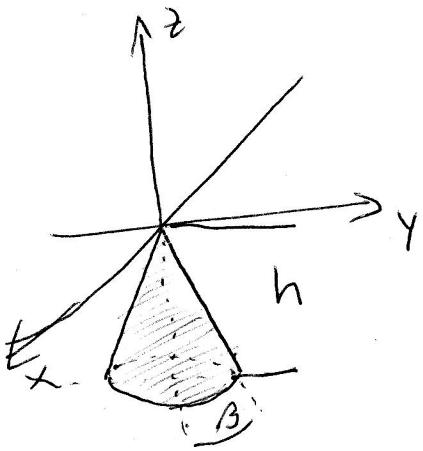
Pauta problema 2



$$\varphi(z) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\, dV'}{|\vec{r}-\vec{r}\,'|}$$

$$\vec{r} = \hat{k}z$$
$$\vec{r}\,' = r'\hat{r} + z'\hat{k} \quad \rightarrow \quad |\vec{r}-\vec{r}\,'| = \left(r'^2 + (z-z')^2\right)^{1/2}$$

$$dV' = 2\pi r' dr' dz' \qquad \text{en cilíndricas}$$

$\rightarrow$ signo finalmente no importa

$$\rightarrow \varphi(z) = \frac{\rho}{4\pi\epsilon_0}\int_{-h}^{0} dz' \int_0^{-z'\tan\beta} \frac{dr' \cdot 2\pi r'}{\left(r'^2 + (z-z')^2\right)^{1/2}}$$

$$\varphi(z) = \frac{\rho}{2\epsilon_0}\int_{-h}^{0} dz' \int_0^{-z'\tan\beta} \frac{r'\, dr'}{\left(r'^2 + (z-z')^2\right)^{1/2}}$$

$$\varphi(z) = \frac{\rho}{2\epsilon_0}\int_{-h}^{0} dz' \left(r'^2 + (z-z')^2\right)^{1/2}\Big|_0^{-z'\tan\beta}$$

$$\varphi(z) = \frac{\rho}{2\epsilon_0}\int_{-h}^{0} dz' \left[\left(z'^2\tan^2\beta + (z-z')^2\right)^{1/2} - |z-z'|\right] \quad , \; z' < 0$$

Ahora, $\Delta W = -\Delta U = -q\Delta\varphi = -q\left(\varphi(0) - \cancelto{0}{\varphi(\infty)}\right) = -q\varphi(0)$

$$\varphi(0) = \frac{\rho}{2\epsilon_0}\int_{-h}^{0} dz' \left(\left(z'^2\tan^2\beta + z'^2\right)^{1/2} + z'\right) \quad , \quad |-z'| = -z'$$

$$\varphi(0) = \frac{\rho}{2\epsilon_0}\int_{-h}^{0} dz' \left(|z'|\sec\beta + z'\right) = \frac{\rho}{2\epsilon_0}\int_{-h}^{0} dz' \,(1-\sec\beta)z'$$

$$= \frac{\rho}{2\epsilon_0}(1-\sec\beta)\frac{z'^2}{2}\Big|_{-h}^{0}$$

$$\varphi(0) = \frac{\rho h^2}{4\epsilon_0}(1-\sec\beta)$$

$$\rightarrow \Delta W = -q\varphi(0) = \frac{q\rho h^2}{4\epsilon_0}(\sec\beta - 1)$$