Pauta auxiliar 5

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P1

- $\int \frac{dx}{a^2+x^2}$ cambio de variable $x = au \Rightarrow dx = adu$

$$= \frac{1}{a}\int \frac{du}{1+u^2} = \frac{1}{a} arctan\left(\frac{x}{a}\right) + C$$

- $\int \frac{xdx}{1+x^2}$ cambio de variable $u = x^2 \Rightarrow du = 2xdx$

$$= \frac{1}{2}\int \frac{du}{1+u} = \frac{1}{2} ln\left|1+x^2\right| + C$$

- $\int \frac{x^2dx}{1+x^2} = \int dx - \int \frac{dx}{1+x^2} = x - arctan(x) + C$
- $\int \frac{xdx}{\sqrt{1+x}} = \int \frac{1+x}{\sqrt{1+x}}dx - \int \frac{dx}{\sqrt{1+x}} = \frac{2}{3}(1+x)^{\frac{3}{2}} - 2\sqrt{1+x} + C$
- $\int \frac{x^2dx}{\sqrt{1+x}}$ cambio de variable $u = 1+x \Rightarrow du = dx$, ademas $x = u-1$

$$\int \frac{u^2-2u+1}{\sqrt{u}}du = \int u^{\frac{3}{2}}du - 2\int u^{\frac{1}{2}}du + \int u^{-\frac{1}{2}}du = \frac{2}{5}(1+x)^{\frac{5}{2}} - \frac{4}{3}(1+x)^{\frac{2}{3}} + 2\sqrt{1+x} + C$$

P2

- $\int \frac{sen(x)cos(x)dx}{\sqrt{1+sen(x)}}$ cambio de variable $u = sen(x) \Rightarrow du = cos(x)dx$

$$= \int \frac{udu}{\sqrt{1+u}}$$

cambio de variable $v = 1+u \Rightarrow dv = du$

$$= \int \frac{v-1}{\sqrt{v}}dv = \int v^{\frac{1}{2}}dv - \int v^{-\frac{1}{2}}dv = \frac{2}{3}(1+sen(x))^{\frac{3}{2}} - 2\sqrt{1+sen(x)} + C$$

- $\int \frac{\sqrt{x}}{\sqrt{1+\sqrt{x}}}dx$ cambio de variable $u = \sqrt{x} \Rightarrow du = \frac{dx}{2x}$

$$= 2\int \frac{u^2}{1+u}$$

cambio de variable $v = 1+u \Rightarrow dv = du$

$$= 2\int \frac{v^2-2v+1}{\sqrt{v}} = \frac{4}{5}(1+\sqrt{x})^{\frac{5}{2}} - \frac{8}{3}(1+\sqrt{x})^{\frac{3}{2}} + 4\sqrt{1+\sqrt{x}} + C$$

P3 $\int cos(ln(x))$ cambio de variable $y = ln(x) \Rightarrow \frac{dx}{x}$ notar que $x = e^y$

$$I = \int cos(y)e^y dy$$

integracion por partes $u = cos(y) \Rightarrow du = -sen(y)dy, dv = e^y dy \Rightarrow v = e^y$

$$I = cos(y)e^y + \int e^y sen(y)dy$$

integracion por partes $u = sen(y) \Rightarrow du = cos(y)dy, dv = e^y dy \Rightarrow v = e^y$

$$I = cos(y)e^y + sen(y)e^y - I \Rightarrow 2I = e^y(cos(y) + sen(y)) + C \Rightarrow I = \frac{x(cos(ln(x)) + sen(ln(x)))}{2} + C$$

P4 $I = \int sen^2(x)dx$ integracion por partes $u = sen(x) \Rightarrow du = cos(x)dx, dv = sen(x)dx \Rightarrow v = -cos(x)$

$$I = -sen(x)cos(x) + \int cos^2(x)dx = -sen(x)cos(x) + \int dx - I$$

$$\Rightarrow I = \frac{x - sen(x)cos(x)}{2} + C$$

P5

- $\int f(x)dx = f(x)/\frac{d}{dx}\left(\right) \Rightarrow f(x) = f'(x) \Rightarrow \frac{f'(x)}{f(x)} = 1 \; \frac{f'(x)}{f(x)} = 1 \Rightarrow \int \frac{f'(x)}{f(x)}dx = \int dx = x + C$
- $\int \frac{f'(x)dx}{f(x)} = x + C \Rightarrow ln(f(x)) = x + C \Rightarrow f(x) = e^{x+C}$