$$A(r) = \frac{\varepsilon_0}{\rho}\mu_0 \vec{M} \times \underbrace{\frac{1}{4\pi\varepsilon_0}\int_V \rho' \frac{\vec{r}-\vec{r}'}{\|\vec{r}-\vec{r}'\|^3}\,dV'}_{\vec{E}(\vec{r})}$$

$$\vec{E} = \begin{cases} \dfrac{\rho}{3\varepsilon_0} r\hat{r} & r \leq R \\ \dfrac{\rho R^3}{3\varepsilon_0}\dfrac{\hat{r}}{r^2} & r \geq R \end{cases}$$

$\boxed{r \leq R}$

$$\vec{A} = \frac{\varepsilon_0}{\rho}\mu_0 M \hat{z} \times \frac{\rho}{3\varepsilon_0} r\hat{r} = \frac{\mu_0 M}{3} r(\hat{z}\times\hat{r})$$

$$\vec{A} = \frac{\mu_0 M r}{3}\sin\theta\,\hat{\phi}$$

$\boxed{r \geq R}$

$$\vec{A} = \frac{\varepsilon_0}{\rho}\mu_0 M \hat{z} \times \frac{\rho R^3}{3\varepsilon_0}\frac{\hat{r}}{r^2} = \frac{\mu_0 M R^3}{3r^2}(\hat{z}\times\hat{r})$$

$$\vec{A} = \frac{\mu_0 M R^3}{3}\frac{\sin\theta}{r^2}\hat{\phi}$$

$$\oint_{\partial S} \vec{B} \cdot d\vec{s} = \mu_0 I_s = \mu_0 L M$$

$$L\,B(r) = \mu_0 L M$$

$$B(\vec{r}) = \begin{cases} \mu_0 M \hat{z} & \rho < R \\ 0 & \rho > 0 \end{cases}$$

[pues en cte en el exterior y en ∞ se hace cero]

$$\nabla \times A = \frac{1}{\rho} \frac{d(\rho A)}{d\rho} \hat{z}$$

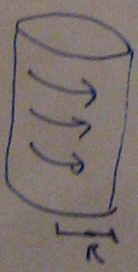
$$\frac{\partial(\rho A)}{\partial \rho} = \mu_0 M \rho$$

$$\rho A = \frac{1}{2} \mu_0 M \rho^2 + \cancel{C}^{\,0}$$

$$A = \frac{1}{2} \mu_0 M \rho + \cancel{\frac{C}{\rho}}^{\,0}$$

$$\vec{A} = \begin{cases} \frac{\mu_0 M}{2} \rho & \rho \leq R \\ \frac{\mu_0 M}{2} R \hat{\phi} & \rho \geq R \end{cases}$$

P2



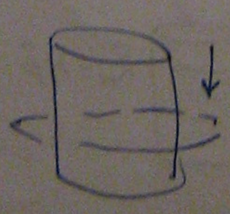
$\vec{M} = M\hat{\phi}$ Encontrar $\vec{A}, \vec{B}, \vec{J}_M, \vec{k}_M$

$$\vec{J}_M = \nabla \times \vec{M} = \frac{1}{\rho} \frac{d(\rho \vec{M})}{\partial \rho} \hat{z} = \frac{M}{\rho} \hat{z}$$

$$\vec{k}_M = M\, \phi \wedge \rho = -M \hat{z}$$

$$B = B_1 + B_2$$

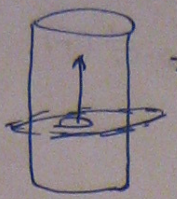
$\vec{B}_2 =$



$$\oint B \cdot d\vec{r} = \mu_0 I_s$$

$$2\pi \rho\, B(\rho) = 2\pi R \mu_0 k_M = -2\pi R M \mu_0$$

$$\vec{B}_2 = \begin{cases} \vec{0} & \rho < R \\ -\mu_0 M \frac{R}{\rho} \hat{\phi} & \rho > R \end{cases}$$



$\vec{J} = \frac{M}{\rho}\hat{z}$

$$\oint_{\partial S} \vec{B}\cdot d\vec{r} = \mu_0 \int_S \vec{J}\cdot d\vec{S}$$

$$\underbrace{\oint_{\partial S}}_{2\pi\rho\, B(\rho)}$$

$r = \min\{\rho, R\}$

$$\mu_0 \int_S \vec{J}\cdot d\vec{S} = \mu_0 \int_0^r \int_0^{2\pi} \frac{M}{R}\hat{z}\ \rho\, d\rho\, d\phi$$

$$= \mu_0\, r\, 2\pi M$$

$$\Longrightarrow B(r) = \mu_0 M \frac{r}{\rho}$$

$$\vec{B}_1(\vec{r}) = \begin{cases} \mu_0 M \hat{\phi} & \rho < R \\ \mu_0 M \frac{R}{\rho} & \end{cases}$$

$\vec{B}(r) = \vec{B}_1 + \vec{B}_2$

$$\vec{B} = \begin{cases} \mu_0 M \hat{\phi} & \rho < R \\ 0 & \rho > R \end{cases}$$

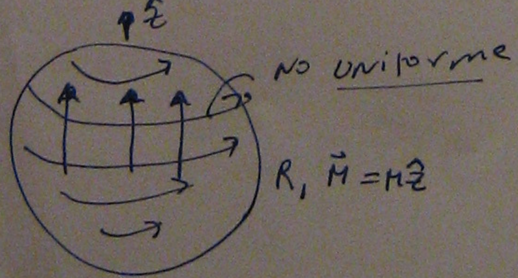
$$A(\vec{r}) = A(\rho)\hat{z} \Rightarrow \nabla\times\vec{A} = -\frac{\partial A}{\partial \rho}\hat{\phi} = \mu_0 M$$

$$A = -\mu_0 M\rho + A_0$$

para que en el exterior sea 0 y el potencial continuo $A_0 = \mu_0 M R$.
Así defino, A_0.

$$\vec{A} = \begin{cases} \mu_0 M (R-\rho)\hat{z} & \rho \le R \\ \vec{0} & \rho \ge R. \end{cases}$$

(P3)



$\vec{J}_M = \nabla\times\vec{M} = 0$

$\vec{K}_M = M\hat{z}\times\hat{r} = M\sin\theta\,\hat{\phi}$

¿ $\underline{\text{dm}}$ $\vec{A}$, $\vec{B}$, $\vec{J}_M$, $\vec{K}_M$

$$A(r) = \frac{\mu_0}{4\pi}\int_V \vec{M}\times\frac{r-r'}{\|r-r'\|^3}$$

$$A(r) = \frac{\mu_0}{4\pi}\vec{M}\times\int_V \frac{r-r'}{\|r-r'\|}dV'$$

$$= \frac{\varepsilon_0}{\rho}\mu_0\vec{M}\times\underbrace{\boxed{\frac{1}{4\pi\varepsilon_0}\int_V \frac{(r-r')}{\|r-r'\|}\rho\, dV'}}_{\vec{E}}$$

$I \longrightarrow \vec{m} \longrightarrow \vec{A} = \frac{\mu_0}{4\pi} \vec{m} \times \frac{\vec{r}}{r^3}$

$$\varphi^{*} = \frac{1}{4\pi} \vec{m} \cdot \frac{\vec{r}}{r^3}$$

$$\vec{M} = \frac{d\vec{m}}{dV} \rightarrow \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint_{\Omega} \vec{M}(\vec{r}\,') \times \frac{r-r'}{\|r-r'\|^3} dv'$$

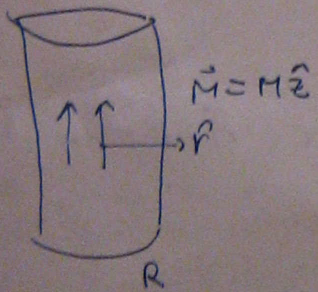
$$\oiint_{\partial\Omega} \vec{f} \cdot d\vec{s} = \iiint_{\Omega} \nabla \cdot \vec{f} \, dv'$$

$$\oiint_{\partial\Omega} \vec{f} \times d\vec{s} = \iiint_{\Omega} - \nabla \times \vec{f} \, dv'$$

$$A(r) = \frac{\mu_0}{4\pi} \iiint_{\Omega} \frac{J_M(r')}{\|r-r'\|} dv' + \frac{\mu_0}{4\pi} \iint_{\partial\Omega} \frac{\vec{K}_M(r')\, ds'}{\|r-r'\|}$$

$$\vec{J}_M = \nabla \times \vec{M}$$
$$\vec{K}_M = \vec{M} \times \hat{n}$$

P1



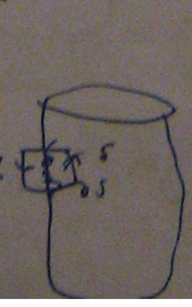
Econtrar $\vec{A}$, $\vec{B}$, $\vec{J}_M$, $\vec{K}_M$

$$\vec{J}_M = \nabla \times \vec{M} = 0$$

$$\vec{K}_M = M\hat{z} \times \hat{r} = M\hat{\phi}$$

Por ley de Ampere

$$\oint_{\partial S} \vec{B} \cdot d\vec{r} = \mu_0 I_s$$



Problem 5.34

(a) $\mathbf{m} = I\mathbf{a} = \boxed{I\pi R^2\,\hat{\mathbf{z}}.}$

(b) $\mathbf{B} \approx \boxed{\dfrac{\mu_0}{4\pi}\dfrac{I\pi R^2}{r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right).}$

(c) On the z axis, $\theta = 0$, $r = z$, $\hat{\mathbf{r}} = \hat{\mathbf{z}}$ (for $z > 0$), so $\boxed{\mathbf{B} \approx \dfrac{\mu_0 I R^2}{2z^3}\,\hat{\mathbf{z}}}$ (for $z < 0$, $\theta = \pi$, $\hat{\mathbf{r}} = -\hat{\mathbf{z}}$, so the field is the same, with $|z|^3$ in place of z^3). The exact answer (Eq. 5.38) reduces (for $z \gg R$) to $B \approx \mu_0 I R^2/2|z|^3$, so they agree.

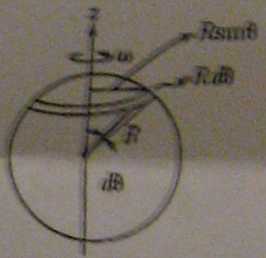
Problem 5.35

For a ring, $m = I\pi r^2$. Here $I \to \sigma v\,dr = \sigma\omega r\,dr$, so $m = \int_0^R \pi r^2 \sigma\omega r\,dr = \boxed{\pi\sigma\omega R^4/4.}$

Problem 5.36

The total charge on the shaded ring is $dq = \sigma(2\pi R\sin\theta)R\,d\theta$. The time for one revolution is $dt = 2\pi/\omega$. So the current in the ring is $I = \dfrac{dq}{dt} = \sigma\omega R^2\sin\theta\,d\theta$. The area of the ring is $\pi(R\sin\theta)^2$, so the magnetic moment of the ring is $dm = (\sigma\omega R^2\sin\theta\,d\theta)\pi R^2\sin^2\theta$, and the total dipole moment of the shell is

$m = \sigma\omega\pi R^4\int_0^\pi \sin^3\theta\,d\theta = (4/3)\sigma\omega\pi R^4$, or $\boxed{\mathbf{m} = \dfrac{4\pi}{3}\sigma\omega R^4\,\hat{\mathbf{z}}.}$

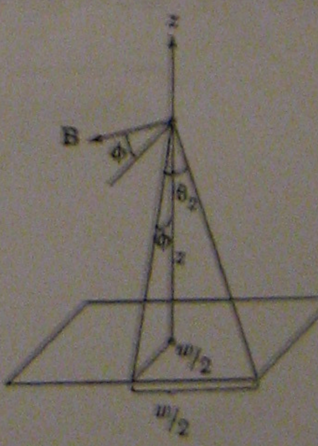


The dipole term in the multipole expansion for $\mathbf{A}$ is therefore $\mathbf{A}_{\text{dip}} = \dfrac{\mu_0}{4\pi}\dfrac{4\pi}{3}\sigma\omega R^4\dfrac{\sin\theta}{r^2}\,\hat{\boldsymbol{\phi}} = \dfrac{\mu_0\sigma\omega R^4}{3}\dfrac{\sin\theta}{r^2}\,\hat{\boldsymbol{\phi}}$, which is also the *exact* potential (Eq. 5.67); evidently a spinning sphere produces a perfect dipole field, with no higher multipole contributions.

Problem 5.37

The field of one side is given by Eq. 5.35, with $s \to \sqrt{z^2 + (w/2)^2}$ and $\sin\theta_2 = -\sin\theta_1 = \dfrac{(w/2)}{\sqrt{z^2 + w^2/2}}$;

$B = \dfrac{\mu_0 I}{4\pi}\dfrac{w}{\sqrt{z^2 + (w^2/4)}\sqrt{z^2 + (w^2/2)}}$. To pick off the vertical component, multiply by $\sin\phi = \dfrac{(w/2)}{\sqrt{z^2 + (w/2)^2}}$; for all four sides, multiply by 4: $\boxed{\mathbf{B} = \dfrac{\mu_0 I}{2\pi}\dfrac{w^2}{(z^2 + w^2/4)\sqrt{z^2 + w^2/2}}\,\hat{\mathbf{z}}.}$ For $z \gg w$, $\mathbf{B} \approx \dfrac{\mu_0 I w^2}{2\pi z^3}\,\hat{\mathbf{z}}$. The field of a dipole $\boxed{m = Iw^2,}$ for points on the z axis (Eq. 5.86, with $r \to z$, $\hat{\mathbf{r}} \to \hat{\mathbf{z}}$, $\theta = 0$) is $\mathbf{B} = \dfrac{\mu_0}{2\pi}\dfrac{m}{z^3}\,\hat{\mathbf{z}}.$ ✓



Problem 5.38

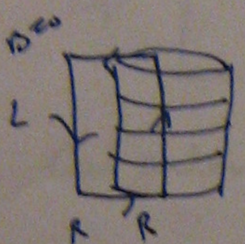
The mobile charges *do* pull in toward the axis, but the resulting concentration of (negative) charge sets up an *electric* field that repels away further accumulation. Equilibrium is reached when the electric repulsion on a mobile charge q balances the magnetic attraction: $\mathbf{F} = q[\mathbf{E} + (\mathbf{v}\times\mathbf{B})] = 0 \Rightarrow \mathbf{E} = -(\mathbf{v}\times\mathbf{B})$. Say the current

^ forma integral queda:

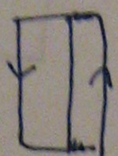
$$\oint_S \nabla \wedge B \, d\vec{s} = \oint_{\partial S} B \, dl = \int_S \mu_0 J(r) \, d\vec{S}$$
$$= I_S \mu_0$$

$$\Rightarrow \boxed{\oint_{\partial S} \vec{B} \cdot d\vec{l} = I_S \mu_0}$$

Ejemplo: Solenoide otravez



$\uparrow B = B(z)\hat{z}$



$$\oint_\gamma \vec{B} \cdot d\vec{l} = \mu_0 \int_S \vec{J} \cdot d\vec{S}$$

S1

$$B \cdot \cancel{x} = \mu_0 I \cdot \frac{N}{L} \cancel{x}$$

$$\vec{B} = \mu_0 I n \hat{z}$$

Vector Potencial:

$$\begin{cases} \nabla \cdot \vec{B} = 0 \\ \nabla \wedge \vec{B} = \mu_0 J \end{cases}$$

Buscamos una expresión talque

$\nabla \cdot (\quad) = 0$ siempre.

esta es $\nabla \cdot (\nabla \wedge \vec{A})$

$\vec{A}$: potencial vectorial y cumple que

$$\boxed{\nabla \wedge \vec{A} = \vec{B}}$$

A no esta determinado univocamente

$A = A' + \nabla \varphi$ donde A' tb es solución válida

dandonos una libertad de escoger A

(Libertad de Gauge)

$\nabla \wedge A = \nabla \wedge (A' + \nabla \varphi)$

$\nabla \wedge A = \nabla \wedge A'$

$\nabla \wedge \vec{B} = \mu_0 \vec{J}$

$\nabla \wedge (\nabla \wedge \vec{A}) = \mu_0 \vec{J}$

$\nabla \cdot (\nabla \vec{A}) = \nabla^2 \vec{A} = \mu_0 \vec{J}$

Escogemos la libertad de Gauge para que $\nabla \vec{A} = 0$ (Gauge de Coulomb)

$$\Rightarrow \boxed{\nabla^2 \vec{A} = -\mu_0 \vec{J}}$$

Calculemos $\vec{A} = \vec{A}(\vec{J})$

$$B(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \vec{J} \wedge \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} dV'$$

$$= \frac{\mu_0}{4\pi} \int_V \vec{J} \wedge \nabla\left(\frac{-1}{|r-r'|}\right) dV'$$

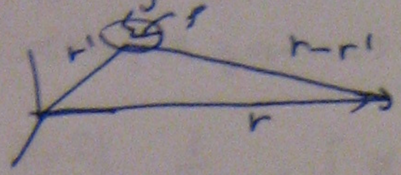
$$\nabla \wedge (\varphi F) = \varphi \nabla \wedge F + \nabla \varphi \wedge F = \varphi \nabla \wedge F - F \wedge \nabla \varphi$$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \left\{ \int \nabla \wedge \left(\frac{J}{|r-r'|}\right) dV' + \int \frac{-1}{|r-r'|} \underbrace{\cancel{\nabla \wedge J}}_{0} \, dV' \right\}$$

$$B = \nabla \wedge \frac{\mu_0}{4\pi} \int_V \frac{J(r')}{|r-r'|} dV'$$

$$\Rightarrow \boxed{\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(r') dV'}{|r-r'|}}$$

omento Magnético . Dipolo Magnético



$$J\,dv = I\,d\vec{r}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\oint \frac{I\,d\vec{r}\,'}{|r-r'|} \qquad |r-r'| >>> \rho$$

entonces.

$$|r-r'| = (r^2 - 2r\cdot r' + r'^2)^{1/2}$$

$$r'^2 << 2r\cdot r' << r^2$$

$$|r-r'| \approx \sqrt{r^2 - 2r\cdot r'} = |r|\sqrt{1 - \frac{2\vec{r}\cdot\vec{r}\,'}{|r|^2}}$$

$$= |r|\left(1 - \frac{\vec{r}\cdot\vec{r}\,'}{|r|^2}\right)$$

$$\frac{1}{|r-r'|} = \frac{1}{|r|}\frac{1}{\left(1 - \frac{\vec{r}\cdot\vec{r}\,'}{|r|^2}\right)}$$

$$\approx \frac{1}{|r|}\left(1 + \frac{\vec{r}\cdot\vec{r}\,'}{|r|^2}\right) + O(\vec{r}\,'^2)$$

$$\vec{A} = \frac{I\mu_0}{4\pi}\left\{\underbrace{\cancel{\oint \frac{d\vec{r}\,'}{|r|}}}_{0} + \oint \frac{d\vec{r}\,'\cdot\vec{r}\cdot\vec{r}\,'}{|r|^3}\right\}$$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\oint \frac{(r\cdot r')}{|r|^3}\cdot dr'$$

Usamos lo siguiente:

1) $d(r'(r\cdot r')) = dr'(r\cdot r') + r'(r\cdot dr')$ r fijo

2) $r\wedge(r'\wedge dr') = r'\cdot(r\cdot dr') - dr'\cdot(r\cdot r')$

estando se tiene

$$d(r'(r\cdot r')) - m(r'\wedge dr') = 2\,dr'(r\cdot r')$$

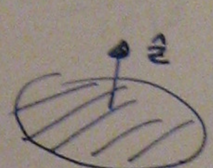
y reemplazamos:

$$\vec{A} = \frac{\mu_0 I}{4\pi}\oint \frac{1}{2|r|^3}\left\{\cancelto{0}{d(r'(r\cdot r'))} - r\wedge(r'\wedge dr')\right\}$$

¿?

$$\vec{A} = \frac{\mu_0 I}{4\pi}\oint \frac{(r'\wedge dr')}{2}\wedge\frac{r}{|r|^3}$$

$$\overrightarrow{Area} = \oint \frac{\vec{r}\wedge d\vec{r}}{2}$$



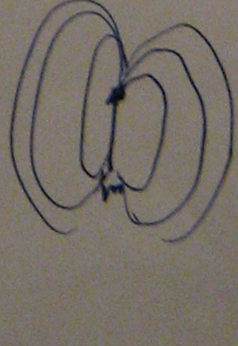
Definimos $\quad \vec{m} = I\ Area\cdot\hat{z}$

$$\boxed{\vec{m} = I\,\frac{1}{2}\oint \vec{r}'\wedge d\vec{r}'}$$

$Area \to 0$

$I \to \infty$

$$\boxed{\vec{m} = \lim_{\substack{I\to 0\\ Area\to 0}} \frac{I}{2}\oint \vec{r}'\wedge d\vec{r}'}$$



$$\boxed{\vec{A} = \frac{\mu_0}{4\pi}\,\frac{(\vec{m}\wedge\vec{r})}{|r|^3}}$$

$$\boxed{B = \frac{\mu_0}{4\pi}\left\{\frac{-\vec{m}}{|r|^3} + \frac{3(\vec{m}\cdot\vec{r})\vec{r}}{|r|^5}\right\}}$$

y el torque

$$\boxed{\tau = 2\vec{m}\wedge\vec{B}}$$

$\nabla \cdot B = 0 \rightarrow \cancel{\nabla \cdot \nabla \wedge A = 0}$

$$\boxed{\nabla \wedge A = B}$$

Libertad de Gauge

$$\boxed{A \longrightarrow A + \nabla\varphi}$$

$\nabla \vec{A} = 0$ (Gauge de Coulomb)

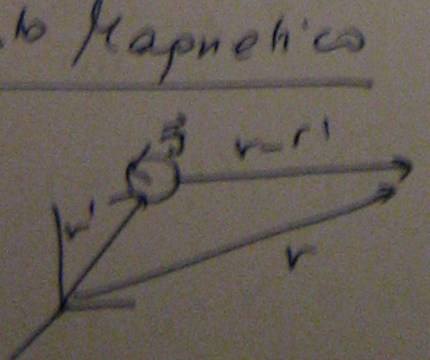
$$\nabla^2 A = -\mu_0 \vec{J}$$

¿Cuánto es $A(J)$?

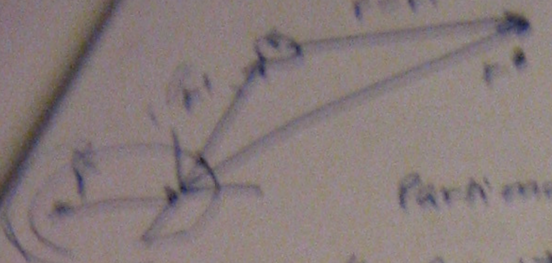
$$\boxed{A(r) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}}{|r-r'|} dV}$$

$$\boxed{A(r) = \frac{\mu_0}{4\pi} \oint \frac{I\, d\vec{r}'}{|r-r'|}}$$

Momento Magnético (Dipolo



Dipolo Magnético y Momento magnético m.

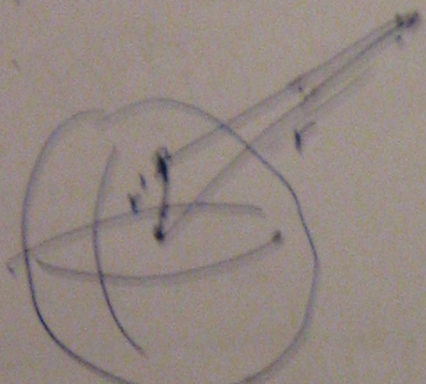


Partimos de:

$$A(r) = \frac{\mu_0}{4\pi} \oint \frac{I\, d\vec{r}'}{|r-r'|}$$

Suponemos $r \gg r'$ y en particular, $|r-r'| \gg \rho$

donde ρ es el radio de la espira. i.e. la espira es muy pequeña comparada con las distancias en estudio.



$r \gg r'$

$$\frac{1}{|r-r'|} = \frac{1}{((r-r')^2)^{\frac{1}{2}}} = (r^2 - 2rr' + r'^2)^{-\frac{1}{2}}$$

$$= |r|^{-1}\Big(\underbrace{1 - \frac{2rr'}{|r|^2} + \frac{r'^2}{|r|^2}}_{=\delta}$$

$$|r|^{-1}\Big\{1 + \frac{1}{2}\frac{2r\cdot r'}{|r|^2} - \underbrace{\frac{1}{2}\frac{r'^2}{|r|^2}}$$

$$\frac{1}{|r-r'|} \approx \frac{1}{|r|}\Big\{1 + \frac{r\cdot r'}{|r|^2}\Big\}$$

$$A(r) = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{r}'}{|r-r'|}$$

$$A(r) = \frac{\mu_0 I}{4\pi}\Big\{\frac{1}{|r|}\cancel{\oint d\vec{r}'}^{\,0} + \frac{1}{|r|^3}\oint \frac{(r\cdot r')\,d\vec{r}'}{1}\Big\}$$

$$A(r) = \frac{\mu_0 I}{4\pi r^3}\oint (r\cdot r')\cdot d\vec{r}'$$

1) $d(r'(r\cdot r')) = dr'(r\cdot r') + d(r\cdot r')\cdot r'$.

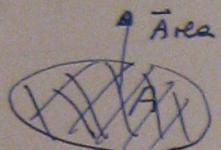
2) $[r' \wedge (dr')] \wedge r = dr'\cdot(r\cdot r') - r$

$$\boxed{\vec{A} = \frac{\mu_0 I}{4\pi}\underbrace{\left[\oint \frac{(r' \wedge dr')}{2}\right]}_{V} \wedge \frac{\vec{r}}{|r|^3}}$$ Pot. dipolo

Area dirigida por vector
mano derecha.

$\frac{1}{2}\, r \wedge dr$ 

Area $= \oint \frac{r \wedge dr}{2}$



$= A\,\hat{z}$

Definimos

$$\boxed{\vec{m} = I \cdot \vec{A}rea} \quad = I A \hat{z}$$

$$\boxed{\vec{m} = \frac{I}{2}\oint r' \wedge dr'}$$ para una espira pequeña.

!! $$\boxed{\vec{A} = \frac{\mu_0}{4\pi}\left(\frac{\vec{m} \wedge \vec{r}}{|r|^3}\right)}$$

Ahora si cambio el origen

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi}\left(\frac{\vec{m} \wedge (r - r')}{|r - r'|^3}\right)}$$

Calculemos el campo magnético

$B = \nabla \wedge A$ Usando propiedades del Algebra vectorial
e identidades del ∇.

$$B = \frac{\mu_0}{4\pi}\left\{ + \vec{m}\left(\nabla \cdot \frac{\vec{r}}{|r|^3}\right) - (\vec{m} \cdot \nabla) \cdot \left(\frac{\vec{r}}{|r|^3}\right)\right.$$

$$\nabla\left(\frac{\vec{r}}{|r|^3}\right) = 4\pi\,\delta(r - r'_0)$$

mostraremos finalmente:

$$B = \frac{\mu_0}{4\pi}\left\{ \vec{m} \cdot 4\pi\,\delta(r) + 3(\vec{m}\cdot\vec{r})\frac{\vec{r}}{r^5} - \frac{\vec{m}}{r^3} \right\}$$

Si examinamos el campo fuera del dipolo (además no guarda sentido)

$$\boxed{B = \frac{\mu_0}{4\pi}\left\{ \frac{3(\vec{m}\cdot\vec{r})\,\vec{r}}{|r|^5} - \frac{\vec{m}}{r^3} \right\}}$$ Dipolo

$$\vec{m} = \frac{I}{2}\oint r' \wedge dr' = I A \hat{z}$$

$\mathcal{E}(c \cdot m)$

- Torque sobre la espira (c.m) B = I $F = I\oint d\vec{r} \wedge B$

$$d\tau = \vec{r} \wedge d\vec{F} = r' \wedge (I\,dr' \wedge B_{ext})$$

$$d\tau = \underbrace{I(r' \wedge dr')}_{*} \wedge B$$

$$\boxed{\tau = 2\vec{m} \wedge B}$$

- Potencial Escalar magnético

Si $\underbrace{J = 0 \Rightarrow \nabla \wedge B = 0}$ $\Rightarrow$ $\boxed{B = -\mu_0 \nabla \varphi^*}$ → Materiales magnéticos y circuitos que contienen J.

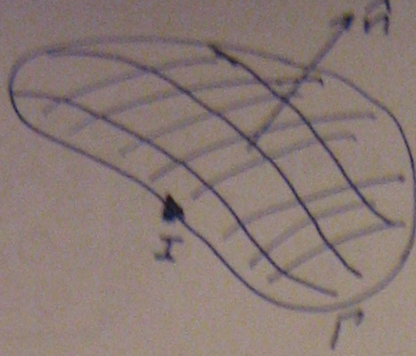
En el caso del dipolo

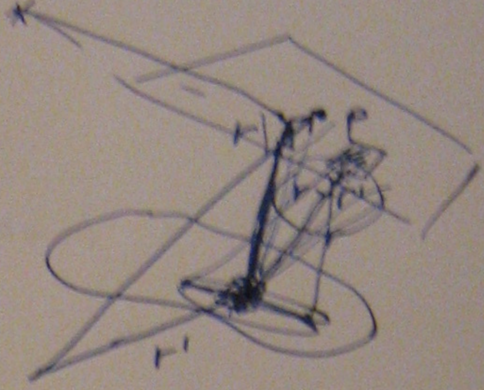
$$B = (-\nabla\varphi^*)\cdot\mu_0$$

Luego: $\boxed{\varphi^* = \frac{1}{4\pi}\frac{m \cdot \vec{r}}{|r|^3}}$ $\varphi^* = \frac{1}{4\pi}\frac{m(r-r')}{|r-r'|^3}$

$$\boxed{d\varphi^* = \frac{1}{4\pi}\,d\vec{m} \cdot \frac{(r-r')}{|r-r'|^3}}$$

en una espira finita





$\vec{m} = I A \hat{z} = I A \hat{n}$

$$\boxed{d\vec{m} = I\, dS \cdot \hat{n} = I\, d\vec{S}}$$

$$\boxed{\varphi^* = \frac{1}{4\pi}\int_S I\, d\vec{s}\,' \cdot \frac{(r-r')}{|r-r'|^3}}$$

$\boxed{\Gamma = \partial S}$

vector desde dS hasta P

Usando como ρ el vector que va desde P al dA.

$\vec{\rho} = -(r-r') = r'-r$

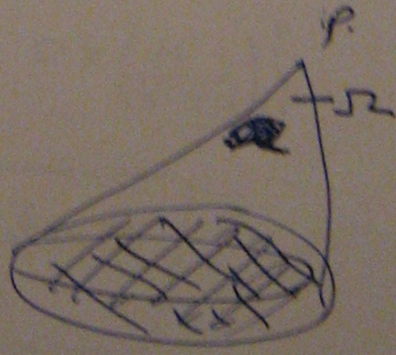
$$\varphi^* = -\frac{1}{4\pi}\int_S I\, d\vec{s}\,\frac{\vec{\rho}}{|\rho|^3}$$

y si ρ no parte del origen sino de una posición r

$r+\rho = r'$

y

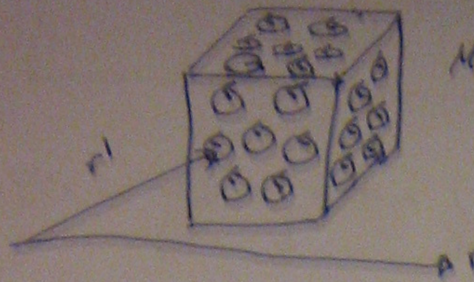
$$\varphi^* = -\frac{I}{4\pi}\int_S d\vec{s}\,'\,\frac{(\vec{\rho}+\vec{r})}{|\vec{\rho}+\vec{r}|^3}$$



Pero

$(\vec{r}-\vec{r}\,')\cdot d\vec{S} =$

$$\boxed{\varphi^* = -\frac{I\Omega}{4\pi}}$$



Materia conformada por espiras.

$\vec{m} = I \cdot A\, \hat{n}$

$d\vec{m} = I\, d\vec{S}$

Magnetización: $\vec{M}$

$$\boxed{\vec{M} = \lim_{v \to 0} \frac{1}{V} \sum_i \vec{m}_i}$$ Magnetización

$$\boxed{d\vec{m} = \vec{M} \cdot dv}$$

Definamos $\boxed{j_m = \vec{M} \times \vec{n}}$ ¿densidad? corriente superficial

$\boxed{J_M = \nabla \times \vec{M}}$ volumétrica

$$A(r) = \frac{\mu_0}{4\pi} \int d\vec{m} \times \frac{(r-r')}{|r-r'|^3} = \frac{\mu_0}{4\pi} \int_V dv' \cdot \vec{M}(r') \times \underbrace{\frac{r-r'}{|r-r'|^3}}_{\nabla'\left(\frac{1}{|r-r'|}\right)}$$

$$\cancel{A(r) = \frac{\mu_0}{4\pi} \left\{ \int dv' \cdot \left(\nabla' \frac{M}{|r-r'|} \right) + \frac{\nabla}{|r-r'|} \right.}$$

$$A(r) = \frac{\mu_0}{4\pi} \left\{ \int dv' \left\{ \frac{\nabla' \times M}{|r-r'|} - \nabla \times \left(\frac{M}{|r-r'|} \right) \right\} \right\}$$

y $$\boxed{\int_V \nabla \wedge \vec{M}\, dv = \oint_{\partial V} d\vec{S} \wedge \vec{M}}$$

$$A(r) = \frac{\mu_0}{4\pi} \left\{ \int_V \frac{\nabla' \wedge \vec{M}}{|r-r'|} - \oint_{S=\partial V} \frac{\vec{n} \wedge \vec{M}}{|r-r'|} \right\}$$

$$\boxed{A(r) = \frac{\mu_0}{4\pi} \left\{ \int \frac{\nabla' \wedge \vec{M}}{|r-r'|} dv + \oint_{S=\partial V} \frac{\vec{M} \wedge \vec{n}}{|r-r'|} \right\}}$$

$$\boxed{A(r) = \frac{\mu_0}{4\pi}\int_S \frac{\vec{j}_m\, ds'}{|r-r'|} + \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}_m \cdot dV}{|r-r'|}}$$

Debido "solo" a la magnetización

$$B(r) = \frac{\mu_0 \nabla}{4\pi} \wedge \int d\vec{m} \wedge \underbrace{\frac{(r-r')}{|r-r'|^3}}_{\nabla'\left(\frac{1}{|r-r'|}\right) = -\nabla\left(\frac{1}{|r-r'|}\right)}$$

tb $\nabla \cdot \left(\frac{r-r'}{|r-r'|^3}\right) = 4\pi\delta($

luego usando identidades vectoriales se llega a que:

$$B(r) = M(r)\mu_0 - \frac{\mu_0}{4\pi}\int dV' \, (M\cdot\nabla)\frac{r-r'}{|r-r'|^3}$$

$$B(r) = \mu_0 M(r) - \frac{\mu_0}{4\pi}\, \nabla \cdot \underbrace{\int \frac{M(r-r')\, dV'}{|r-r'|^3}}_{\varphi^*}$$

$$\boxed{B(r) = \mu_0 M(r) - \mu_0 \nabla \varphi^*}$$

Calculemos una expresión útil para φ^*

$$\boxed{\begin{aligned} M\cdot\vec{n} &= \sigma_M \\ -\nabla'\vec{M} &= \rho_M \end{aligned}}$$

se deduce que:

$$\boxed{\varphi^* = \frac{1}{4\pi}\left\{\int_S \frac{\sigma_M\, ds'}{|r-r'|} + \int_V \frac{\rho_M\, dV'}{|r-r'|}\right\}}$$

Pot escalar

$$B(r) = \frac{\mu_0}{4\pi}\left\{\int_V \rho_M \frac{(r-r')}{|r-r'|^3}\, dV'\right\} + \frac{\mu_0}{4\pi}\int_S \sigma_M \frac{(r-r')}{|r-r'|^3}\, ds' + \mu_0 M(r)$$

Fuentes de campo Magnético y $\vec{H}$

$$\boxed{B(r) = \frac{\mu_0}{4\pi}\int_V \frac{J \wedge (r-r')}{|r-r'|^3}\,dv' + \mu_0 M(r) - \mu_0 \nabla\varphi^*}$$

$$\frac{\mu_0}{4\pi}\oint_\Gamma I\,d\vec{r}\wedge\frac{(r-r')}{|r-r'|^3}$$

$$\boxed{J_{TOT} = \underset{\downarrow\; J_{libre}}{J} + \underbrace{J_M + \dot{J}_m}_{M}}$$

$$\boxed{H = \frac{B(r)}{\mu_0} - M(r)}$$

$$\boxed{H = \frac{1}{4\pi}\int_V \frac{J \wedge (r-r')}{|r-r'|^3}\,dv' - \nabla\varphi^*}$$

$$\nabla\wedge H = \nabla\wedge\Big(\underbrace{\frac{B(r)}{\mu_0}}_{J + \underbrace{J_M}_{J_M + \dot{J}_m} - J_M} - M\Big) = J$$

$$\boxed{\nabla\wedge H = J}$$ Ley de Ampere

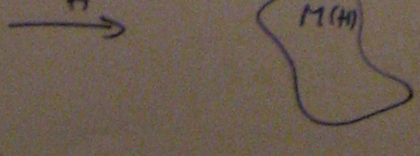
$$\boxed{\nabla\cdot B = 0}$$

Integralmente:

$$\int_S (\nabla\wedge H)\,\vec{n}\cdot dS = \int_S J\cdot d\vec{s}$$

$$\boxed{\oint_{C=\partial S} H\cdot d\vec{l} = I}$$ (verdadero)

Medios lineales e Isótropos



$$\boxed{M = \chi_m H}$$

$\chi_m > 0$ Paramagnético

$\chi_m < 0$ (diamagnético)

Luego

$$H = \frac{B}{\mu_0} - M = \frac{B}{\mu_0} - \chi_m H$$

$$B = \underbrace{\mu_0 (1 + \chi_m)}_{\mu} H$$

$$\boxed{B = \mu H}$$

Inducción Magnético-Eléctrica : ley de Faraday Lenz

$\nabla \cdot D = \rho$ $\quad$ $\nabla \cdot B = 0$ $\quad$ Estática

$\nabla \wedge E = 0$ $\quad$ $\nabla \wedge H = J$

Definimos

$$\varnothing = \int_S \vec{B} \cdot d\vec{s}$$ flujo de campo magnético

$$\boxed{V = -\frac{d\varnothing}{dt}}$$ ley de Faraday

$$\oint_\Gamma E \cdot d\vec{l} = V = -\frac{d\varnothing}{dt} = \mathcal{E}$$

$$\boxed{\oint_{\Gamma = \partial S} E \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{s}}$$ Inducción E-M Caso General.

→ Si S no depende de t !!

$$\boxed{\nabla \wedge E = -\frac{\partial B}{\partial t}}$$ L. Faraday Lenz.

$$\boxed{\nabla \wedge E + \frac{\partial B}{\partial t} = 0}$$