ISE 536–Fall06: Linear Programming and Extensions November 29, 2006

Lecture 24: Sample Final Exam

Lecturer: Fernando Ordóñez *Final December 11, 2006*

Final is on Monday, December 11, 2006
From 8:00am to 10:00am
Room KAP 148
Closed Book, no calculators
3 sheets with notes allowed.

1. Consider the following two linear programming problems:

$$
\begin{array}{llcll}
\min & 3x_{13} + 5x_{23} & & \min & y_{13} \\
\text{s.t.} & x_{12} + x_{13} = 4 & & \text{s.t.} & y_{12} + y_{13} - y_{21} = 1 \\
 & x_{12} - x_{23} = 0 & \text{and} & & -y_{12} + y_{21} + y_{23} = 1 \\
 & x_{13} + x_{23} = 4 & & & y_{13} + y_{23} = 2 \\
 & x_{12}, x_{13}, x_{23} \geq 0 & & & y_{12}, y_{13}, y_{21}, y_{23} \geq 0
\end{array}
$$

We want to minimize the total cost of both problems with the additional constraint that $x_{23} = y_{23}$. We solve this problem using Dantzig-Wolfe decomposition. We start the algorithm with the basic feasible solution to the master problems that uses the following extreme points from problem 1:

$$x_1^1 = (x_{12}^1, x_{13}^1, x_{23}^1) = (4, 0, 4) \quad \text{and} \quad x_1^2 = (x_{12}^2, x_{13}^2, x_{23}^2) = (0, 4, 0)$$

and the following extreme point from problem 2:

$$y_2^1 = (y_{12}^1, y_{13}^1, y_{21}^1, y_{23}^1) = (1, 0, 0, 2)\ .$$

(a) What are the numerical values of the variables in the master problem?

(b) Write down the basis for the master problem associated to the current basic feasible solution to the master problem.

(c) Calculate the vector (y, σ_1, σ_2) of simplex multipliers (optimal dual variables).

(d) Solve a subproblem associated with the second problem and use its solution to carry out a simplex iteration of the master problem. Specify the values for the new basic variables in the master problem, as well as the corresponding solution to each of the two problems above.

P1)

Sample Final Solutions

$$\min \; 3x_{13} + 5x_{23} + y_{13}$$

$$x_{23} = y_{23}$$

$x^0 = (x_{12}, x_{13}, x_{23}) \in P_1$

$y = (y_{12}, y_{13}, y_{21}, y_{23}) \in P_2$

$x^1 = (4, \underset{x_{13}}{0}, \underset{x_{23}}{4})$, $x^2 = (0, 4, 0)$ $\qquad y^1 = (1, \underset{y_{12}}{0}, 0, 2)$

$$\min \; \lambda_1^1 (3 \cdot 0 + 5 \cdot 4) + \lambda_1^2 (3 \cdot 4 + 5 \cdot 0) + \lambda_2^1 (1 \cdot 0)$$

$$\lambda_1^1 \cdot 4 + \lambda_1^2 \cdot 0 = \lambda_2^1 \cdot 2 \qquad x_{23} - y_{23} = 0$$

$$\lambda_1^1 + \lambda_1^2 = 1$$

$$\lambda_2^1 = 1$$

$$\lambda_1^i, \lambda_2^1 \geq 0$$

a)

$\boxed{\lambda_2^1 = 1} \Rightarrow 4\lambda_1^1 + \lambda_1^2 \cdot 0 = 2 \Rightarrow \boxed{\begin{matrix}\lambda_1^1 = 1/2 \\ \lambda_1^2 = 1/2\end{matrix}}$

b)

$$\begin{bmatrix} 4 & 0 & -2 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \lambda_1^1 \\ \lambda_1^2 \\ \lambda_2^1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$B = \begin{bmatrix} 4 & 0 & -2 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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c) $y^t = c_B^t B^{-1} \iff y^t B = c_B^t \iff B^t y = c_B$

$c_B = (20, 12, 0)$

$$\begin{bmatrix} 4 & 1 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \begin{pmatrix} y \\ \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} 20 \\ 12 \\ 0 \end{pmatrix} \Rightarrow \boxed{\sigma_1 = 12}$$

$\Rightarrow 4y = 20 - 12 = 8 \Rightarrow \boxed{y = 2}$

$\boxed{\sigma_2 = 4}$

d) $\min \; y_{13} + (2) y_{23} - 4$

$y_{12} + \underline{y_{13}} - y_{21} = 1$

$-y_{12} + y_{21} + \underline{y_{23}} = 1$

$\underline{y_{13}} + \underline{y_{23}} = 2$

$y_{ij} \geq 0$

$y_{13} = 2 \quad y_{23} = 0$

$y_{12} - y_{21} = -1 \qquad y_{12} = y_{21} - 1$

$-y_{12} + y_{21} = 1$

$y_{21} = 1, \; y_{12} = 0$

$y = (0.2 \; 1 \; 0) \qquad d_i = -2 \leq 0$

$\cancel{\begin{matrix} 4 & 1 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{matrix}}$

basic vars are λ_1^1, λ_1^2 and λ_2^2.

$(0, 1, 1)$

	20	12	0	2
0	4	0	−2	0
1	1	1	0	0
1	0	0	1	1

corresp. solution to (P1) is

$x^2 = (0, 4, 0)$

to (P_2) is

$y^2 = (0, 2, 1, 0)$

is not $x_{23} = y_{23} = 0$

0	0	12	10	2
0	1	0	−1/2	0
1	0	1	1/2	0
1	0	0	1	1

−12	0	0	4	2
0	1	0	−1/2	0
1	0	1	1/2	0
1	0	0	1	1

−14	0	0	2	0
0	1	0	−1/2	0
1	0	1	1/2	0
1	0	0	1	1

$\lambda_1^1 = 0$

$\lambda_1^2 = 1$

$\lambda_2^1 = 0$

$\lambda_2^2 = 1$