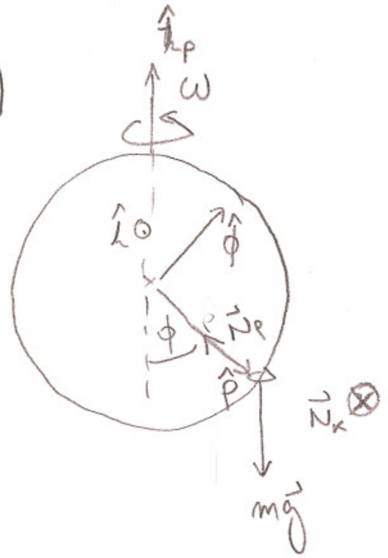
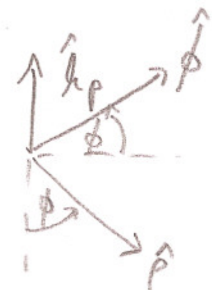
P1)



$$\vec{R}_0 = 0 \qquad \vec{r}\,' = R\hat{\rho}$$
$$\vec{v}_0 = 0 \qquad \vec{v}\,' = R\dot{\phi}\,\hat{\phi}$$
$$\vec{a}_0 = 0 \qquad \vec{a}\,' = -R\dot{\phi}^2\hat{\rho} + R\ddot{\phi}\,\hat{\phi}$$
$$\vec{\omega} = \omega\hat{k}_p$$



$$\hat{k}_p = -\hat{\rho}\cos\phi + \hat{\phi}\sin\phi$$

$$\vec{\omega} = \omega(-\cos\phi\,\hat{\rho} + \sin\phi\,\hat{\phi})$$

$$\vec{\omega}\times\vec{r}\,' = \omega(-\cos\phi\,\hat{\rho} + \sin\phi\,\hat{\phi}) \otimes R\hat{\rho}$$
$$= -\omega R\sin\phi\,\hat{k}$$
$$\vec{\omega}\times\vec{\omega}\times\vec{r}\,' = \omega(-\cos\phi\,\hat{\rho} + \sin\phi\,\hat{\phi}) \otimes (-\omega R\sin\phi\,\hat{k})$$
$$= -\omega^2 R\sin\phi\,(\sin\phi\,\hat{\rho} + \cos\phi\,\hat{\phi})$$
$$2\vec{\omega}\times\vec{v}\,' = 2\omega(-\cos\phi\,\hat{\rho} + \sin\phi\,\hat{\phi}) \otimes R\dot{\phi}\,\hat{\phi}$$
$$= -2R\omega\dot{\phi}\cos\phi\,\hat{k}$$

$$\vec{F} = -N_p\hat{\rho} + mg(\cos\phi\,\hat{\rho} - \sin\phi\,\hat{\phi}) - N_K\hat{k}$$
$$\vec{a} = (-R\dot{\phi}^2 - \omega^2 R\sin^2\phi)\hat{\rho} + (R\ddot{\phi} - \omega^2 R\sin\phi\cos\phi)\hat{\phi} +$$
$$- 2R\omega\dot{\phi}\cos\phi\,\hat{k}$$

a) ① $\hat{\rho}$) $-N_p + mg\cos\phi = -mR(\dot{\phi}^2 + \omega^2\sin^2\phi)$

② $\hat{\phi}$) $-mg\sin\phi = mR(\ddot{\phi} - \omega^2\sin\phi\cos\phi)$

③ $\hat{k}$) $-N_K = -2mR\omega\dot{\phi}\cos\phi$

b) EN ② $\ddot{\phi} = 0$

$$-mg\sin\phi = -mR\omega^2\sin\phi\cos\phi$$

$\Rightarrow \sin\phi = 0 \quad \Rightarrow \quad \phi_1 = 0$

$\phi_2 = \pi$

$\Rightarrow \sin\phi \neq 0 \quad \Rightarrow \cos\phi_3 = \dfrac{g}{R\omega^2} \quad \Rightarrow \quad \dfrac{g}{R\omega^2} \leq 1 \quad \Rightarrow \quad \omega^2 \geq \dfrac{g}{R}$

c) $$\ddot{\phi} = \omega^2 \sin\phi\cos\phi - \frac{g}{R}\sin\phi = g(\phi)$$

$$g'(\phi) = \omega^2\left[\cos^2\phi - \sin^2\phi\right] - \frac{g}{R}\cos\phi$$

$$g'(\phi) = \omega^2\left[2\cos^2\phi - 1\right] - \frac{g}{R}\cos\phi$$

DONDE

$$\omega^2_{peq} = -g'(\phi)\Big|_{\phi^*}$$

$\underline{\phi^* = 0}$

$$g'(0) = \omega^2 - \frac{g}{R}$$

$\underline{\phi^* = \pi}$ $$g'(\pi) = \omega^2 + \frac{g}{R}$$

$\underline{\cos\phi^* = \dfrac{g}{R\omega^2}}$

$$g'\left(\arccos\left(\frac{g}{R\omega^2}\right)\right) = \omega^2\left[2\frac{g^2}{R^2\omega^4} - 1\right] - \frac{g^2}{R^2\omega^2}$$

$$= \frac{2g^2 - R^2\omega^4 - g^2}{R^2\omega^2} = \frac{g^2 - R^2\omega^4}{R^2\omega^2}$$

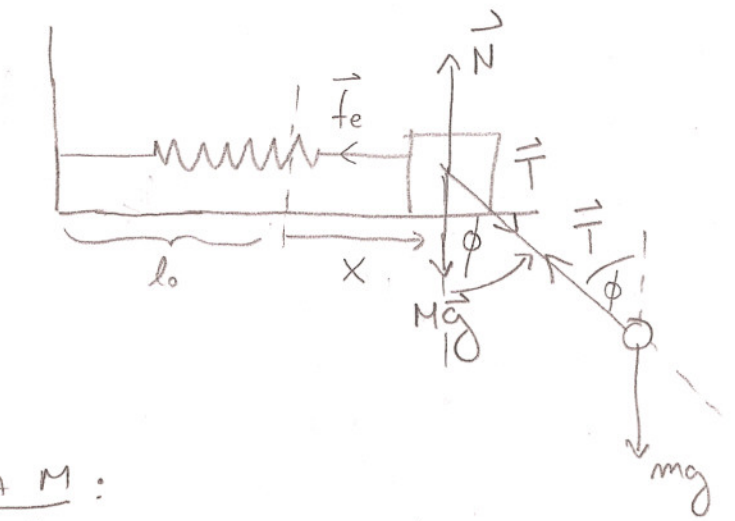
PARA QUE SEAN EQUILIBRIOS ESTABLES $\omega^2_{peq} > 0$

$\underline{\phi^* = 0}$ $\quad \dfrac{g}{R} - \omega^2 > 0 \quad \Rightarrow \quad \boxed{\omega^2 < \dfrac{g}{R}} \quad \Rightarrow \quad \omega^2 = \dfrac{g}{R}$ FRECUENCIA CRÍTICA

$\underline{\phi^* = \pi}$ INESTABLE

$\underline{\phi^* = \arccos\left(\dfrac{g}{R\omega^2}\right)}$ $\quad \dfrac{R^2\omega^4 - g^2}{R^2\omega^2} > 0 \quad \Rightarrow \quad \omega^2 > \dfrac{g}{R}$

P2



$$\vec{r}_m = (x + L\sin\phi)\hat{\imath} - L\cos\phi\,\hat{\jmath}$$
$$\vec{v}_m = (\dot{x} + L\cos\phi\,\dot{\phi})\,\hat{\imath} + L\sin\phi\,\dot{\phi}\,\hat{\jmath}$$
$$\vec{a}_m = (\ddot{x} + L(-\sin\phi\,\dot{\phi}^2 + \cos\phi\,\ddot{\phi}))\hat{\imath} + L(\cos\phi\,\dot{\phi}^2 + \sin\phi\,\ddot{\phi})\hat{\jmath}$$

PARA M:

$$① \ \hat{x})\ -kx + T\sin\phi = M\ddot{x}$$

PARA m:

$$② \ \hat{\imath})\ -T\sin\phi = m(\ddot{x} + L(-\sin\phi\,\dot{\phi}^2 + \cos\phi\,\ddot{\phi}))$$
$$③ \ \hat{\jmath})\ T\cos\phi - mg = mL(\cos\phi\,\dot{\phi}^2 + \sin\phi\,\ddot{\phi})$$

$② \cdot \cos\phi + ③ \sin\phi$:

$$④ \quad -mg\sin\phi = m\ddot{x}\cos\phi + mL\ddot{\phi}$$

$① + ②$:

$$⑤ \quad -kx = (m+M)\ddot{x} + mL(-\sin\phi\,\dot{\phi}^2 + \cos\phi\,\ddot{\phi})$$

ptos DE eq: $\dot{x} = \ddot{x} = \dot{\phi} = \ddot{\phi} = 0$

$$\Rightarrow x_{eq} = 0$$
$$\Rightarrow \sin\phi_{eq} = 0 \Rightarrow \phi_{eq} = 0 \ \vee \ \phi_{eq} = \pi$$

TAYLOR EN TORNO A LOS PUNTOS DE EQUILIBRIO

$$\sin\phi\Big|_{\phi\approx 0} \approx \cancelto{0}{\sin 0} + (\cos 0)\cdot\phi - \cancelto{0}{\frac{(\sin 0)}{2}}\phi^2 = \phi$$

$$\sin\phi \Big|_{\phi\approx\pi} \approx \cancelto{0}{\sin\pi} + \cos\pi(\phi-\pi) - \cancelto{0}{\frac{\sin\pi}{2}}(\phi-\pi)^2 = \pi-\phi$$

$$\cos\phi \Big|_{\phi\approx 0} = \cos 0 - (\sin 0)\phi - \frac{\cos 0}{2}\phi^2 = 1-\frac{\phi^2}{2}$$

$$\cos\phi \Big|_{\phi\approx\pi} = \cos\pi - (\sin\pi)(\phi-\pi) - \frac{\cos\pi}{2}(\phi-\pi)^2 = \frac{(\phi-\pi)^2}{2} - 1$$

ASÍ EN ④ Y ⑤, EN TORNO A $\phi\approx 0$ ($\phi\approx\pi$ NO LO ANALIZAREMOS)

⑥ $$-mg\phi = m\ddot{x}\left(1-\frac{\phi^2}{2}\right) + mL\ddot{\phi}$$

⑦ $$-kx = (m+M)\ddot{x} + mL\left(-\phi\dot{\phi}^2 + \left(1-\frac{\phi^2}{2}\right)\ddot{\phi}\right)$$

LOS TÉRMINOS POR SOBRE SEGUNDO ORDEN SE ELIMINAN:

⑧ $$-mg\phi = m\ddot{x} + mL\ddot{\phi}$$

⑨ $$-kx = (m+M)\ddot{x} + mL\ddot{\phi}$$

⑨ - ⑧

$$-kx + mg\phi = M\ddot{x} \Rightarrow \ddot{x} = -\frac{k}{M}x + \frac{m}{M}g\phi \quad ⑩$$

EN ⑧

$$-mg\phi = -\frac{mk}{M}x + \frac{m^2}{M}g\phi + mL\ddot{\phi}$$

$$\Rightarrow \ddot{\phi} = \frac{k}{LM}x - \frac{g}{L}\left(1+\frac{m}{M}\right)\phi \quad ⑪$$

$$\Rightarrow \begin{bmatrix}\ddot{x}\\ \ddot{\phi}\end{bmatrix} = \underbrace{\begin{bmatrix}-\frac{k}{M} & \frac{mg}{M} \\ \frac{k}{LM} & -\frac{g}{ML}(M+m)\end{bmatrix}}_{A}\begin{bmatrix}x\\ \phi\end{bmatrix}$$

CALCULAMOS $\det(A-\lambda I)$

$$\left(\frac{k}{M}+\lambda\right)\left(\frac{g}{ML}(M+m)+\lambda\right) - \frac{m}{M^2}\frac{gk}{L} = 0$$

$$\Rightarrow \lambda_{1,2} = \frac{-KL - g(m+M) \pm \sqrt{(g(m+M)+KL)^2 - 4MgKL}}{2ML}$$

POR LO TANTO LAS FRECUENCIAS PROPIAS SON:

$$\omega_i^2 = -\lambda_i$$

$$\omega_{1,2}^2 = \frac{KL + (m+M)g \pm \sqrt{(g(m+M)+KL)^2 - 4MgKL}}{2ML}$$

AHORA CONSIDERANDO QUE:

$$M = 2m$$

$$\frac{g}{L} = \frac{K}{M} = \omega_0^2$$

$$\Rightarrow \omega_1 = \frac{\omega_0^2}{2}$$

$$\Rightarrow \omega_2 = 2\omega_0^2$$

SABEMOS QUE:

$$\ddot{x} = -\omega_i^2 x$$

REEMPLAZANDO EN (10)

$\omega_1^2 = \omega_0^2/2$:

$$-\frac{\omega_0^2}{2} x = -\frac{K}{M} x + \frac{mg}{M}\phi$$

$$\frac{\omega_0^2}{2} x = \frac{\omega_0^2}{2} L\phi \Rightarrow x = L\phi$$

$\underline{\omega_2^2 = 2\omega_0^2}$:

$$-2\omega_0^2 X = -\omega_0^2 X + \frac{\omega_0^2}{2} L \phi$$

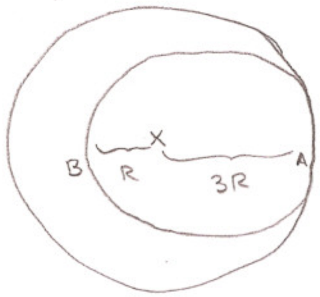
$$X = -\frac{1}{2} L \phi$$

———— o ————

ESTO A TRAVÉS DE ENERGÍA SÓLO CAMBIA LA ECUACIÓN ORIGINAL

$$E = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m \left(\dot{x}^2 + 2L\dot{x}\cos\phi\,\dot{\phi} + L^2 \dot{\phi}^2 \right) + \frac{1}{2} k x^2 - mgL\cos\phi$$

P3



ÓRBITA CIRCUNFERENCIAL:

$$\hat{\rho})\quad -\frac{GMm}{(3R)^2} = m(\cancel{\ddot{\rho}}^{\,0} - 3R\dot{\theta}^2)$$

$$\frac{GM}{(3R)^3} = \dot{\theta}^2$$

$$\Rightarrow v_i = \sqrt{\frac{GM}{3R}}$$

ÓRBITA ELÍPTICA:

$$E_A = \frac{1}{2} m v_A^2 - \frac{GMm}{3R} \qquad l_A = m3Rv_A$$

$$E_B = \frac{1}{2} m v_B^2 - \frac{GMm}{R} \qquad l_B = mRv_B$$

$$E_A = E_B$$
$$l_A = l_B$$

$$\Rightarrow \frac{v_A^2}{2} - \frac{GM}{3R} = \frac{v_B^2}{2} - \frac{GM}{R}$$

$$v_B = 3v_A$$

$$\Rightarrow \frac{v_A^2}{2} - \frac{GM}{3R} = \frac{9v_A^2}{2} - \frac{GM}{R}$$

$$\frac{2GM}{3R} = 4v_A^2 \Rightarrow v_A = \sqrt{\frac{GM}{6R}} \Rightarrow \Delta v = v_i - v_A$$

$$\boxed{\Delta v = \sqrt{\frac{GM}{3R}}\left(1 - \frac{1}{\sqrt{2}}\right)}$$

b) $$\rho(\theta) = \frac{h^2/c}{1+\sqrt{1+\frac{2\epsilon h^2}{c^2}}\cos\theta}$$

DONDE

$$h = \frac{\ell}{m} = 3R\sqrt{\frac{GM}{6R}} = \sqrt{\frac{3GMR}{2}}$$

$$\epsilon = \frac{E}{m} = \frac{1}{2}v_A^2 - \frac{GM}{3R} = \frac{GM}{12R} - \frac{GM}{3R} = -\frac{GM}{4R}$$

$$c = GM$$

$$\Rightarrow \rho(\theta) = \frac{3R}{2+\cos\theta}$$

c) $$v_B = 3v_A \Rightarrow \boxed{v_B = \sqrt{\frac{3GM}{2R}}}$$

d) EL PERIODO DE UNA ELIPSE SEGÚN LA 3^{ERA} LEY DE KEPLER ES:

$$T^2 = \frac{(2\pi)^2 a^3}{c}$$

DONDE $$a = \frac{R_{MAX} + R_{MIN}}{2} = \frac{3R+R}{2} = 2R$$

$$c = GM$$

$$\Rightarrow t^* = \pi\sqrt{\frac{(2R)^3}{GM}}$$