P51 c) Ocupando

$$y_1 = \operatorname{senh} x - \operatorname{Tan}\frac{KL}{2}\cosh x \qquad \frac{l}{2} > x > 0$$

$$y_2 = -\operatorname{senh} x - \operatorname{Tan}\frac{KL}{2}\cosh x \qquad -\frac{l}{2} < x < 0$$

nos damos cuenta que $y_1(x) = y_2(-x)$
por lo tanto la función $y(x)$ es par

$$\int_{-l/2}^{l/2} y_m(x)\, y_n(x)\,(\sigma - m\delta(x))\,dx = 2\int_0^{l/2} y_m\, y_n\,(\sigma - m\delta(x))$$

probando pimero para $n=m$

$$I = 2\int_0^{l/2} \left(\operatorname{senh} x - \operatorname{Tan}\frac{KL}{2}\cosh x\right)^2 (\sigma - m\delta(x))\,dx$$

$$= \left(-\operatorname{tg}^2\frac{KL}{2}\cdot\frac{1}{2} + \frac{\sigma\left(\sec^2\frac{KL}{2}\right)(-kl + \operatorname{sen} kl)}{4K}\right) 2$$

ojo que se usa: $\int_0^{B} \delta(x) = \frac{1}{2}$

Ahora usando: $\sec^2\frac{KL}{2} = 1 + \operatorname{Tg}^2\frac{KL}{2}$

$$\operatorname{sen} KL = \operatorname{sen}\frac{kL}{2}\cos\frac{kL}{2}\cdot 2$$

$$I = 2\left(-tg^2\frac{kL}{2}\cdot\frac{m}{2} + \frac{\sigma}{4k}\left\{-kl\cdot\sec^2\frac{kL}{2} + 2\frac{\operatorname{sen} kl/2}{\cos kl/2}\right\}\right)$$

$$I = 2\left(-tg\frac{kL}{2}\cdot\frac{m}{2} + \frac{\sigma(-l)}{4}\left(\operatorname{Tan}^2\frac{kL}{2}+1\right) + \frac{\sigma}{4k}\cdot 2\operatorname{Tan}\frac{kL}{2}\right)$$

por ultimo usando

$$\operatorname{Tan}\frac{kL}{2} = \cancel{\frac{m}{\sigma l}}\cdot\cancel{\ } \frac{\sigma l}{m}\cdot\frac{2c}{\omega l} = \frac{\sigma\cdot 2c}{m\omega}$$

$$I = 2\left(-\left(\frac{\sigma c 2}{m\omega}\right)^2\frac{m}{2} + \frac{\sigma}{4k}\cdot 2\cdot\left(\frac{\sigma c 2}{m\omega}\right) - \frac{\sigma l}{4}\left(\left(\frac{\sigma c 2}{m\omega}\right)^2 + 1\right)\right)$$

$$I = 2\left(-\frac{\sigma^2 4 c^2}{m\omega^2} + \frac{\sigma^2 c}{km\omega} - \frac{\sigma^3 l c^2}{m^2\omega^2} + -\frac{\sigma l}{4}\right)$$

$I \neq 0$ No da 1 ya que no nos dimos una funcion normalizada

definiendo $y' = \frac{1}{\sqrt{I}}y$ se verifica que si da 1

ahora para $n \neq m$, llamamo $h_m = h$ y $h_n = h$

$$\int_0^{l/2} 2\left(+\operatorname{sen} hx \mp \operatorname{Tan}\frac{kL}{2}\cos hx\right)\left(\operatorname{sen} hx - \operatorname{Tan}\frac{kL}{2}\cdot\cos hx\right)\left(\sigma - m\delta(x)\right)$$

$$= \cancel{\ldots}$$

$$= 2\left(-\operatorname{Tan}\frac{hL}{2}\operatorname{Tan}\frac{kL}{2}\cdot\frac{m}{2} + \frac{\sigma}{(h^2-k^2)}\left(-h\operatorname{Tan}\frac{hL}{2} + k\operatorname{Tan}\frac{hL}{2}\right)\right)$$

Una vez más usando

$$\tan\frac{kl}{2} = \frac{\sigma\, 2c}{m\,\omega} = \frac{\sigma\, 2\,\cancel{k}}{m\,K}$$

$\Longrightarrow$

$$\int_{l/2}^{l/2} y_m(x)\, y_n(x)\, m(x)\, dx = 2\left(-\frac{\sigma^2 4}{m^2 K h}\cdot\frac{m}{2} + \frac{\sigma}{h^2-K^2}\left(\frac{\sigma 2}{mK}h - \frac{\sigma 2}{mh}K\right)\right.$$

$$= 2\left(-\frac{\sigma^2 \cancel{4}2}{m^{\cancel{2}}Kh}\, m + \frac{\sigma^2 2}{(h^2-K^2)}\left(\frac{h^2-K^2}{mKh}\right)\right)$$

$$= 2\left(-\frac{\sigma^2 2\cancel{m}}{mKh} + \frac{\sigma^2 2}{mKh}\right) = 0 \quad \checkmark$$

Se cumple por lo tanto que

$$\int_{-l/2}^{l/2} y_m(x)\, y_n(x)\, m(x)\, dx = 0 \text{ si } n \neq m$$

$$\int_{-l/2}^{l/2} \frac{1}{I}\, y_m^2(x)\, m(x)\, dx = 1$$

$$\Longrightarrow \boxed{\int_{-l/2}^{l/2} y'_m(x)\, y'_n(x)\, m(x)\, dx = \delta_{mn}}$$