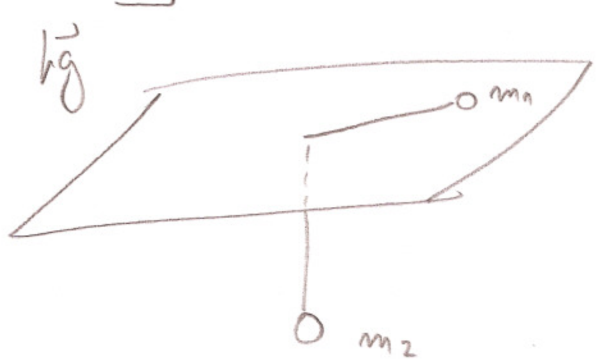
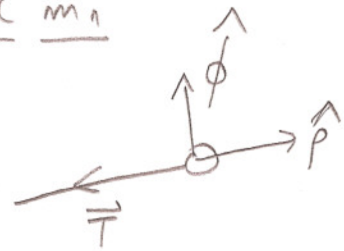
P1)



DCL m1

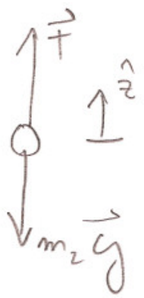


$$\hat{\rho})\quad -T = m_1(\ddot{\rho} - \rho\dot{\phi}^2) \quad (1)$$

$$\hat{\phi})\quad 0 = m_1 a_\phi \Rightarrow \rho^2\dot{\phi} = cte \quad (2)$$

$$\hat{k})\quad -m_1 g + N = 0 \quad (3)$$

DCL m2



$$\hat{z})\quad T - m_2 g = m_2 \ddot{z} \quad (4)$$

$$\ddot{z} = \ddot{\rho}$$

$$(1)+(4) \Rightarrow \quad -m_2 g = (m_1+m_2)\ddot{\rho} - m_1 \rho \dot{\phi}^2$$

$$\rho^2\dot{\phi} = \rho_0 v_0 = h = \frac{l}{m}$$

$$\dot{\phi} = \frac{\rho_0 v_0}{\rho^2} = \frac{h}{\rho^2}$$

$$-m_2 g = (m_1+m_2)\ddot{\rho} - m_1 \frac{h^2}{\rho^3}$$

$$(m_1+m_2)\ddot{\rho} = -\frac{\partial V_{eff}}{\partial \rho} = m_1 \frac{h^2}{\rho^3} - m_2 g$$

$$V_{eff}(\rho) = \frac{m_1 h^2}{2\rho^2} - m_2 g(l-\rho) \leftarrow$$ [calculado a través de Energía]

$$\frac{\partial V_{eff}}{\partial \rho} = 0 \text{ pto eq} \Rightarrow m_1 \frac{h^2}{\rho^3} - m_2 g = 0 \Rightarrow \left(\frac{m_1 h^2}{m_2 g}\right)^{1/3} = \rho_0$$

P2)

$$U = \begin{cases} U_1 & x<0 \\ U_2 & x>0 \end{cases}$$

$$-\frac{dU}{dy} = F_y = 0$$

$$F_y = m\ddot{y} = 0 \Rightarrow v_y = cte \quad 1$$

$$v_y = v_1 \sin\theta_1 = v_2 \sin\theta_2 \quad 1$$

Por conservación de la Energía

$$\frac{1}{2} m v_1^2 + U_1 = \frac{1}{2} m v_2^2 + U_2 \quad 2$$

$$v_2^2 = v_1^2 + \frac{2(U_1-U_2)}{m}$$

$$v_2 = \sqrt{v_1^2 + \frac{2(U_1-U_2)}{m}} \quad 1$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_2}{v_1} = \frac{\sqrt{v_1^2 + \frac{2(U_1-U_2)}{m}}}{v_1} \quad 1$$

P3)

$$U(x) = \frac{1}{2} k \left(\sqrt{x^2+l^2} - l_0 \right)^2$$

$$\frac{dU}{dx} = \frac{k \left(\sqrt{x^2+l^2} - l_0 \right) x}{\sqrt{x^2+l^2}}$$

$$\frac{dU}{dx} = 0 \Rightarrow x\left(\sqrt{x^2+l^2} - l_0\right) = 0$$

$$\Rightarrow x = 0 \quad \vee \quad x^2 = l_0^2 - l^2$$

$$\Rightarrow x = \pm \sqrt{l_0^2 - l^2}$$

es válido si $l_0^2 \geq l^2$

$$\boxed{l_0 \geq l}$$

$$\frac{d^2U}{dx^2} = k \frac{\left(\sqrt{x^2+l^2} - l_0\right)}{\sqrt{x^2+l^2}} + \frac{k l_0 x^2}{(x^2+l^2)^{3/2}}$$

$x=0$)
$$\left.\frac{d^2U}{dx^2}\right|_{x=0} = k\left(\frac{l-l_0}{l}\right) \Rightarrow x=0 \text{ es ESTABLE si } l > l_0$$

$$\Rightarrow \omega^2_{peq}\Big|_{x=0} = \frac{k}{m}\left(1-\frac{l_0}{l}\right)$$

$x^2 = l_0^2 - l^2$)
$$\left.\frac{d^2U}{dx^2}\right|_{x^2=l_0^2-l^2} = k\frac{(l_0^2-l^2)}{l_0^2} \Rightarrow x = \sqrt{l_0^2-l^2} \text{ si } l_0 > l$$

$$\Rightarrow \omega^2_{peq} = \frac{\left.\frac{d^2U}{dx^2}\right|_{x^2=l_0^2-l^2}}{m} = \frac{k}{m}\left(1-\frac{l^2}{l_0^2}\right)$$

$\Rightarrow x=0$ es pto de eq si $l_0 \leq l$, ya que de esta forma no existe otra eq.