$$\begin{aligned}
I_0 &= \int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx = \arcsin\left(\frac{x}{2}\right)\Big|_{x=0}^{1} \\
&= \arcsin\left(\frac{1}{2}\right) \\
&= \frac{\pi}{6} \\
I_1 &= \int_0^1 \frac{x}{\sqrt{4-x^2}}\,dx = -\sqrt{4-x^2}\Big|_{x=0}^{1} \\
&= \sqrt{4}-\sqrt{3}
\end{aligned}$$

Recordando que

$$\frac{d}{dx}\sqrt{4-x^2} = -\frac{x}{\sqrt{4-x^2}},$$

e integrando por partes I_{n+2}, se obtiene que

$$\begin{aligned}
I_{n+2} &= \int_0^1 \frac{x^{n+2}}{\sqrt{4-x^2}}dx \\
&= \int_0^1 x^{n+1}\,\frac{x}{\sqrt{4-x^2}}dx \\
&= (n+1)\int_0^1 x^n\sqrt{4-x^2}\,dx - x^{n+1}\sqrt{4-x^2}\big|_{x=0}^{1} \\
&= (n+1)\int_0^1 \frac{x^n(4-x^2)}{\sqrt{4-x^2}}\,dx - \sqrt{3} \\
&= 4(n+1)I_n - (n+1)I_{n+2} - \sqrt{3}
\end{aligned}$$

de donde, despejando I_{n+2} se encuentra la relación

$$I_{n+2} = 4\left(\frac{n+1}{n+2}\right)I_n - \frac{\sqrt{3}}{n+2}, \quad n\in\mathbb{N}.$$