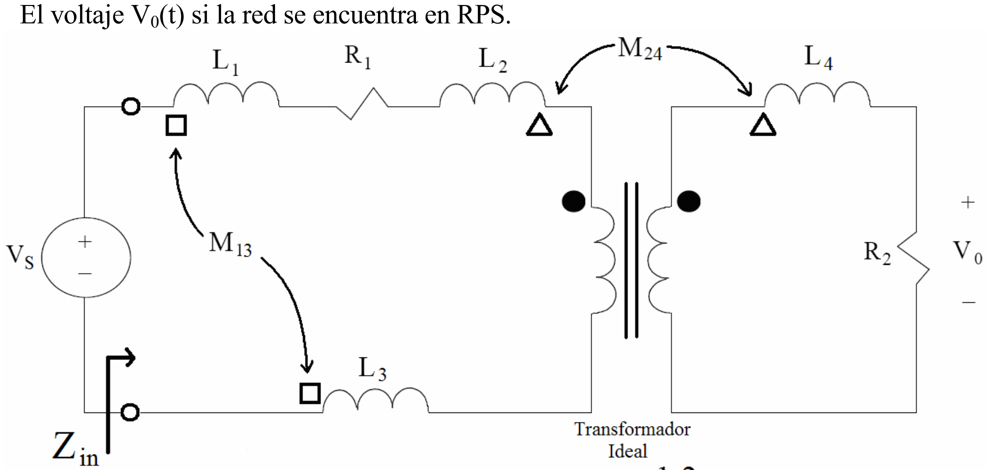


$L_1=2[H]$ $L_2=5[H]$ $M=3[H]$ $R=1[\Omega]$

2.- Para la red lineal e invariante de la figura, determine:

a) La impedancia de entrada Z_{in}

b) El voltaje $V_0(t)$ si la red se encuentra en RPS.

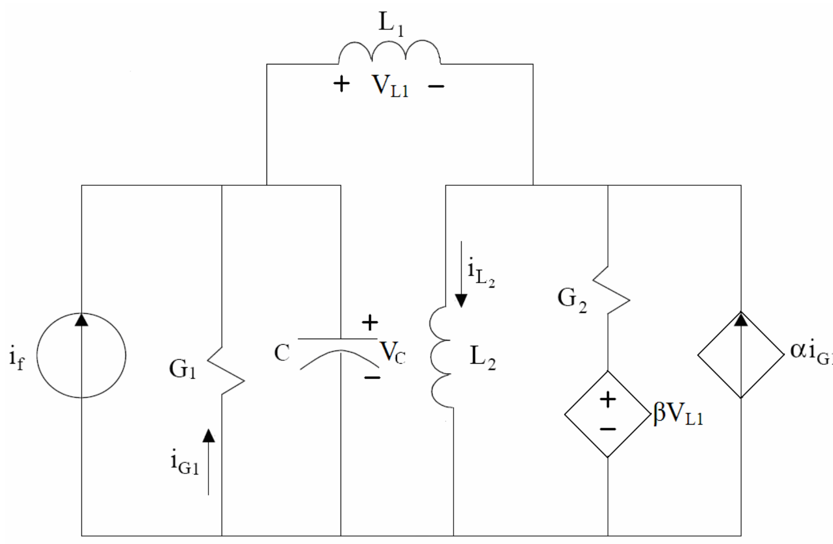


$n_1{:}n_2=1{:}2$

$V_S = 20\cos(2t+\pi/3)$

$L_1=3[H]$ $L_2=6[H]$ $L_3=2[H]$ $L_4=2[H]$ $M_{13}=1[H]$ $M_{24}=1[H]$ $R_1=4[\Omega]$ $R_2=6[\Omega]$

3.- Considere la red lineal e invariante de la Figura. Obtenga las ecuaciones correspondientes al método nodal $[Y_N(s)][E(s)]=[I_S(s)]$ siguiendo el método matricial sistemático, detallando su procedimiento paso a paso y utilizando transformada de Laplace. Incorpore las condiciones iniciales como fuentes.

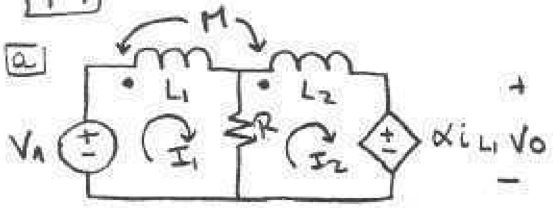


$V_C(0)=2[V]$ $i_{L1}(0)=1[A]$ $i_{L2}(0)=2[A]$

Señal	***Forma de onda***	***Transformada***
Impulso	$\delta(t)$	1
Escalón	$u(t)$	$\frac{1}{S}$
Rampa	$t\cdot u(t)$	$\frac{1}{S^2}$
Exponencial	$\left(e^{-\alpha\cdot t}\right)\cdot u(t)$	$\frac{1}{S+\alpha}$
Rampa Amortiguada	$\left(te^{-\alpha\cdot t}\right)\cdot u(t)$	$\frac{1}{(S+\alpha)^2}$
Seno	$[sen(\beta\cdot t)]\cdot u(t)$	$\frac{\beta}{S^2+\beta^2}$
Coseno	$[\cos(\beta\cdot t)]\cdot u(t)$	$\frac{S}{S^2+\beta^2}$
Seno Amortiguado	$[e^{-\alpha\cdot t}sen(\beta\cdot t)]\cdot u(t)$	$\frac{\beta}{(S+\alpha)^2+\beta^2}$
Coseno Amortiguado	$[e^{-\alpha\cdot t}\cos(\beta\cdot t)]\cdot u(t)$	$\frac{(S+\alpha)}{(S+\alpha)^2+\beta^2}$
Polos complejos simples	$2\cdot\lvert K\rvert\cdot e^{\alpha\cdot t}\cdot\cos(\beta\cdot t+\angle K)\cdot u(t)$	$\frac{K}{(S+\alpha-j\beta)}+\frac{\overline{K}}{(S+\alpha+j\beta)}$
Polos complejos dobles	$2\cdot\lvert K\rvert\cdot t\cdot e^{\alpha\cdot t}\cdot\cos(\beta\cdot t+\angle K)\cdot u(t)$	$\frac{K}{(S+\alpha-j\beta)^2}+\frac{\overline{K}}{(S+\alpha+j\beta)^2}$

P1

a)



$$\Rightarrow V_1 = sL_1 I_1 + sM I_2 + R(I_1 - I_2)$$

$$-R(I_1 - I_2) + sL_2 I_2 + sM I_1 + \alpha I_1 = 0$$

$$\Rightarrow \begin{bmatrix} sL_1 + R & -R + sM \\ -R + sM + \alpha & R + sL_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}$$

y como $\alpha i_{L_1} = \alpha I_1$

$\Rightarrow V_o = \alpha I_1$ e $I_1 = \dfrac{\Delta_1}{\Delta}$

$\Rightarrow \boxed{V_o = \alpha \dfrac{\Delta_1}{\Delta}}$

$$\Rightarrow \begin{bmatrix} 2s + 1 & -1 + 3s \\ -1 + 3s + \alpha & 1 + 5s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix}$$

$$\Delta = (2s+1)(1+5s) + (1-3s)(-1+3s+\alpha) \Rightarrow \Delta = s^2 + s(13-3\alpha) + (1+\alpha)$$

$$\Delta_1 = V_1 \cdot (5s+1) \qquad \Rightarrow V_o = \frac{\alpha V_1 (5s+1)}{s^2 + s(13-3\alpha) + (1+\alpha)}$$

$$\Rightarrow \boxed{\frac{V_o(s)}{V_1(s)} = H(s) = \frac{\alpha (5s+1)}{s^2 + s(13-3\alpha) + \alpha}}$$

b) Polos reales en -3 y -1 $\Rightarrow$ denominador de $H(s) = (s+3)(s+1) = s^2 + 4s + 3$

$$\Rightarrow 13 - 3\alpha = 4 \Rightarrow \boxed{\alpha = 3} \qquad \Rightarrow H(s) = \frac{15s + 3}{(s+3)(s+1)}$$

c) Respuesta al impulso $\Rightarrow V_1(t) = \delta(t) \Rightarrow V_1(s) = 1$

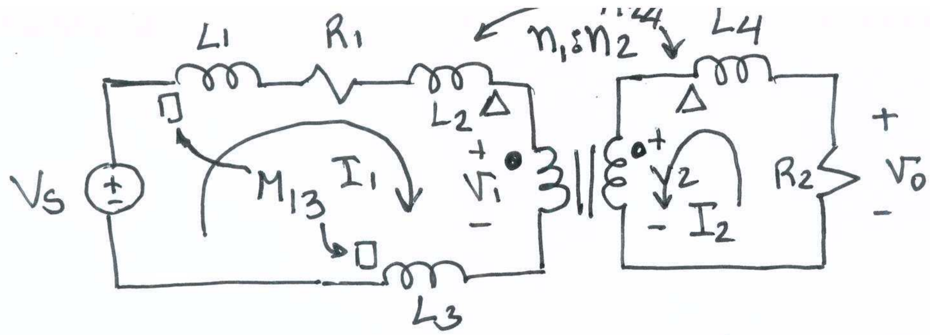
$$\Rightarrow H(s) = \frac{V_o(s)}{\underbrace{V_1(s)}_{1}} = V_o(s) \qquad \Rightarrow \mathcal{L}^{-1}[H(s)] = \mathcal{L}^{-1}[V_o(s)]$$ ← La respuesta al impulso!

$$H(s) = \frac{A}{(s+3)} + \frac{B}{(s+1)} \qquad A = H(s) \cdot (s+3)\Big|_{s=-3} = \frac{-45+3}{-2} \Rightarrow \boxed{21 = A}$$

$$B = H(s) \cdot (s+1)\Big|_{s=-1} = \frac{-15+3}{2} \Rightarrow \boxed{B = -6}$$

$$\Rightarrow H(s) = 21 \cdot \frac{1}{(s+3)} - 6 \cdot \frac{1}{(s+1)} \qquad \Rightarrow \boxed{h(t) = \left[21e^{-3t} - 6e^{-t}\right] \cdot u(t)}$$

2)



con los direcciones de corrientes virtuales de la figura

$$V_S = j\omega L_1 I_1 - j\omega M_{13} I_1 + R_1 I_1 + j\omega L_2 I_1 + j\omega M_{24} I_2 + V_1 + j\omega L_3 I_1 - j\omega M_{13} I_1$$

$$0 = R_2 I_2 + j\omega L_4 I_2 + j\omega M_{24} I_1 + V_2$$

ordenando un poco

$$V_S = \left[R_1 + j\omega(L_1 + L_2 + L_3 - 2M_{13})\right] I_1 + j\omega M_{24} I_2 + V_1$$

$$0 = (R_2 + j\omega L_4) I_2 + j\omega M_{24} I_1 + V_2$$

Ecuaciones del transformador $\quad \dfrac{V_1}{V_2} = \dfrac{n_1}{n_2} \qquad \dfrac{I_1}{I_2} = -\dfrac{n_2}{n_1}$

$$\Rightarrow I_2 = -\frac{n_1}{n_2} I_1$$

$$V_2 = \frac{n_2}{n_1} V_1$$

$$V_S = \left[R_1 + j\omega(L_1 + L_2 + L_3 - 2M_{13})\right] I_1 - j\omega M_{24} \frac{n_1}{n_2} I_1 + V_1$$

$$0 = -(R_2 + j\omega L_4)\frac{n_1}{n_2} I_1 + j\omega M_{24} I_1 + \frac{n_2}{n_1} V_1$$

$$\Rightarrow V_1 = \left[\frac{n_1}{n_2}(R_2 + j\omega L_4) I_1 - j\omega M_{24} I_1\right]\frac{n_1}{n_2}$$

$$V_S = \left[R_1 + j\omega(L_1 + L_2 + L_3 - 2M_{13}) - 2j\omega M_{24}\frac{n_1}{n_2} + \left(\frac{n_1}{n_2}\right)^2 (R_2 + j\omega L_4)\right] I_1$$

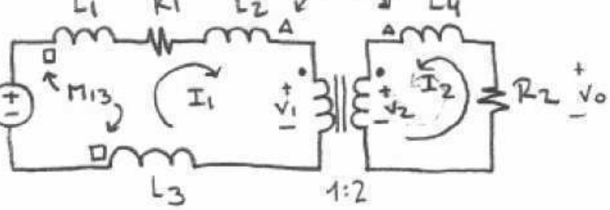
$$Z_{in} = \frac{V_S}{I_1} = R_1 + \left(\frac{n_1}{n_2}\right)^2 R_2 + j\omega\left(L_1 + L_2 + L_3 - 2M_{13} - 2\frac{n_1}{n_2} M_{24} + \left(\frac{n_1}{n_2}\right)^2 L_4\right.$$

b) $V_0 = -I_2 R_2 \qquad -I_2 = \dfrac{n_1}{n_2} I_1 \quad$ e $\quad I_1 = \dfrac{V_S}{Z_{in}}$

$$\Rightarrow V_0 = \frac{n_1}{n_2}\frac{V_S}{Z_{in}} R_2 \qquad V_0(t) = \frac{|V_S|}{|Z_{in}|}\cdot\frac{n_1}{n_2} R_2 \cos(\omega t + \angle V_S - \angle Z_{in})$$

P2

a



$$\begin{cases} v_{L_1} = j6I_1 - j2I_1 = j4I_1 \\ v_{L_2} = j12I_1 + j2I_2 \\ v_{L_3} = j4I_1 - j2I_1 = j2I_1 \\ v_{L_4} = j4I_2 + j2I_1 \end{cases}$$

$$(20\angle 60^\circ) = v_{L_1} + 4I_1 + v_{L_2} + v_{L_3} + V_1$$

$$V_2 = -v_{L_4} - 6I_2$$

$$\frac{V_1}{V_2} = \frac{1}{2} \Rightarrow \boxed{V_1 = \frac{V_2}{2}} \ (1) \qquad \frac{I_1}{I_2} = \frac{-2}{1} \Rightarrow \boxed{I_2 = -\frac{I_1}{2}} \ (2)$$

$$\Rightarrow (20\angle 60^\circ) = j4I_1 + 4I_1 + j12I_1 + j2I_2 + j2I_1 + \overset{(1)}{\frac{V_2}{2}} \quad (3)$$

$V_2 = -j4I_2 - j2I_1 - 6I_2$ en (3):

$$(20\angle 60^\circ) = I_1\left(j(4 + 12 + 2)\right) + 4I_1 + j2I_2 - j2I_2 - jI_1 - 3I_2$$

Reordenando y aplicando (2):

$$\Rightarrow I_1 = \frac{(20\angle 60^\circ)}{(11/2 + j17)} \Rightarrow \boxed{I_1 = 1{,}095 - j0{,}234}$$

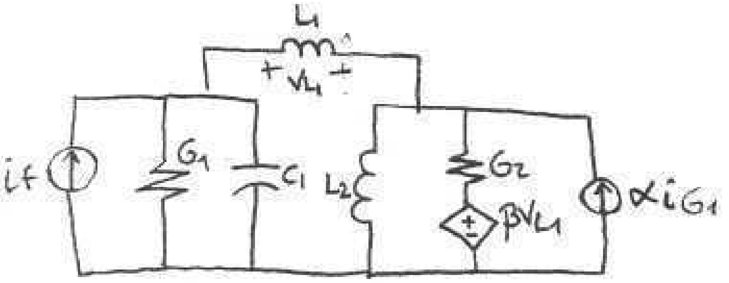
$$Z_{in} = \frac{V_s}{I_1} = \frac{(20\angle 60^\circ)}{(1{,}095 - j0{,}234)} \Rightarrow \boxed{Z_{in} = 5{,}5 + j17}$$

b

$$V_{R_2} = -V_o = R_2 \cdot I_2 = 6 \cdot -\frac{(1{,}095 - j0{,}234)}{2}$$

$$\Rightarrow \boxed{\begin{aligned} V_o &= 3{,}285 - j0{,}702 \\ &= (3{,}359 \angle -12{,}06^\circ) \end{aligned}} \Rightarrow \boxed{V_o(t) = 3{,}359\cos(2t - 12{,}06^\circ)}$$

P3 Método matricial de Nodos, incluyendo las c.i. en los elementos.
(dominio de Laplace)

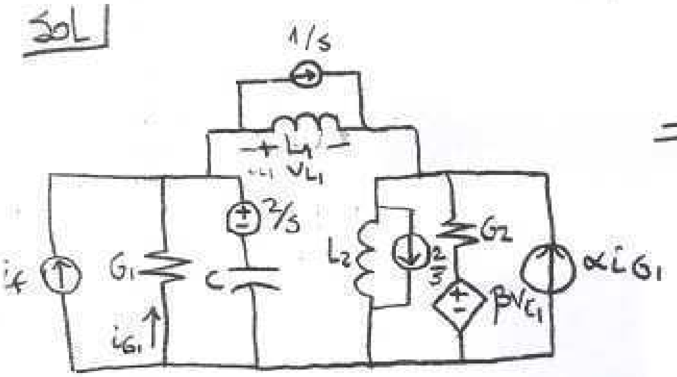


$V_C(0) = 2\ [V]$

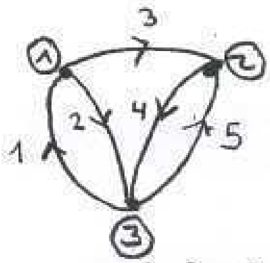
$i_{L_1}(0) = 1\ [A]$

$i_{L_2}(0) = 2\ [A]$

Sol



$\Rightarrow$



$$\Rightarrow A = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline ① & -1 & 1 & 1 & 0 & 0 \\ ② & 0 & 0 & -1 & 1 & -1 \\ ③ & 1 & -1 & 0 & -1 & 1 \end{array}$$

← elimino esta fila.

$$\Rightarrow Y_n \cdot \vec{e} = \vec{I}_s$$

$$\Rightarrow \left[A \cdot Y_B \cdot A^t\right] \vec{e} = A \cdot Y_B \vec{V}_s - A \vec{J}_s$$

← $A \cdot Y_B \cdot A^t$

$$\Rightarrow \underbrace{\begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} G_1 & 0 & 0 & 0 & 0 \\ 0 & sC & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{sL_1} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{sL_2} & 0 \\ 0 & 0 & 0 & 0 & G_2 \end{bmatrix}} \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$$

$$\underbrace{\underbrace{\begin{bmatrix} -G_1 & sC & \frac{1}{sL_1} & 0 & 0 \\ 0 & 0 & \frac{-1}{sL_1} & \frac{1}{sL_2} & -G_2 \end{bmatrix}}_{A \cdot Y_B} \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}}$$

← A^t

$$\begin{bmatrix} G_1 + sC + \frac{1}{sL_1} & \frac{-1}{sL_1} \\ \frac{-1}{sL_1} & \frac{1}{sL_1} + \frac{1}{sL_2} + G_2 \end{bmatrix} = Y_n$$

$\underline{A \cdot Y_B \cdot \vec{V_s}}$:

$$\underbrace{\begin{bmatrix} -G_1 & sc & \frac{1}{sL_1} & 0 & 0 \\ 0 & 0 & \frac{-1}{sL_1} & \frac{1}{sL_2} & -G_2 \end{bmatrix}}_{A \cdot Y_B} \overset{\swarrow \vec{V_s}}{\begin{bmatrix} 0 \\ 2/s \\ 0 \\ 0 \\ -\beta V_{L_1} \end{bmatrix}} = \begin{bmatrix} 2c \\ \beta G_2 V_{L_1} \end{bmatrix} = A \cdot Y_B \cdot \vec{V_s}$$

$$\underline{A \cdot \vec{J_s}} = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_f \\ 0 \\ 1/s \\ 2/s \\ \alpha i_{G_1} \end{bmatrix} = \begin{bmatrix} -i_f + 1/s \\ \frac{-1}{s} + \frac{2}{s} - \alpha i_{G_1} \end{bmatrix}$$

$$\Rightarrow A \cdot Y_B \cdot \vec{V_s} - A \vec{J_s} = \begin{bmatrix} 2c \\ \beta G_2 V_{L_1} \end{bmatrix} - \begin{bmatrix} -i_f + \frac{1}{s} \\ \frac{1}{s} - \alpha i_{G_1} \end{bmatrix} = \begin{bmatrix} 2c + i_f - \frac{1}{s} \\ \beta G_2 V_{L_1} - \frac{1}{s} + \alpha i_{G_1} \end{bmatrix}$$

$$= \begin{bmatrix} 2c + i_f - \frac{1}{s} \\ \beta G_2 (e_1 - e_2) - \frac{1}{s} - \alpha e_1 G_1 \end{bmatrix}$$

$\Rightarrow \underline{Y_n \cdot \vec{e} = \vec{I_s}}$:

$$\begin{bmatrix} G_1 + sc + \frac{1}{sL_1} & \frac{-1}{sL_1} \\ \frac{-1}{sL_1} & \frac{1}{sL_1} + \frac{1}{sL_2} + G_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 2c + i_f - \frac{1}{s} \\ \beta G_2 (e_1 - e_2) - \frac{1}{s} - \alpha e_1 G_1 \end{bmatrix}$$ Y despejando:

$$\boxed{\begin{bmatrix} G_1 + sc + \frac{1}{sL_1} & \frac{-1}{sL_1} \\ -\beta G_2 + \alpha G_1 - \frac{1}{sL_1} & \frac{1}{sL_1} + \frac{1}{sL_2} + G_2 + \beta G_2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} 2c + i_f - \frac{1}{s} \\ \frac{-1}{s} \end{bmatrix}}$$