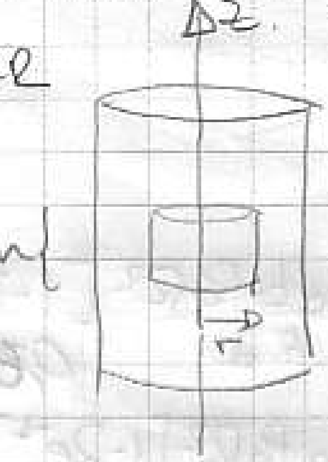
Pauta P2 Control 1 FI2A2-5

a) Usamos Gauss (Suponemos $\vec{E}(\vec{r}) = E(r)\hat{r}$)

$r<R$



$$\oiint \vec{E}\cdot d\vec{S} = Q_{encl}/\varepsilon_0$$

$$E\cdot 2\pi r h = \int_0^h\int_0^{2\pi}\int_0^r \frac{\rho_0}{\varepsilon_0}\, e^{-r/R}\, r\,dr\,d\theta\,dz$$

$$= \frac{\rho_0}{\varepsilon_0}\, 2\pi h \cdot \underbrace{\int_0^r r e^{-r/R}\,dr}_{I}$$

$I = \int_0^r r\cdot e^{-r/R}\,dr$ por integración por partes

$u = r, \quad dv = e^{-r/R}$

$du = 1, \quad v = -R\cdot e^{-r/R}$

$$\Rightarrow I = r\cdot(-Re^{-r/R})\Big|_0^r - \int_0^r 1\cdot(-Re^{-r/R})\,dr$$

$$= -Rr\cdot e^{-r/R} + R\int_0^r e^{-r/R}\,dr$$

$$= -rRe^{-r/R} + R\cdot\left[R - R\cdot e^{-r/R}\right]$$

$$I = R^2 - Re^{-r/R}(R+r)$$

$$\Rightarrow E\cdot \cancel{2\pi} r \cancel{h} = \frac{\rho_0}{\varepsilon_0}\cdot \cancel{2\pi h}\cdot\left[R^2 - R(R+r)e^{-r/R}\right]$$

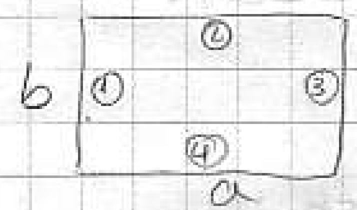
$$\Rightarrow \boxed{\vec{E} = \frac{\rho_0}{\varepsilon_0 r}\left[R^2 - R(R+r)\,e^{-r/R}\right]\hat{r}} \quad 1{,}5 \text{ pts.}$$

$r>R$ $\quad I|_{r=R} = \left[R^2 - R(R+R)\cdot e^{-R/R}\right] = R^2\left[1-2e^{-1}\right]$

$$\Rightarrow E\cdot \cancel{2\pi} r \cancel{h} = \frac{\rho_0}{\varepsilon_0}\cdot \cancel{2\pi h}\cdot R^2\left[1-2e^{-1}\right]$$

$$\boxed{\vec{E} = \frac{\rho_0 R^2}{\varepsilon_0 r}\left[1-2e^{-1}\right]\hat{r}} \quad 1{,}5 \text{ pts.}$$

b) La fuerza se calcula como $\vec{F} = \int dQ \cdot \vec{E}$
En este caso, se integran 4 zonas pues



$$\vec{F} = \int_{①} + \int_{②} + \int_{③} + \int_{④}$$

$\longrightarrow \hat{\imath}$

$$\int_{①} = \int_0^b \lambda_0 dy \cdot \frac{\rho_0 R^2}{\varepsilon_0 d}\left[1-2e^{-1}\right]\hat{\imath} = \frac{\lambda_0 b \rho_0 R^2}{\varepsilon_0 d}\left[1-2e^{-1}\right]\hat{\imath}$$

0,6 pts

$$\int_{②} = \int_d^{a+d} \lambda_0 dx \, \frac{\rho_0 R^2}{\varepsilon_0 x}\left[1-2e^{-1}\right]\hat{\imath} = \frac{\lambda_0 \rho_0 R^2}{\varepsilon_0}\left[1-2e^{-1}\right] \ln\left(\frac{a+d}{d}\right)$$

0,6 pts

$$\int_{③} = \int_0^b \lambda_0 dy \, \frac{\rho_0 R^2}{\varepsilon_0 (a+d)}\left[1-2e^{-1}\right]\hat{\imath} = \frac{\lambda_0 b \rho_0 R^2}{\varepsilon_0 (a+d)}\left[1-2e^{-1}\right]\hat{\imath}.$$

0,6 pts

$$\int_{④} = \int_a^{a+d} \lambda_0 dx \, \frac{\rho_0 R^2}{\varepsilon_0 x}\left[1-2e^{-1}\right]\hat{\imath} = \frac{\lambda_0 \rho_0 R^2}{\varepsilon_0}\left[1-2e^{-1}\right] \ln\left(\frac{a+d}{a}\right)\hat{\imath}$$

0,6 pts

$$\rightarrow \boxed{\vec{F} = \frac{\lambda_0 \rho_0 R^2\left[1-2e^{-1}\right]}{\varepsilon_0}\left(\frac{b}{d} + \frac{b}{a+d} + 2\ln\left(\frac{a+d}{a}\right)\right)\hat{\imath}}$$

0,6 pts