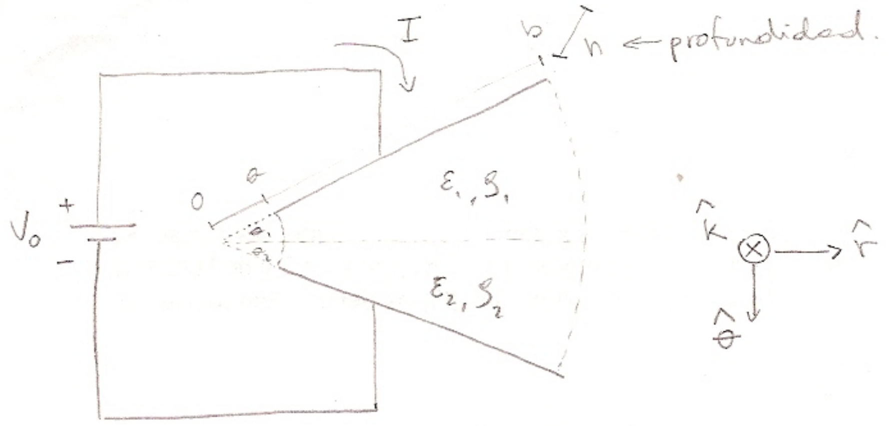

En régimen permanente $\frac{\partial \rho}{\partial t} = 0 \Rightarrow \nabla \cdot \vec{J} = 0$

Utilizando coord cilíndricas y aprovechando la simetría del problema $\Rightarrow \vec{J} = J(r,\theta)\hat{\theta}$, luego:

$$\nabla \cdot \vec{J} = \frac{1}{r}\frac{\partial J}{\partial \theta} = 0 \Rightarrow \vec{J} = J(r)\,\hat{\theta}$$

Por otro lado, sabemos:

$$V_0 = -\int \vec{E} \cdot d\vec{l} = -\int_{\theta_1}^{-\theta_2} \vec{E} \cdot r\,d\theta\,\hat{\theta}$$

Como $\vec{J} = \rho \vec{E}$, descomponiendo la integral en cada medio, queda:

$$V_0 = -\left[\int_{\theta_1}^{0} \frac{J_1(r)}{\rho_1}\hat{\theta} \cdot r\,d\theta\,\hat{\theta} + \int_{0}^{-\theta_2} \frac{J_2(r)}{\rho_2}\hat{\theta} \cdot r\,d\theta\,\hat{\theta}\right]$$

$$V_0 = -\left[\frac{J_1(r)\,r}{\rho_1}(0-\theta_1) + \frac{J_2(r)\,r}{\rho_2}(-\theta_2 - 0)\right]$$

$$V_0 = \frac{J_1(r)\,r\,\theta_1}{\rho_1} + \frac{J_2(r)\,r\,\theta_2}{\rho_2}$$

Por condiciones de borde $J_{1n} = J_{2n}$ (válido cuando $\frac{\partial \rho}{\partial t} = 0$).

$$V_0 = J(r)\,r\left(\frac{\theta_1}{\rho_1} + \frac{\theta_2}{\rho_2}\right)$$

b) $$\vec{J}(r) = \frac{V_0}{r\left(\frac{\theta_1}{\rho_1} + \frac{\theta_2}{\rho_2}\right)}\,\hat{\theta}$$

Luego, se puede calcular $\vec{E}$ en cada material:

$$\vec{E}_1(r) = \frac{V_0}{r\,\rho_1\left(\frac{\theta_1}{\rho_1}+\frac{\theta_2}{\rho_2}\right)}\,\hat{\theta}$$

$$\vec{E}_2(r) = \frac{V_0}{r\,\rho_2\left(\frac{\theta_1}{\rho_1}+\frac{\theta_2}{\rho_2}\right)}\,\hat{\theta}$$

a) $$\vec{E}(r,\theta) = \begin{cases} \vec{E}_1(r) & \text{si } \theta\in[0,\theta_1] \\ \vec{E}_2(r) & \text{si } \theta\in[-\theta_2,0] \end{cases}$$

c) Se sabe que la densidad de carga superficial en la interfaz, está dada por:

"El que sale menos el que entra" $\rightarrow$ $D_{2n} - D_{1n} = \sigma_L \quad / \ \vec{D} = \varepsilon\vec{E}$

$$\varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = \sigma_L \quad / \ \vec{J} = \rho\vec{E}$$

$$\frac{\varepsilon_2}{\rho_2} J_{2n} - \frac{\varepsilon_1}{\rho_1} J_{1n} = \sigma_L \quad / \ J_{1n} = J_{2n} = J$$

$$\left(\frac{\varepsilon_2}{\rho_1} - \frac{\varepsilon_1}{\rho_2}\right) J = \sigma_L$$

$$\Rightarrow \quad \sigma_L(r) = \left(\frac{\varepsilon_2}{\rho_1} - \frac{\varepsilon_1}{\rho_2}\right)\cdot\frac{V_0}{r\left(\frac{\theta_1}{\rho_1}+\frac{\theta_2}{\rho_2}\right)}$$

d) La potencia se puede calcular como

$$P = \iiint_{\Omega} \vec{E} \cdot \vec{J} \, dV \qquad \text{ó} \qquad P = VI$$

(si se conocen los campos) (si se conocen V e I)

De la primera forma:

$$P = \int_0^h \int_a^b \int_{-\theta_2}^{\theta_1} E(r,\theta) \hat{\theta} \cdot J(r) \hat{\theta} \; r \, d\theta \, dr \, dz$$

Sea $\xi = \dfrac{V_0}{\frac{\theta_1}{\sigma_1} + \frac{\theta_2}{\sigma_2}} \Rightarrow J(r) = \dfrac{\xi}{r}$; $E_1(r) = \dfrac{\xi}{\sigma_1 r}$; $E_2(r) = \dfrac{\xi}{\sigma_2 r}$

$$P = h \cdot \int_a^b \Bigg[\underbrace{\int_0^{\theta_1} \frac{\xi}{\sigma_1 r} \cdot \frac{\xi}{r} \, r \, d\theta}_{\frac{\xi^2}{\sigma_1 r}(\theta_1 - 0)} + \underbrace{\int_{-\theta_2}^{0} \frac{\xi}{\sigma_2 r} \cdot \frac{\xi}{r} \cdot r \, d\theta}_{\frac{\xi^2}{\sigma_2 r}(0 - -\theta_2)} \Bigg] dr$$

$$P = h\xi^2 \int_a^b \left(\frac{\theta_1}{\sigma_1 r} + \frac{\theta_2}{\sigma_2 r} \right) dr$$

$$P = \underbrace{h \xi^2}_{\frac{V_0^2}{\left(\frac{\theta_1}{\sigma_1} + \frac{\theta_2}{\sigma_2}\right)^2}} \left(\frac{\theta_1}{\sigma_1} + \frac{\theta_2}{\sigma_2} \right) \ln\left(\frac{b}{a}\right)$$

$$P = \frac{h V_0^2 \ln\left(\frac{b}{a}\right)}{\frac{\theta_1}{\sigma_1} + \frac{\theta_2}{\sigma_2}}$$

De la segunda forma:

$$I = \iint \vec{J} \cdot \vec{dS} = \int_a^b \int_0^h \frac{V_0}{r\left(\frac{\theta_1}{\sigma_1} + \frac{\theta_2}{\sigma_2}\right)} \hat{\phi} \cdot dz\, dr\, \hat{\phi}$$

e) $$I = \frac{h V_0 \ln\left(\frac{b}{a}\right)}{\frac{\theta_1}{\sigma_1} + \frac{\theta_2}{\sigma_2}}$$

$$P = VI = \frac{h V_0^2 \ln\left(\frac{b}{a}\right)}{\frac{\theta_1}{\sigma_1} + \frac{\theta_2}{\sigma_2}}$$

f) Por ley de Ohm

$$R = \frac{V}{I} = \frac{V_0}{\frac{h V_0 \ln\left(\frac{b}{a}\right)}{\frac{\theta_1}{\sigma_1} + \frac{\theta_2}{\sigma_2}}} = \frac{\frac{\theta_1}{\sigma_1} + \frac{\theta_2}{\sigma_2}}{h \ln\left(\frac{b}{a}\right)}$$