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Limits on Lithospheric Stress Imposed by Laboratory Experiments

W. F. BRACE

Department of Earth and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

D. L. KOHLSTEDT

Department of Materials Science and Engineering, Cornell University, Ithaca, New York 14853

Laboratory measurements of rock strength provide limiting values of lithospheric stress, provided that one effective principal stress is known. Fracture strengths are too variable to be useful; however, rocks at shallow depth are probably fractured so that frictional strength may apply. A single linear friction law, termed Byerlee's law, holds for all materials except clays, to pressures of more than 1 GPa, to temperatures of 500°C, and over a wide range of strain rates. Byerlee's law, converted to maximum or minimum stress, is a good upper or lower bound to observed in situ stresses to 5 km, for pore pressure hydrostatic or subhydrostatic. Byerlee's law combined with the quartz or olivine flow law provides a maximum stress profile to about 25 or 50 km, respectively. For a temperature gradient of 15°K/km, stress will be close to zero at the surface and at 25 km (quartz) or 50 km (olivine) and reaches a maximum of 600 MPa (quartz) or 1100 MPa (olivine) for hydrostatic pore pressure. Some new permeability studies of crystalline rocks suggest that pore pressure will be low in the absence of a thick argillaceous cover.

INTRODUCTION

Stresses in the earth cannot exceed the strength of rocks. Therefore measurements of rock strength can be used to set limits on stress in the earth. Here we review what is known about strength of rocks, particularly in the upper, colder parts of the lithosphere, where brittle failure limits the stress. Following the recent work of Goetze and Evans [1979], we give limiting stresses as a function of depth, noting the strong influence of pore pressure. We test these stresses to about 4 km using published in situ stress levels and plausible assumptions about pore pressure.

It is important to keep in mind the limitations of laboratory measurements in any discussion of lithospheric stress. They provide only one thing, namely, the maximum stress difference which rocks can support at some pressure, temperature, and strain rate. We will call this maximum stress difference, or strength, S, where $S = |(\sigma_1 - \sigma_3)|$. Provided one of the principal stresses, σ_1 or σ_3, is known at some point in the earth, the maximum or minimum value of the other can be calculated from laboratory values of S. We will note below the difficulties involved in estimating any one of the principal stresses.

LABORATORY STRENGTH MEASUREMENTS

Brittle Fracture

For the present discussion we need only consider the magnitude of S for rocks and how it varies with the pressure, temperature, and strain rate encountered in crustal rocks. The subject has been most recently reviewed by *Paterson* [1978], *Jaeger and Cook* [1976], and *Ohnaka* [1973]. Assuming crystalline rocks are of greatest interest, the room temperature results generally show that (1) S has a marked dependence on pressure, (2) the pressure dependence, although of the same sign, varies unsystematically with rock type, and (3) S at any one pressure can vary by a factor of up to 5, depending on rock type.

Temperature and strain rate dependences of S also vary among the different crystalline rocks [*Griggs et al.,* 1960]. Fortunately, these effects are rather small for brittle behavior. *Paterson* [1978] showed that thermal weakening, if considered thermally activated, has an activation energy of less than 1 kcal mol^{-1}. *Brace and Jones* [1971] found a weak strain rate sensitivity; S increased 5–20% per 10^3 increase in strain rate.

Given the fivefold variation in S at any pressure and the other smaller, although equally unsystematic, variations with temperature and strain rate, we conclude that laboratory measurements of fracture strength are almost useless in the present context. One way out of this situation may be to consider the influence of fractures.

Evidence for fractures (joints, faults, bedding foliation, and the like) in rocks to 5- or 10-km depth was summarized by *Brace* [1972]. The evidence was of two types, direct observation in mines or drill holes and inference from measured electrical or fluid conductivity. Without repeating all the arguments, the evidence for a water-filled, interconnected pore space seemed compelling at least to about 5 km. Recent deep resistivity measurements [*Nekut et al.,* 1977] suggest that the same conclusion holds to 20 km, and a recent study of crustal permeability argues for high conductivities to 8 km [*Brace,* 1980]. High fluid conductivity necessitates interconnecting fractures, since porosity in crystalline terrains must be a percent or less.

The lower depth limit for interconnected pore space is unknown. Plastic flow appears to be one limiting factor [*Brace,* 1972], and 'healing' of cracks another [*Batzle and Simmons,* 1977]. Nothing is known about depths at which the latter becomes significant. Plastic flow of silicate rocks appears to begin at temperatures around 400°–500°C [*Goetze and Brace,* 1972; *Tullis and Yund,* 1977; *Paterson,* 1978]. Perhaps it is safe to assume that interconnection of fractures and other pore space disappears below depths at which these temperatures are reached.

If there is a region of fractured rock, extending to perhaps 25 km, we need to consider the effect on S. Fortunately, experimentalists have addressed this question in recent years, with a result that will be very useful for the present discussion.

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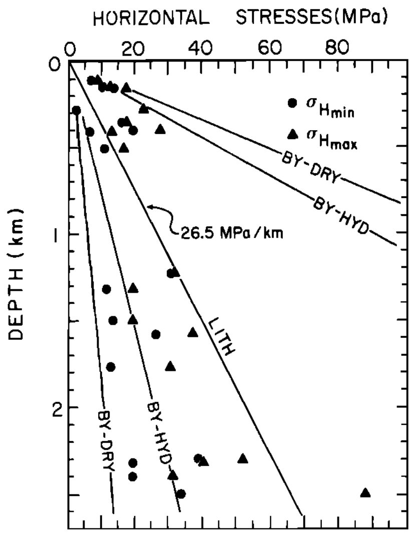

Fig. 1. Total horizontal stresses measured in southern Africa [*McGarr and Gay*, 1978]. The vertical total stress gradient (26.5 MPa/km) is shown along with Byerlee's law (BY) for hydrostatic pore pressure (HYD) and zero pore pressure (DRY).

Rock Strength During Frictional Sliding

If rock is fractured, frictional sliding will occur on the fractures before the stress reaches the level to cause brittle fracture of the mass as a whole. S of fractured rock is determined therefore by frictional resistance. Frictional resistance has been measured in the laboratory for wide ranges of rock type, pressure, and temperature. One might expect at least as wide a variation in friction as in fracture strength, with additional factors such as surface roughness playing a role. However, this does not turn out to be the case under geologic conditions. One of the most significant recent discoveries in rock mechanics is the virtual independence of friction on rock type, displacement, and surface conditions [*Barton*, 1976; *Byerlee*, 1978]. First noted by *Byerlee* [1968], frictional resistance can be fitted by a simple bilinear relation over a range of normal stresses from about 3 MPa to 1.7 GPa:

$$\tau = 0.85\bar{\sigma}_n \qquad 3 < \bar{\sigma}_n < 200 \text{ MPa}$$
$$\tau = 60 \pm 10 + 0.6\bar{\sigma}_n \qquad \bar{\sigma}_n > 200 \text{ MPa} \qquad (1)$$

where τ and $\bar{\sigma}_n$ are shearing and normal stress at which frictional resistance is overcome on a fracture. This simple relationship, which we shall refer to as Byerlee's law, seems to hold for all geologic materials except certain clay minerals. More recent work on shearing resistance of pure clays [*Wang et al.*, 1980] supports this conclusion, although in all of the experiments done with clay, the samples were probably undrained, a condition which would lower the strength.

The effect of temperature on friction was reported by *Stesky et al.* [1974] for a gabbro and granite. The stresses are independent of temperature to 500°C for the granite and 400°C for the gabbro. Other silicate rocks will probably behave similarly. The effect of sliding rate is also small, comparable to the effect on fracture strength [*Stesky*, 1975; *Dieterich*, 1972] up to the same temperature levels.

The role of pore pressure needs special attention. Most mechanical behavior of rocks is determined by effective stress $\bar{\sigma}_{ij}$, where

$$\bar{\sigma}_{ij} = \sigma_{ij} - \alpha P_p \delta_{ij}$$

the σ_{ij} are the total macroscopic stresses, and P_p is the pore pressure (see reviews by *Paterson* [1978] and *Brace* [1972]). Empirically, α is close to 1 for fracture and friction of silicate rocks, although it may depart significantly for some elastic and transport phenomena. For example, α of 4 has been reported for permeability in sandstone [*Zoback and Byerlee*, 1975]. Thus in (1), pore pressure is included by noting that

$$\bar{\sigma}_n = \sigma_n - P_p \qquad (2)$$

We will adopt this concept in what follows. Measured in situ stress is usually reported as total stress, and (2) will be used to relate laboratory strength criteria such as (1) to calculated lithospheric stresses.

ESTIMATE OF LITHOSPHERIC STRESS

Lithospheric Stress From Byerlee's Law

Byerlee's law is expressed in (1) in terms of the stress components on the fracture. In terms of the principal effective stresses it becomes [*Jaeger and Cook*, 1976, p. 14]

$$\bar{\sigma}_1 \simeq 5\bar{\sigma}_3 \qquad \bar{\sigma}_3 < 110 \text{ MPa}$$
$$\bar{\sigma}_1 \simeq 3.1\bar{\sigma}_3 + 210 \qquad \bar{\sigma}_3 > 110 \text{ MPa} \qquad (3)$$

A plausible assumption in fractured rock is that fractures of all orientations exist. Regardless of their orientation then, once the principal stresses reach the values in (3), frictional sliding occurs somewhere. In fractured rock therefore these values represent the maximum permissible stresses, regardless of rock type (neglecting clays), temperature to 500°C, and details of fracture geometry and gouge thickness.

To compare reported in situ stresses with the limiting values given by (3), one principal stress and the pore pressure must be known. Usually, the vertical stress, which is known, is taken to be either σ_1 or σ_3, and the pore pressure to be hydrostatic. The recent compilation of *McGarr and Gay* [1978] is particularly convenient for comparing measured lithospheric stresses with the limiting stresses predicted by Byerlee's law. The comparison will be at best approximate owing to the appreciable errors in measured stresses and the scatter in rock friction values.

Horizontal stresses σ_H in southern Africa (Figure 1) are available from a number of overcoring measurements in deep mines [*McGarr and Gay*, 1978]. The two solid lines marked BY-HYD in this figure are computed from Byerlee's law, assuming hydrostatic pore pressure; one line corresponds to horizontal σ_1, and the other to horizontal σ_3. Most of the measured values are contained within these lines, as they should be if Byerlee's law correctly gives the limiting value of lithospheric stress. However, a better fit is obtained if pore pressure is assumed to be zero, giving the two curves marked BY-DRY. Since these deep mines are often reported to be 'dry' [*McGarr et al.*, 1975], perhaps $P_p = 0$ is reasonable. If P_p had the maximum value, equal to the total vertical stress σ_V, then the horizontal stresses should fall along the lines marked LITH.

Stresses have been measured in the United States primarily by hydraulic fracturing in sedimentary basins (Figure 2). Except for two points, all measurements of the minimum compression fall correctly to the right of Byerlee's law with hydrostatic pore pressure. The two points refer to crystalline rocks

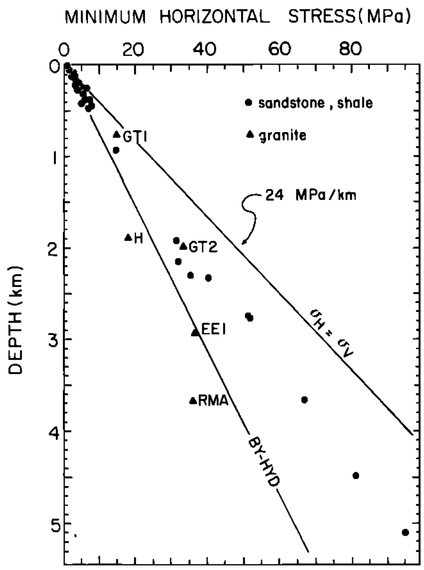

Fig. 2. The minimum total horizontal stress measured in the United States. Symbols as in Figure 1 [*McGarr and Gay*, 1978].

in South Dakota (H) and near Denver (RMA); they could be explained by slightly subhydrostatic pore pressure.

Measurements from deep Canadian mines in crystalline rocks are collected in Figure 3 [*McGarr and Gay*, 1978]. Byerlee's law for hydrostatic pressure (BY-HYD) is seen to enclose most of the data points, as does the upper branch in the African example (Figure 1). This result would imply imminent thrust faulting in these two areas at these depths. Proximity to the lower branch (the deeper African measurements, for example) implies imminent normal faulting.

We conclude that Byerlee's law satisfactorily brackets most measured values of horizontal lithospheric stress to about 4 km for hydrostatic pore pressure. In one area where the stresses measured exceed the hydrostatic branch of Byerlee's law, dry rocks and therefore reduced pore pressure are plausible. *McGarr and Gay* [1978] made a similar comparison of measured stresses and an experimentally derived failure criterion. They used the Coulomb criterion which, in the same form as (1), was

$$\tau = 0.6\bar{\sigma}_n$$

and found that over a fairly broad region of the Gulf Coast this criterion was met for hydrostatic pore pressure.

Extrapolation Below 4 km

Byerlee's law appears to predict satisfactorily maximum or minimum horizontal stress to about 4 km, and it is of interest to extrapolate the law to greater depths. Eventually, temperature effects will play a greater role. To take these into account, we need to consider rock plasticity. In this paper we use experimental flow laws for quartz and olivine based on the assumption that properties of these minerals will control the stress levels in the lower crust and upper mantle. Although this assumption is plausible for olivine and the upper mantle, the use of quartz is questionable for crustal rocks. Feldspar almost certainly plays a major role; however, its ductile behavior is poorly understood. Quartz is apparently more ductile than feldspar at lower crustal temperatures [*Tullis and Yund*, 1977], so that the quartz flow law will give at best a lower bound to the crustal stress.

Strength in the ductile regime is probably independent of pore pressure, so this complication can be avoided. However, strength is dependent on strain rate and temperature. Current understanding of the flow law of olivine was summarized by *Goetze* [1978]:

$$\dot{\epsilon} = 7 \times 10^4(\sigma_1 - \sigma_3)^3 \exp\left(\frac{-0.52 \text{ MJ/mol}}{RT}\right) \qquad \sigma_1 - \sigma_3 < 200 \text{ MPa}$$

$$\dot{\epsilon} = 5.7 \times 10^{11} \exp\left[\frac{-0.54 \text{ MJ/mol}}{RT}\left(1 - \frac{\sigma_1 - \sigma_3}{8500}\right)^2\right] \qquad \sigma_1 - \sigma_3 > 200 \text{ MPa} \tag{4}$$

where $\dot{\epsilon}$ is in s^{-1} and $\sigma_1 - \sigma_3$ is in megapascals. The plastic deformation behavior of quartz appears to depend markedly on OH^- content. We consider here experimental results on dry samples which should provide an upper limit to the strength of quartzite. Recent experiments of *Christie et al.* [1979] on dry quartzite at 800° and 900°C combined with the data of *Heard and Carter* [1968] yield the flow law

$$\dot{\epsilon} = 5 \times 10^6(\sigma_1 - \sigma_3)^3 \exp\left(\frac{-0.19 \text{ MJ/mol}}{RT}\right) \qquad \sigma_1 - \sigma_3 < 1000 \text{ MPa} \tag{5}$$

where $\dot{\epsilon}$ is in s^{-1} and $\sigma_1 - \sigma_3$ is in megapascals.

The maximum and minimum horizontal stresses that can be supported as a function of depth are shown in Figure 4. The curving portions, marked QTZ and OL, come from the quartz flow law (equation (5)) and the olivine flow law (equation (4)) for a strain rate of 10^{-15} s^{-1} using a linear geotherm $T = 350 + 15z$ [*Goetze and Evans*, 1979], where z is depth in kilometers and T is temperature in degrees kelvin. This geotherm for old oceanic lithosphere is quite close, to 40 km, to that for a stable continental interior according to *Lachenbruch and Sass* [1977]. The linear portions, marked BY-HYD, are from Byerlee's law (equation (3)) with hydrostatic pore pressure. Maximum stresses are shown in terms of the horizontal minus the

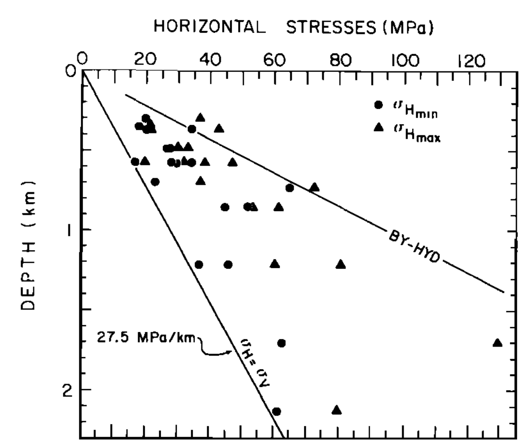

Fig. 3. Total horizontal stresses measured in Canada [*McGarr and Gay*, 1978]. Symbols as in Figure 1.

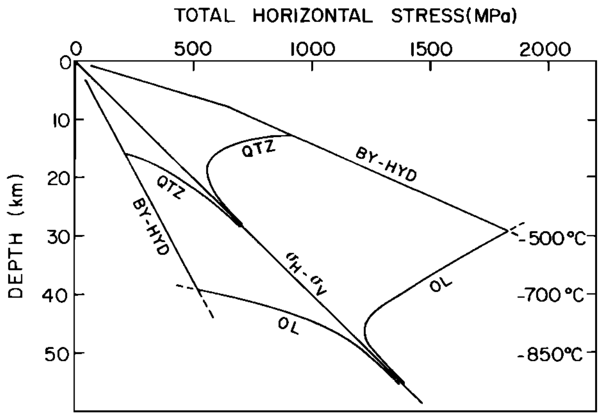

Fig. 4. Limiting values of total horizontal stress as a function of depth, based on Byerlee's law with hydrostatic pore pressure (BY-HYD) and the quartz (QTZ) and olivine (OL) flow laws adjusted to a strain rate of 10^{-15} s^{-1}. The temperature profile T(°K) = 350 + 15z(km).

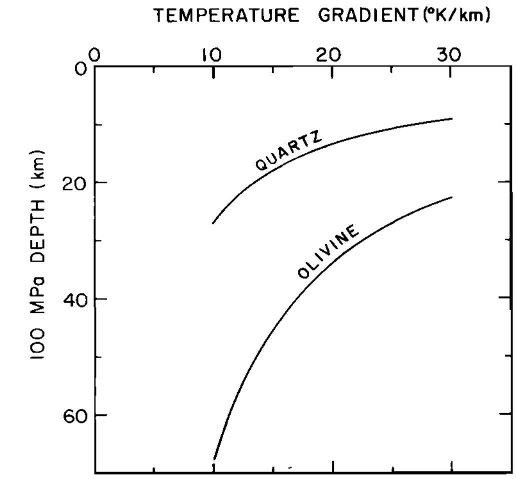

Fig. 6. Depth below which strength is less than 100 MPa as a function of temperature gradient.

vertical stress in Figure 5. The influence of pore pressure other than hydrostatic is shown by the dashed lines with different λ values; λ is the ratio of the pore pressure to the total vertical stress.

DISCUSSION

Figure 5 illustrates graphically the limits within which lithospheric stress must lie on the basis of the assumptions (1) that rocks are fractured and that friction on fractures controls the stress at shallow depths, (2) that the creep properties of quartz or olivine control the stress below about 15 or 25 km, respectively, and (3) that the effective stress principle operates for friction but not for creep. These limits should be refined when feldspar rheology becomes better understood. For the moment, several implications are worth noting.

1. For a temperature gradient of 15°K/km a region of high strength exists between the surface and about 25 km for a quartz rheology and between the surface and about 50 km depth for an olivine rheology. The lower depth is temperature dependent. These depths are decreased to 10 and 20 km, respectively, if the temperature gradient is increased to 30°K/km. The variation with temperature gradient of the depth below which the strength is less than 100 MPa is shown in Figure 6.

2. The lithosphere has close to zero strength at the surface and at depths below about 25 km (quartz rheology) or 50 km (olivine rheology). This combined frictional plus ductile behavior is important in any discussion of bending in a lithospheric slab [*Goetze and Evans,* 1979].

3. If rocks are dry (λ = 0), then according to Figure 5 the maximum strength is 300 MPa or 850 MPa for quartz depending on whether horizontal extension or compression was occurring, and 700 MPa or 1500 MPa for olivine. If the pore pressure is hydrostatic (λ = 0.42), these values for maximum strength decrease to 200 MPa or 600 MPa for quartz and 450 MPa or 1100 MPa for olivine.

4. The level of pore pressure (as shown by the value of λ) has great influence on crustal stress levels. For example, the stress drops from its maximum possible value when the rocks are dry (λ = 0) to zero when the pore pressure is equal to the lithostatic pressure (λ = 1), that is, when the total stress is hydrostatic and equal to the vertical stress.

What is known about pore pressure level at depth? Unfortunately, almost nothing, so that this parameter is almost totally unconstrained at this time. One new inference may be drawn from a recent analysis of crustal permeability [*Brace,* 1980]. Crystalline rocks to about 8 km appear to have a permeability comparable to that of sandstone. *Bredehoeft and Hanshaw* [1968] found that pore pressure under these circumstances could not be much different than hydrostatic, for times relevant to most geologic phenomena. The same conclusion applies here for crystalline rocks which outcrop at the surface. They concluded that high pore pressure required a thick argillaceous section to act as an impermeable blanket. This conclusion may be generally true not only for sedimentary but also for crystalline rocks.

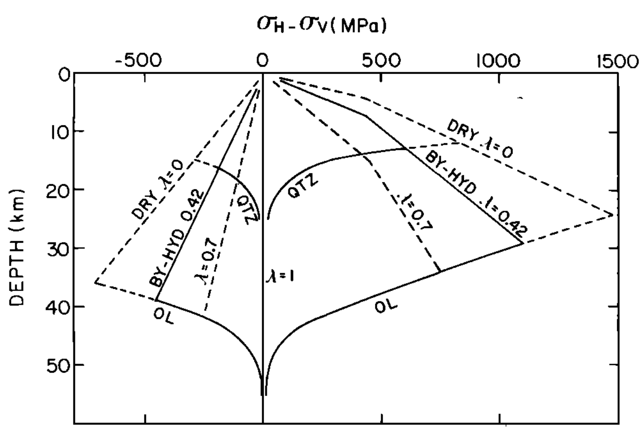

Fig. 5. Difference between maximum or minimum horizontal stress and the vertical stress as a function of depth. Values of λ give pore pressure level. See also Figure 4.

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