

CLASE AUXILIAR 9

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Ejercicios

P1 Calcule:

- $\int \frac{x dx}{\sqrt{x^2 + 1 + (\sqrt{x^2 + 1})^3}}$
- $\int_0^4 \frac{dx}{\sqrt{2x^{\frac{3}{2}} - x^2}}$ *Indicación* : use el cambio de variable $x = u^2$

P2 Calcule:

- $J = \int_{-2a}^{2a} x\sqrt{4a^2 - x^2} dx - \int_0^{2a} x\sqrt{a^2 - (x-a)^2} dx$
- Demostrar que: $(m+1)I_{m,n} + nI_{m+1,n-1} = 2^n$ dado

$$I_{m,n} = \int_0^1 x^n (1+x)^m dx$$

P3 Calcular $I = \int \frac{a_1 \operatorname{sen}^2(x) + 2b_1 \operatorname{sen}(x)\cos(x) + c_1 \cos^2(x)}{a \operatorname{sen}(x) + b \cos(x)}$ Para lo cual:

- Demostrar que

$$I = A \operatorname{sen}(x) + B \cos(x) + C \int \frac{dx}{a \operatorname{sen}(x) + b \cos(x)}$$

- Calcular $\int \frac{dx}{a \operatorname{sen}(x) + b \cos(x)}$

P4

- Sea f integrable en $[a, b]$. Demostrar que $\exists x \in [a, b]$ tal que:

$$\int_a^x f(t) dt = \int_x^b f(t) dt$$

- Sea $G(x) = \int_0^x \frac{\operatorname{sen}(t)}{t} dt$, demostrar:

$$1 + \int_0^{\pi/2} G(x) dx = \frac{\pi}{2} \int_0^{\pi/2} \frac{\operatorname{sen}(t)}{t} dt$$

- $\forall \alpha \in (-\pi, \pi), \alpha \neq 0$ probar que

- $I(\alpha) = \int_0^1 \frac{dx}{1 + 2x \cos(\alpha) + x^2} = \frac{\alpha}{2 \operatorname{sen}(\alpha)}$