P1

a) usamos $\vec{F} = m\vec{a}$ en la coordenada $\hat{r}$

$$-m\rho\dot{\theta}^2 = -\frac{GmM}{\rho^2} \Rightarrow \rho^2\dot{\theta}^2 = \frac{GM}{\rho}$$

pero $\vec{v} = \rho\dot{\theta}\hat{\theta} \Rightarrow$ $\boxed{v' = \sqrt{\frac{GM}{\rho}}}$ rapidez del satélite antes

aumenta la velocidad

$$v = \alpha v' \Rightarrow v = \alpha\sqrt{\frac{GM}{\rho}}$$

y la Energía es (por conservación, calculamos E cuando recién aumenta la rapidez)

$$E = \frac{1}{2}mv^2 - \frac{GMm}{\rho} = \frac{1}{2}m\alpha^2\frac{GM}{\rho} - \frac{GMm}{\rho}$$

$$\Rightarrow \boxed{E = \frac{GMm}{\rho}\left(\frac{\alpha^2}{2} - 1\right)}$$

b) Sabemos que

$$e^2 = 1 + \frac{2El^2}{(GM)^2m^3}$$

dado que $\sum\vec{\tau} = 0 \Rightarrow \vec{l} = cte \Leftrightarrow |l| = cte$

$$\vec{l} = \vec{r}\times m\vec{v} \Rightarrow l = m\rho\alpha\sqrt{\frac{GM}{\rho}}$$

$$\Rightarrow \boxed{l^2 = m^2\alpha^2\rho GM}$$

por lo tanto

$$e^2 = 1 + \frac{2\left(\frac{G\cancel{M}\cancel{m}}{\cancel{\rho}}\left[\frac{\alpha^2}{2}-1\right]\right)\left(\cancel{m^2}\alpha^2\cancel{\rho}\cancel{GM}\right)}{\cancel{(GM)^2}\cancel{m^3}}$$

$$\Rightarrow \boxed{e^2 = 1 + 2\alpha^2\left(\frac{\alpha^2}{2}-1\right)}$$ ✓

para órbita parabólica, $e=1$

$$\Rightarrow 2\alpha^2\left(\frac{\alpha^2}{2}-1\right)=0 \Rightarrow \alpha = 0 \quad (\underline{\text{NO}} \text{ sirve})$$

$$\boxed{\alpha^* = \sqrt{2}}$$

es el que se busca

<u>obs</u>: en órbita parabólica, $E=0$

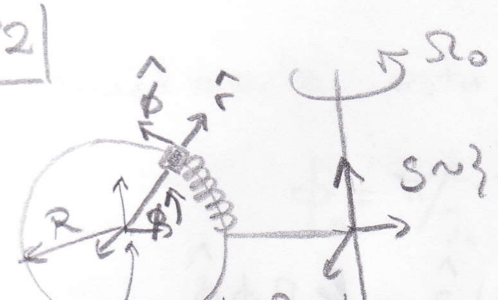
pero $E = \frac{GMm}{\rho}\left(\frac{\alpha^2}{2}-1\right) \Rightarrow E = 0 \Leftrightarrow \boxed{\alpha = \sqrt{2}}$

c) $r(\phi) = \frac{R}{1+e\cos\phi}$, con $R = \frac{l^2}{GMm^2} = \frac{(\cancel{m^2}\alpha^2\rho \cancel{GM})}{\cancel{GM}\cancel{m^2}} = \alpha^2\rho$

$$\Rightarrow r(\phi) = \frac{\alpha^2\rho}{1+\sqrt{1+2\alpha^2\left(\frac{\alpha^2}{2}-1\right)}\cos\phi}$$

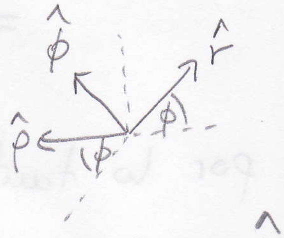
$$\Rightarrow r_{min} = r(\phi=0) = \rho \quad y \quad r_{max} = r(\phi=\pi) = \frac{\rho}{\frac{2}{\alpha^2}-1}$$

P2



notemos que

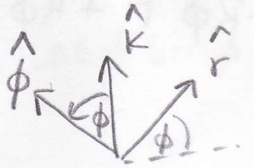
$$\vec R = 2R\hat\rho \Rightarrow \boxed{\ddot{\vec R} = -2R\Omega_0^2\hat\rho}$$



$$\hat\rho = -\cos\phi\,\hat r + \sin\phi\,\hat\phi$$

$$\vec\Omega = \Omega_0\hat k = cte \Rightarrow \boxed{\vec\Omega = \Omega_0\hat k} \quad \boxed{\dot{\vec\Omega} = 0}$$

fuerzas no inerciales:



$$\hat k = \cos\phi\,\hat\phi + \sin\phi\,\hat r$$

▷ $\vec\Omega \times \vec r\,' = (\Omega_0\hat k)\times(R\hat r) = \Omega_0 R(\hat k \times \hat r)$

$$= \Omega_0 R(\cos\phi\,\hat\phi + \sin\phi\,\hat r)\times\hat r$$

$$= -\Omega_0 R\cos\phi\,\hat z$$

$\Rightarrow \vec\Omega\times\vec\Omega\times\vec r\,' = -\Omega_0^2 R\cos\phi\;\hat k\times\hat z$ / $\hat k\times\hat z = (\cos\phi\,\hat\phi + \sin\phi\,\hat r)\times\hat z$

$$= \cos\phi\,\hat r - \sin\phi\,\hat\phi$$

$$\Rightarrow \boxed{\vec\Omega\times\vec\Omega\times\vec r\,' = -\Omega_0^2 R\cos^2\phi\,\hat r + \Omega_0^2 R\cos\phi\sin\phi\,\hat\phi}$$

▷ $\vec\Omega\times\vec v\,' = \Omega_0 R\dot\phi\;\hat k\times\hat\phi$ / $\hat k\times\hat\phi = \sin\phi\,\hat z$

$$\Rightarrow \boxed{\vec\Omega\times\vec v\,' = \Omega_0 R\dot\phi\sin\phi\,\hat z}$$

$\dot{\vec{\Omega}} \times \vec{r}\,' = \phi$, pues $\dot{\vec{\Omega}} = \phi$

$$\vec{F}_{reales} = -mg\hat{k} + \vec{N} + \vec{F}_{resorte}$$

$$= -mg(\cos\phi\,\hat{\phi} + \sin\phi\,\hat{r}) + N_r\hat{r} + N_z\hat{z} - kR\phi\hat{\phi}$$

por lo tanto, sabiendo que $\vec{a}\,' = -R\dot{\phi}^2\hat{r} + R\ddot{\phi}\hat{\phi}$, se tiene

que la ecuación vectorial es

$$\boxed{\begin{aligned} m(-R\dot{\phi}^2\hat{r} + R\ddot{\phi}\hat{\phi}) = {} & -mg(\cos\phi\,\hat{\phi} + \sin\phi\,\hat{r}) + N_r\hat{r} + N_z\hat{z} - kR\phi\hat{\phi} \\ & + 2mR\Omega_0^2(-\cos\phi\,\hat{r} + \sin\phi\,\hat{\phi}) - 2m\Omega_0 R\dot{\phi}\sin\phi\,\hat{z} \\ & + m\Omega_0^2 R\cos^2\phi\,\hat{r} - m\Omega_0^2 R\cos\phi\sin\phi\,\hat{\phi} \end{aligned}}$$

a) reposo relativo en A $\Rightarrow$ veamos que nos dice $\hat{\phi}$

$\hat{\phi}]$ $\quad mR\ddot{\phi} = -mg\cos\phi - kR\phi + 2mR\Omega_0^2\sin\phi - m\Omega_0^2 R\cos\phi\sin\phi$

reposo relativo en $\phi = \pi/2 \Rightarrow \dot{\phi} = \ddot{\phi} = 0$

Entonces

$$0 = 0 - kR\pi/2 + 2mR\Omega_0^2$$

$$\Rightarrow \boxed{\Omega_0^2 = \frac{k\pi}{4m}}$$

b) inicialmente esta en A y tiene velocidad inicial

$$\Rightarrow \quad \phi_0 = \pi/2 \quad y \quad \dot{\phi}_0 \neq 0$$

finalmente está en B

$$\Rightarrow \quad \phi_{final} = \pi \quad y \quad \dot{\phi}_{final} = 0$$

usamos la ec. de movimiento de $\hat{\phi}$

$$\ddot{\phi} = -\frac{g}{R}\cos\phi - \frac{k}{m}\phi + 2\Omega_0^2 \sin\phi - \Omega_0^2 \cos\phi \sin\phi$$

integramos, primero haciendo $\ddot{\phi} = \dot{\phi}\frac{d\dot{\phi}}{d\phi}$

$$\int_{\dot{\phi}_0}^{0} \dot{\phi}\, d\dot{\phi} = -\frac{g}{R}\int_{\pi/2}^{\pi} \cos\phi\, d\phi - \frac{k}{m}\int_{\pi/2}^{\pi} \phi\, d\phi + 2\Omega_0^2 \int_{\pi/2}^{\pi} \sin\phi\, d\phi - \Omega_0^2 \int_{\pi/2}^{\pi} \cos\phi \sin\phi\, d\phi$$

$$-\frac{\dot{\phi}_0^2}{2} = \frac{g}{R} - \frac{k}{2m}\underbrace{\left(\pi^2 - \frac{\pi^2}{4}\right)}_{\frac{3\pi^2}{4}} + 2\Omega_0^2 + \Omega_0^2 \cdot \frac{1}{2}$$

$$\Rightarrow \boxed{\dot{\phi}_0^2 = \frac{3k\pi^2}{4m} - 5\Omega_0^2 - \frac{2g}{R}}$$

c) veamos las componentes de la ec. vectorial

$\hat{r}\rfloor \quad -mR\dot{\phi}^2 = -mg\sin\phi + N_r - 2mR\Omega_0^2\cos\phi + mR\Omega_0^2\cos^2\phi \quad (1)$

$\hat{z}\rfloor \quad 0 = N_z - 2m\Omega_0 R\dot{\phi}\sin\phi \quad (2)$

en el punto A, $\phi = \frac{\pi}{2}$, $\dot{\phi} = \left(\frac{3k\pi^2}{4m} - 5\Omega_0^2 - \frac{2g}{R}\right)^{1/2}$

$(1) \Rightarrow \quad -mR\left(\frac{3k\pi^2}{4m} - 5\Omega_0^2 - \frac{2g}{R}\right) = -mg + N_r$

$\Rightarrow \boxed{N_r = 3mg + 5mR\Omega_0^2 - \frac{3Rk\pi^2}{4}}$

$(2) \Rightarrow \boxed{N_z = 2m\Omega_0 R\left(\frac{3k\pi^2}{4m} - 5\Omega_0^2 - \frac{2g}{R}\right)^{1/2}}$

$\Rightarrow \boxed{\vec{N}(\text{en A}) = \left(3mg + 5mR\Omega_0^2 - \frac{3Rk\pi^2}{4}\right)\hat{r} + 2m\Omega_0 R\left(\frac{3k\pi^2}{4m} - 5\Omega_0^2 - \frac{2g}{R}\right)^{1/2}\hat{z}}$

Normal en el punto A

en el punto B, $\phi = \pi$ y $\dot{\phi} = 0$

$(2) \rightarrow \quad N_z = 0$

$(1) \Rightarrow \quad N_r = -3mR\Omega_0^2$

$\Rightarrow \boxed{\vec{N}(\text{en B}) = -3mR\Omega_0^2\hat{r}}$