Ingeniería Eléctrica
FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS
UNIVERSIDAD DE CHILE

FI 2002
ELECTROMAGNETISMO
Clase 15
Corriente Eléctrica-III

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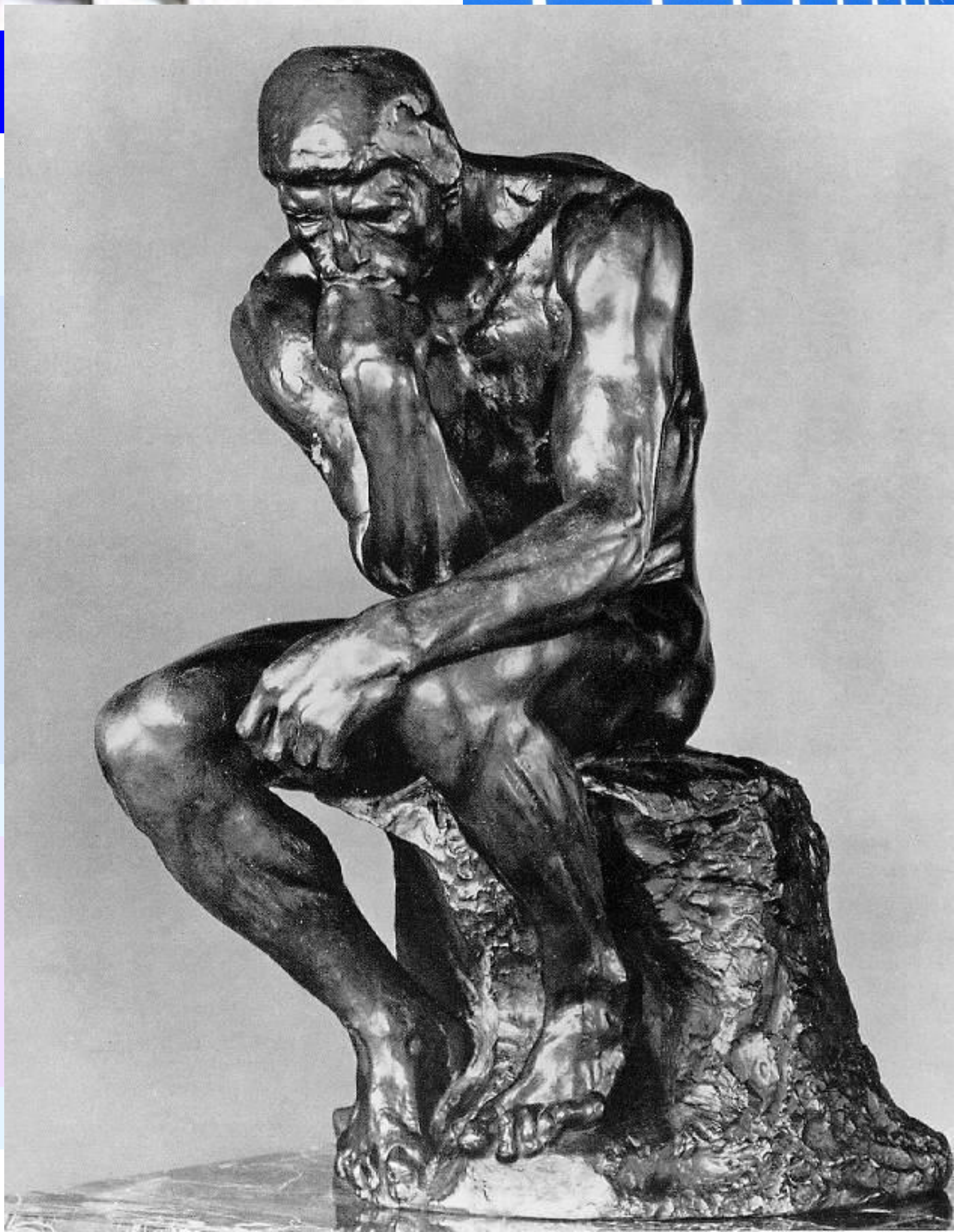
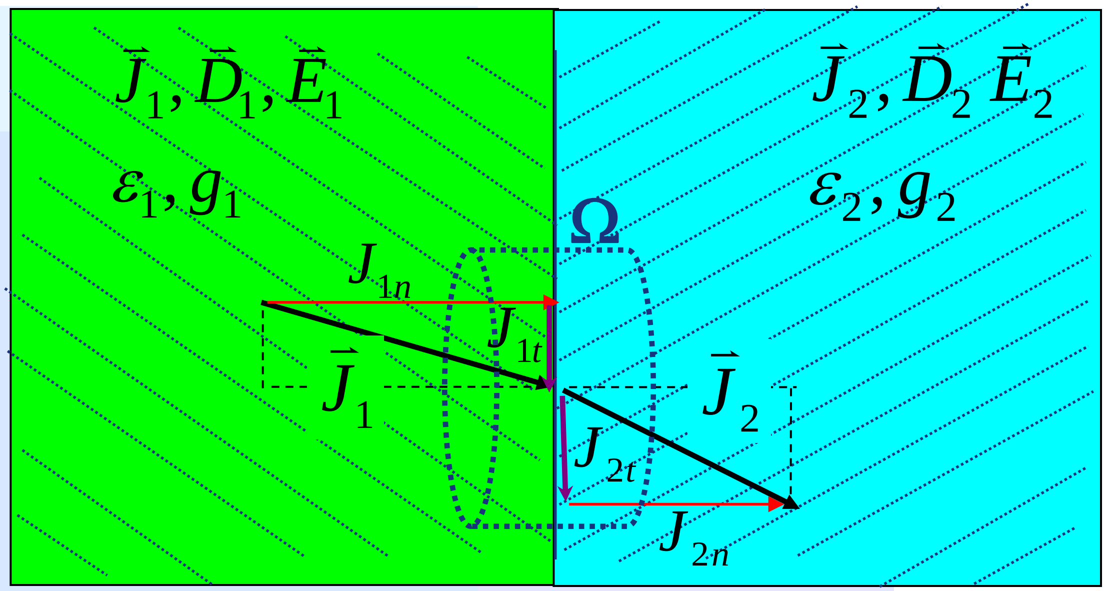


Auguste Rodin, “El Pensador”

Condiciones de Borde para $\vec{J}$

$$\nabla \times \vec{E} = 0$$

$$E_{1t} = E_{2t} \Rightarrow \frac{J_{1t}}{g_1} = \frac{J_{2t}}{g_2}$$



$$\nabla \bullet \vec{D} = \rho$$

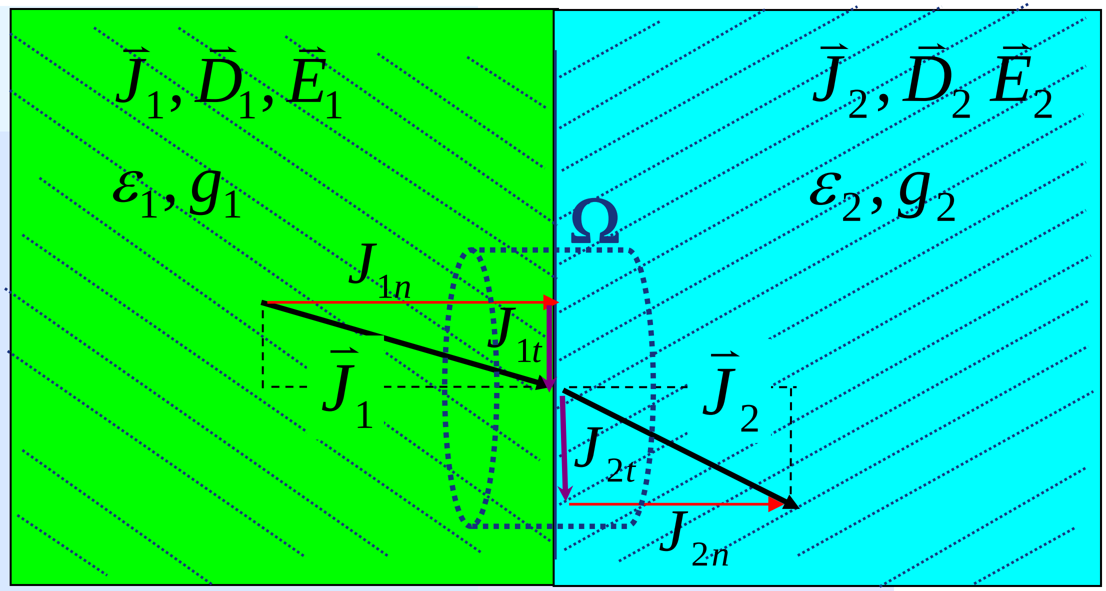
$$D_{1N} - D_{2N} = \sigma_{libre}$$

$$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \sigma_l \quad \Rightarrow \quad \varepsilon_1 \frac{J_{1n}}{g_1} - \varepsilon_2 \frac{J_{2n}}{g_2} = \sigma_l$$

Condiciones de Borde para $\vec{J}$

$$\frac{J_{1t}}{g_1} = \frac{J_{2t}}{g_2}$$

$$\varepsilon_1 \frac{J_{1n}}{g_1} - \varepsilon_2 \frac{J_{2n}}{g_2} = \sigma_l$$



I. Situación Estacionaria $\frac{\partial \rho(t)}{\partial t} = 0$

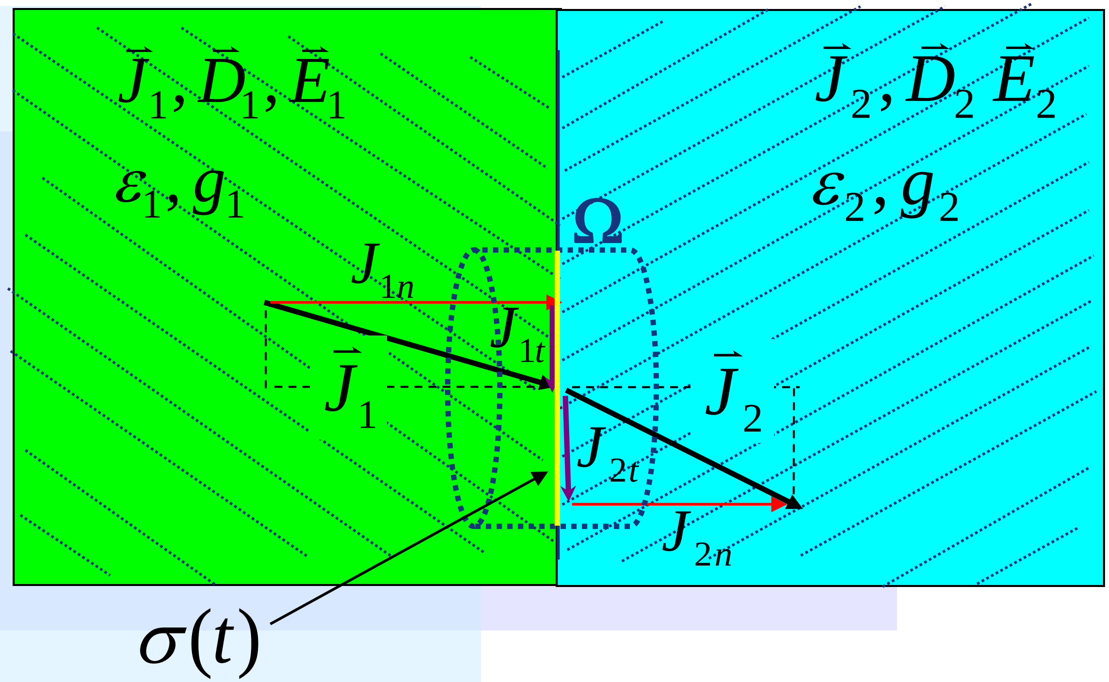
$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad \Rightarrow \nabla \cdot \vec{J} = 0 \qquad \oiint_{S(\Omega)} \vec{J} \bullet d\vec{s} = 0 \Rightarrow J_{1n} = J_{2n}$$

Condiciones de Borde para $\vec{J}$

II. Situación transitoria

$$\frac{\partial \rho(t)}{\partial t} \neq 0$$

$$\oiint_{S(\Omega)} \vec{J} \cdot d\vec{S} = J_{2n}\Delta S - J_{1n}\Delta S$$



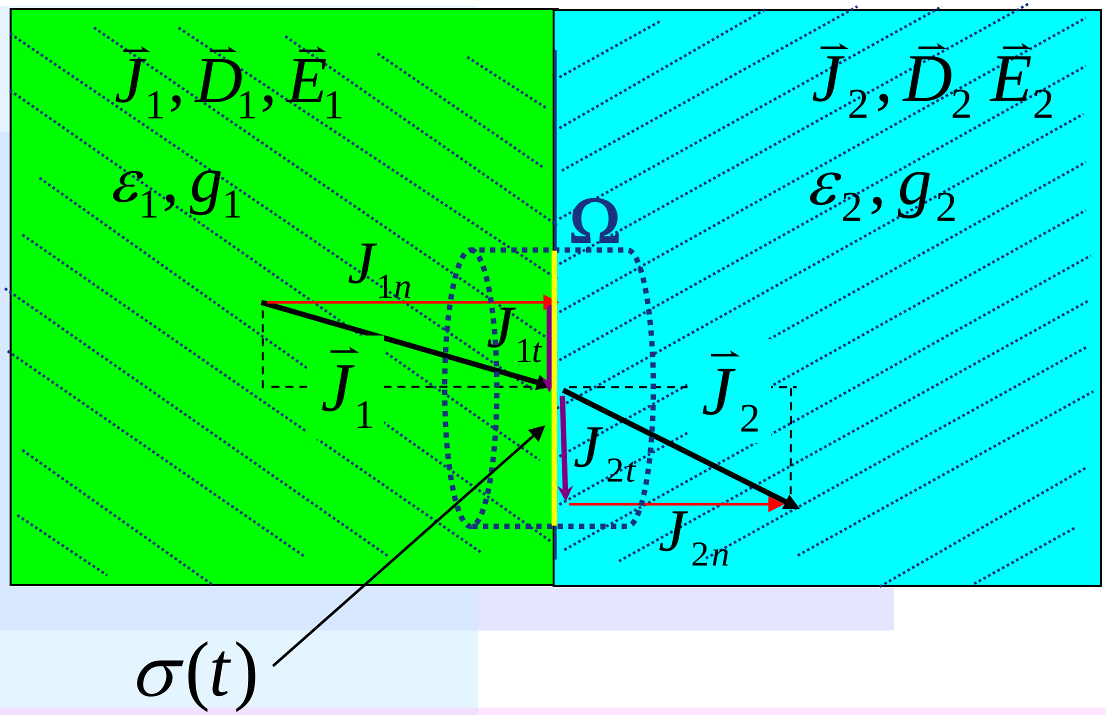
Haciendo tender la altura del cilindro a cero $\frac{\partial Q}{\partial t}$ se acumula sólo en la superficie que limita los medios

Condiciones de Borde para $\vec{J}$

$$\Rightarrow \frac{\partial Q}{\partial t} = \frac{\partial}{\partial t}\left(\sigma \cdot \Delta S\right)$$

$$\Rightarrow J_{2n}\Delta S - J_{1n}\Delta S + \frac{\partial \sigma}{\partial t}\Delta S = 0$$

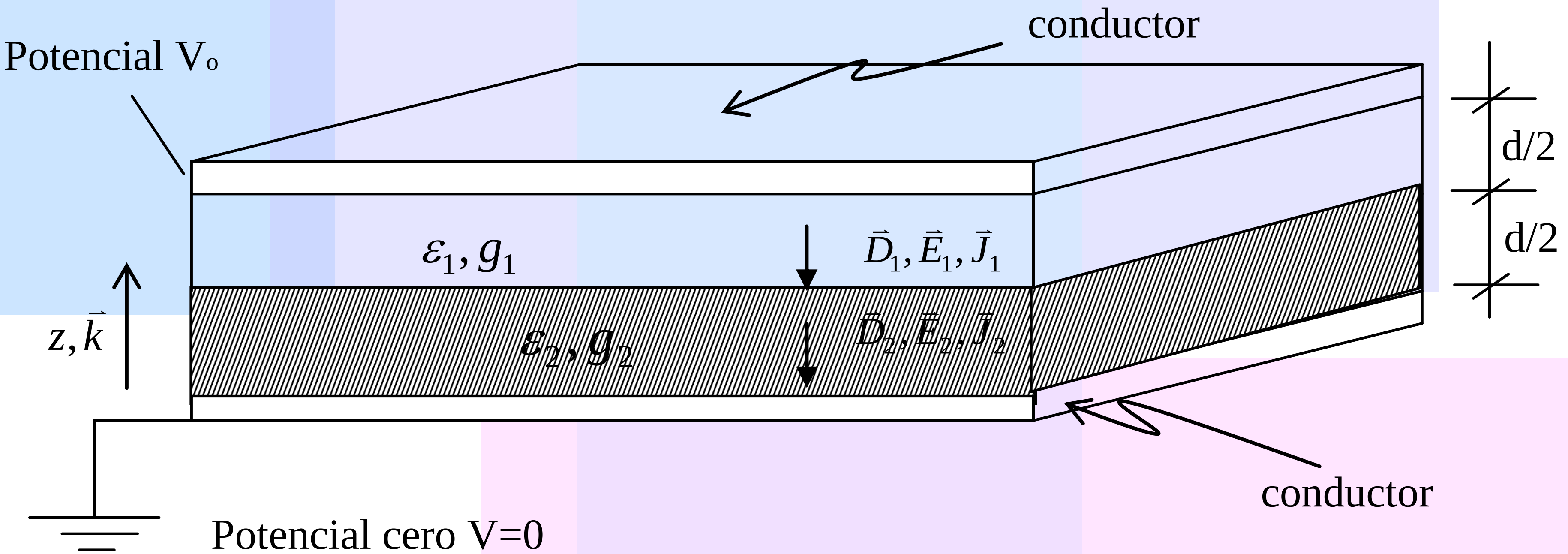
$$\Rightarrow J_{2n} - J_{1n} + \frac{\partial \sigma}{\partial t} = 0$$



$$\therefore \quad J_{2n} - J_{1n} = \frac{\partial \sigma(t)}{\partial t}$$

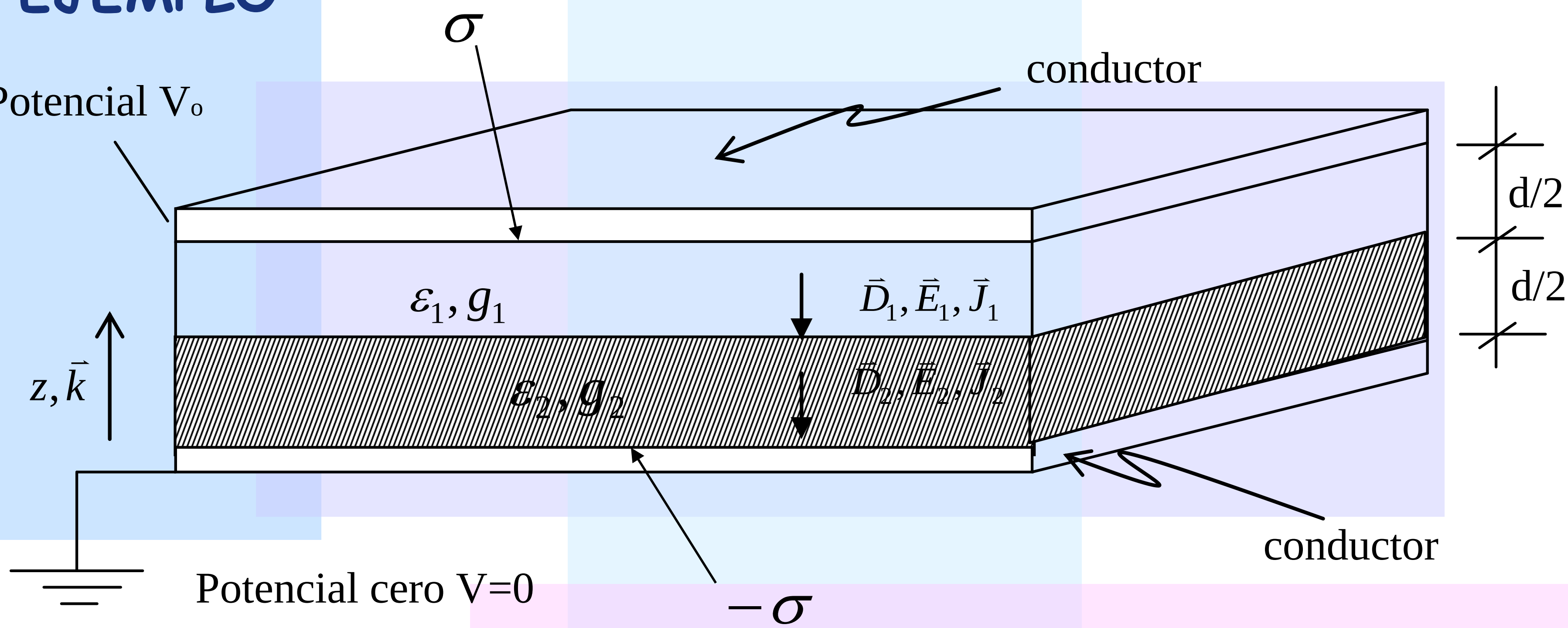
Condiciones de Borde para $\vec{J}$

EJEMPLO: Encontrar campos y densidad de carga en conductor (situación estacionaria)



Condiciones de Borde para $\vec{J}$

EJEMPLO



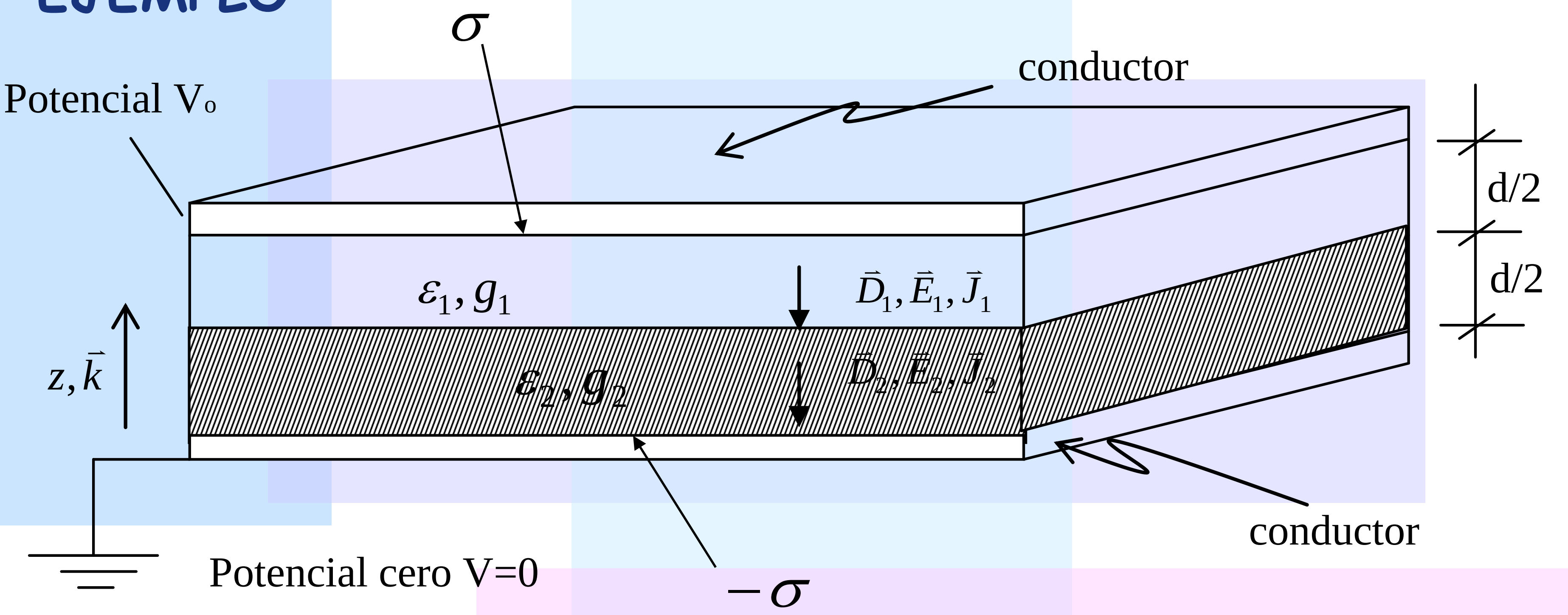
$$J_{1n} = J_{2n} \Rightarrow \frac{E_1}{g_1} = \frac{E_2}{g_2}$$

y

$$\Delta V = -\int \vec{E} \bullet dz\hat{k} \Rightarrow V_0 = E_1 \frac{d}{2} + E_2 \frac{d}{2}$$

Condiciones de Borde para $\vec{J}$

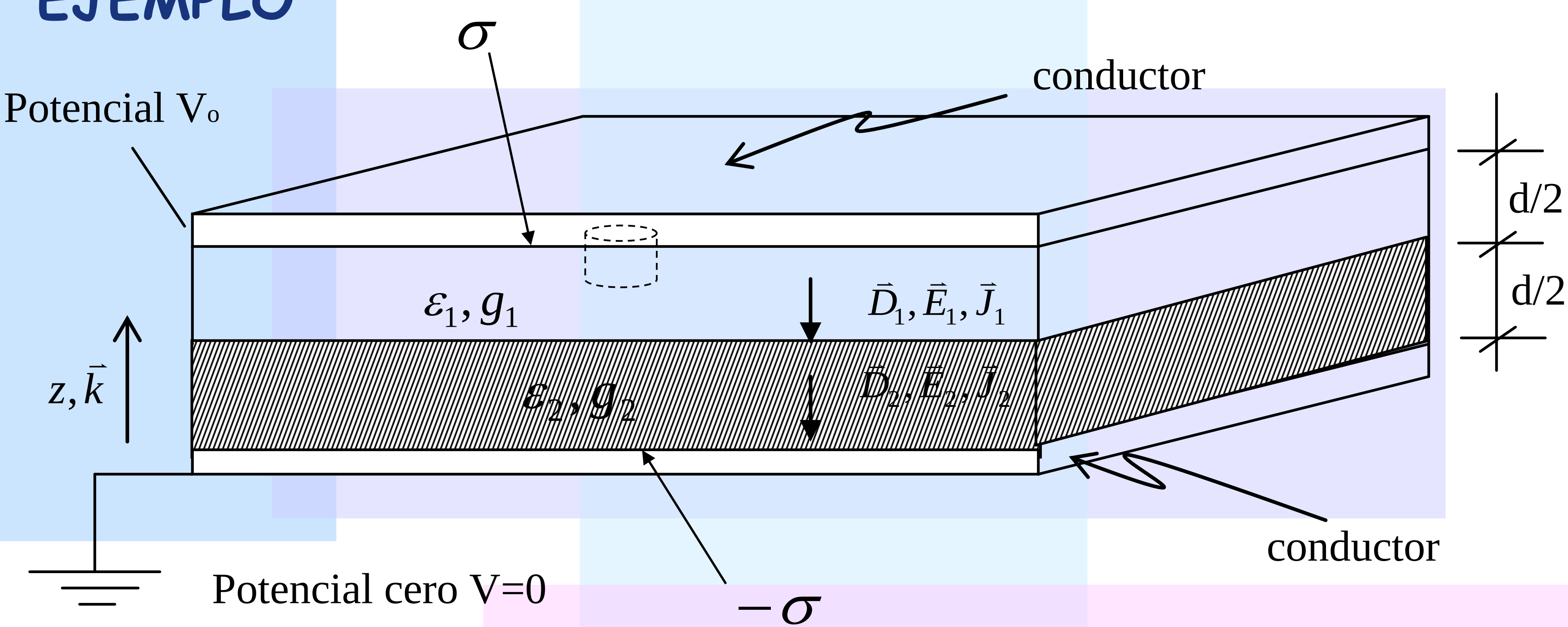
EJEMPLO



$$\Rightarrow E_1 = \frac{2g_1}{d(g_1 + g_2)} V_0 \qquad E_2 = \frac{2g_2}{d(g_1 + g_2)} V_0$$

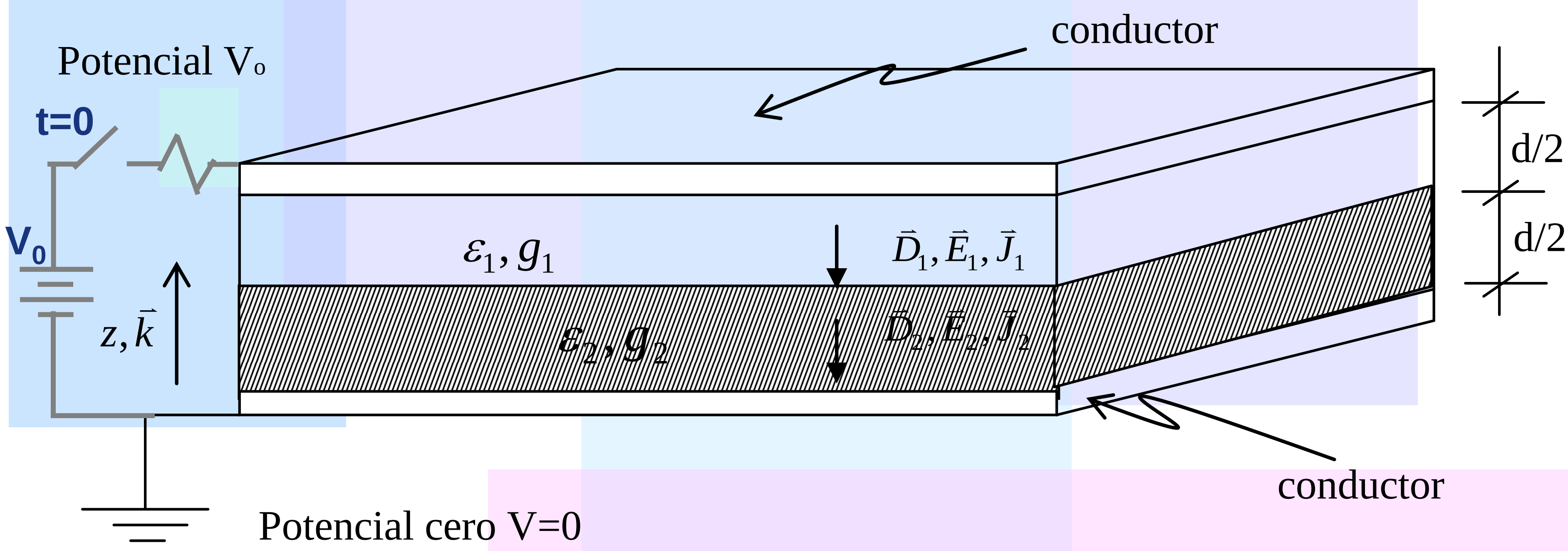
Condiciones de Borde para $\vec{J}$

EJEMPLO



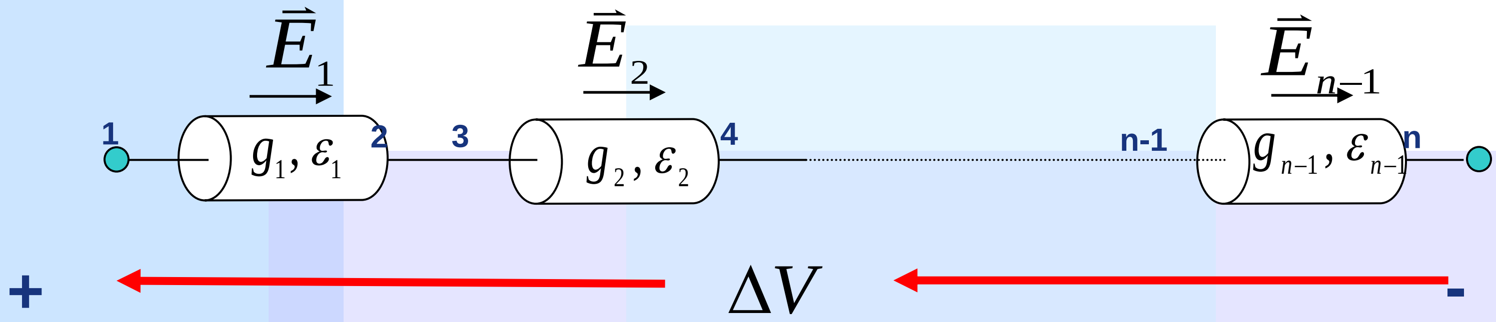
$$\iint_S \vec{D} \cdot d\vec{s} = Q_T \quad \Rightarrow \quad \sigma = \varepsilon_1 E_1 = \frac{2\varepsilon_1 g_1}{d(g_1 + g_2)} V_0$$

EJEMPLO Propuesto



Encontrar la distribución de carga superficial en función del tiempo.

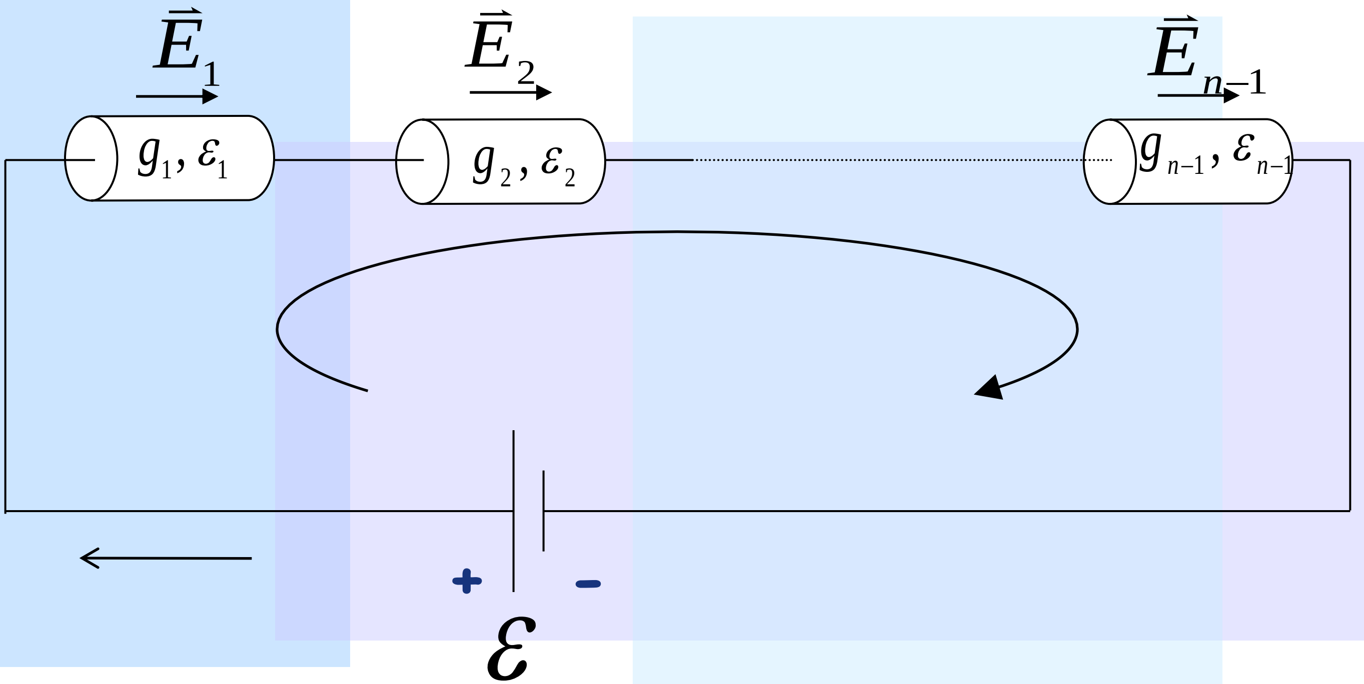
Ley de Voltajes de Kirchoff



$$\Delta V = \int_1^2 \vec{E}_1 \cdot d\vec{l} + \int_3^4 \vec{E}_2 \cdot d\vec{l} + \ldots + \int_{n-1}^{n} \vec{E}_{n-1} \cdot d\vec{l} \quad (5.47)$$

$$\Delta V = \sum E_i l_i = (V_1 - V_2) + (V_3 - V_4) + \ldots + (V_{n-1} - V_n) \quad (5.48)$$

Ley de Voltajes de Kirchoff

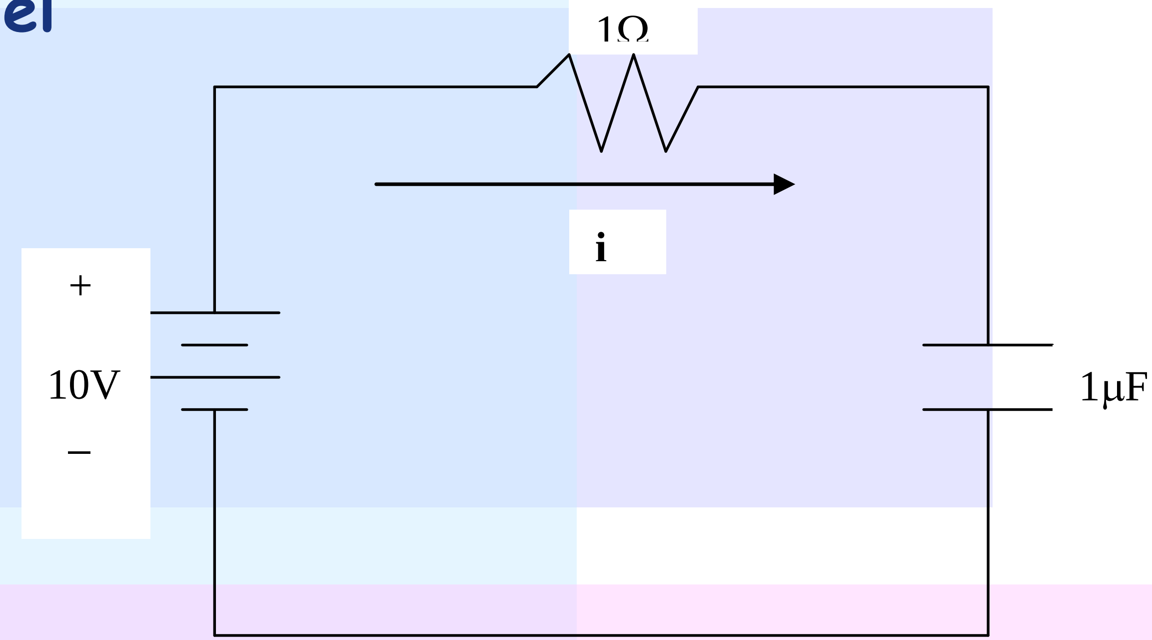


$$\Delta V = \varepsilon \quad \Rightarrow \sum \Delta V_i - \varepsilon = 0$$

Ley de Voltajes de Kirchoff

Ejemplo

Encontrar el valor de la corriente en función del tiempo si inicialmente el condenador tiene una carga Q_0

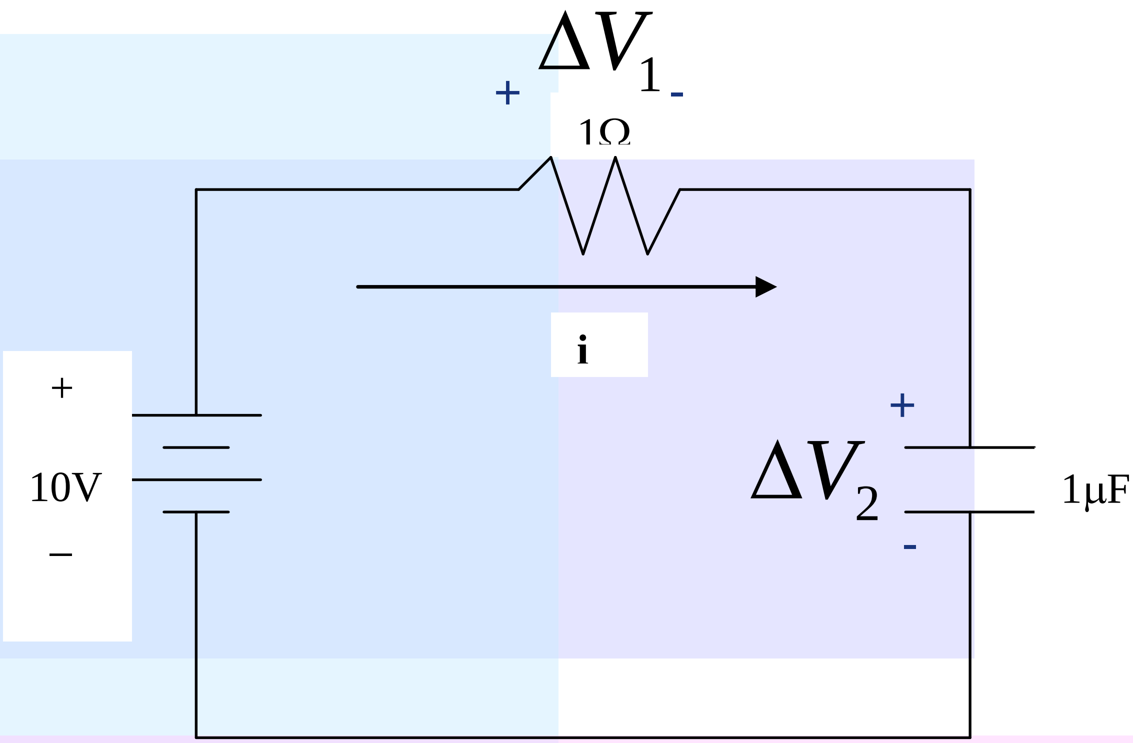


Ejemplo

$$\varepsilon = 10,$$

$$\Delta V_1 = Ri,$$

$$\Delta V_2 = V_c$$



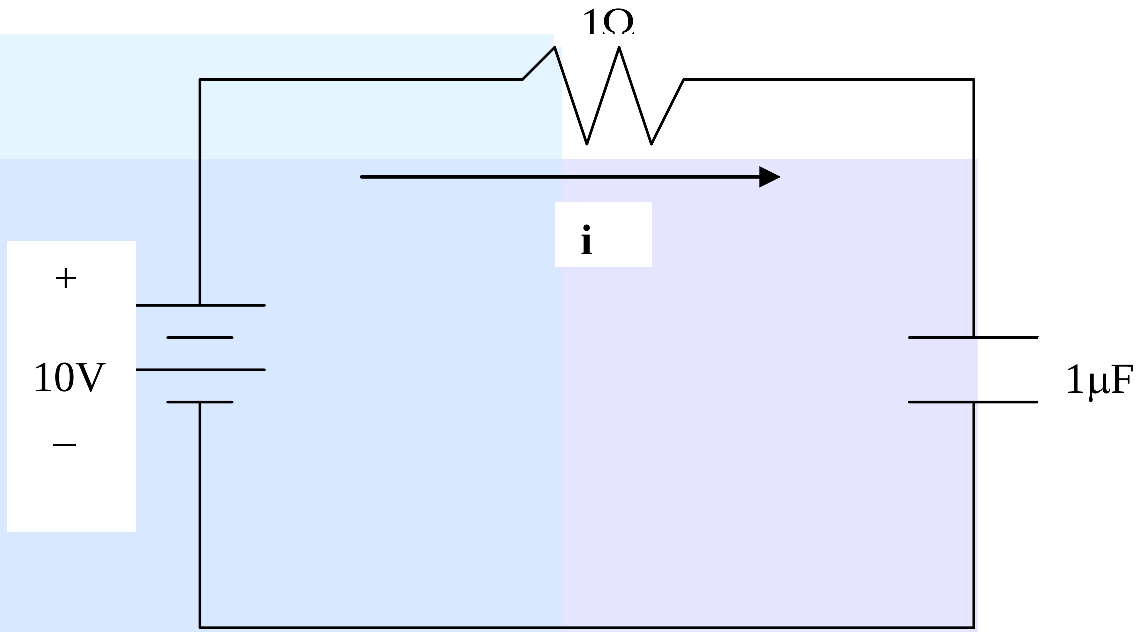
$$\sum \Delta V_i - \varepsilon = 0 \Rightarrow \quad Ri + V_c - 10 = 0$$

Ejemplo

$$i = i_c = \frac{dq_c}{dt}$$

$$q_c = CV_c$$

$$\Rightarrow i = C\frac{dV_c}{dt}$$

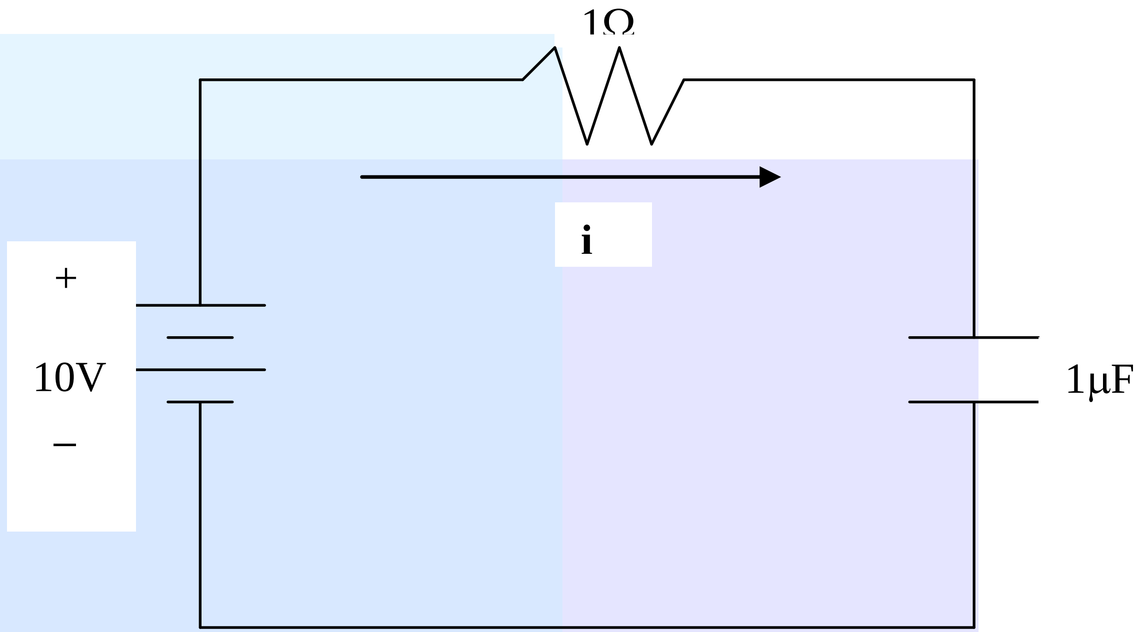


Teníamos

$$Ri + V_c - 10 = 0 \Rightarrow RC\frac{dV_c}{dt} + V_c = 10 \quad \Rightarrow 10^{-6}\frac{dV_c}{dt} + V_c = 10$$

Ejemplo

$$\Rightarrow 10^{-6}\frac{dV_c}{dt} + V_c = 10$$



Solución homogénea

$$10^{-6}\frac{dV_c}{dt} + V_c = 0$$

$$V_c = Ae^{-10^6 t}$$

Solución particular

$$V_c = 10$$

Solución

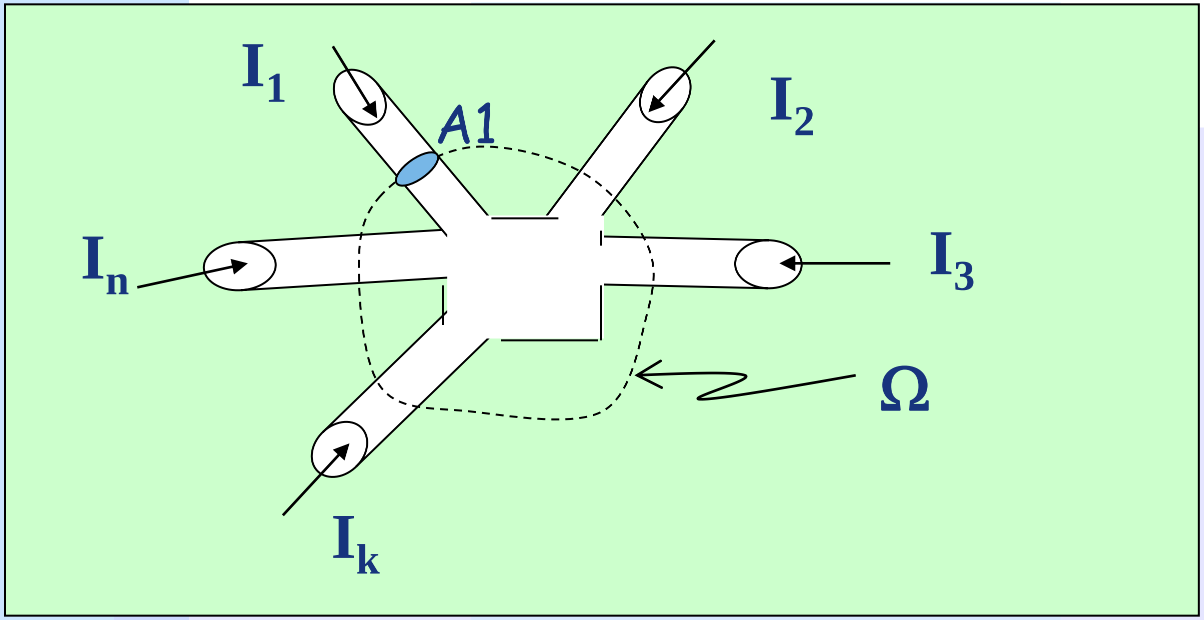
$$V_c = Ae^{-10^6 t} + 10$$

C.I.

$$Q_0 = CV_c(t=0)$$

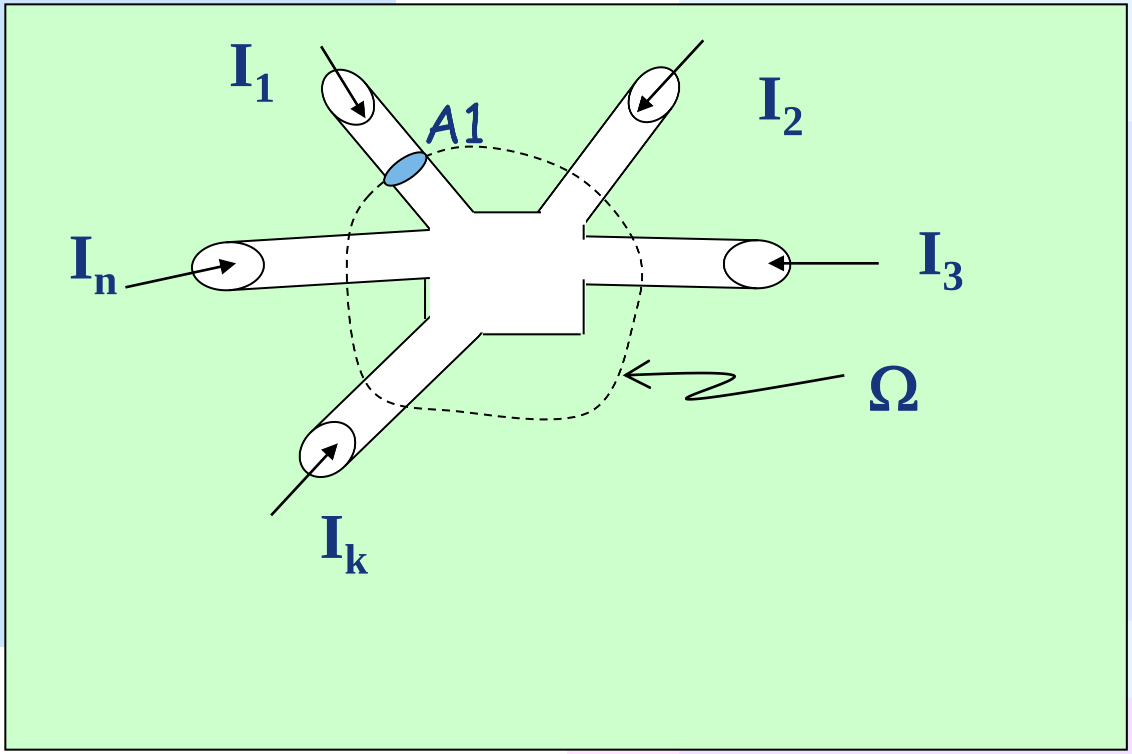
$$\Rightarrow V_c(t=0) = 10^{-6} Q_0$$

Ley de Corrientes de Kirchoff



$$\frac{\partial \rho}{\partial t} = 0 \Rightarrow \nabla \cdot \vec{J} = 0 \quad \Rightarrow \iiint_{\Omega} \nabla \cdot \vec{J} dV = 0 \Rightarrow \oiint_{S(\Omega)} \vec{J} \bullet d\vec{s} = 0$$

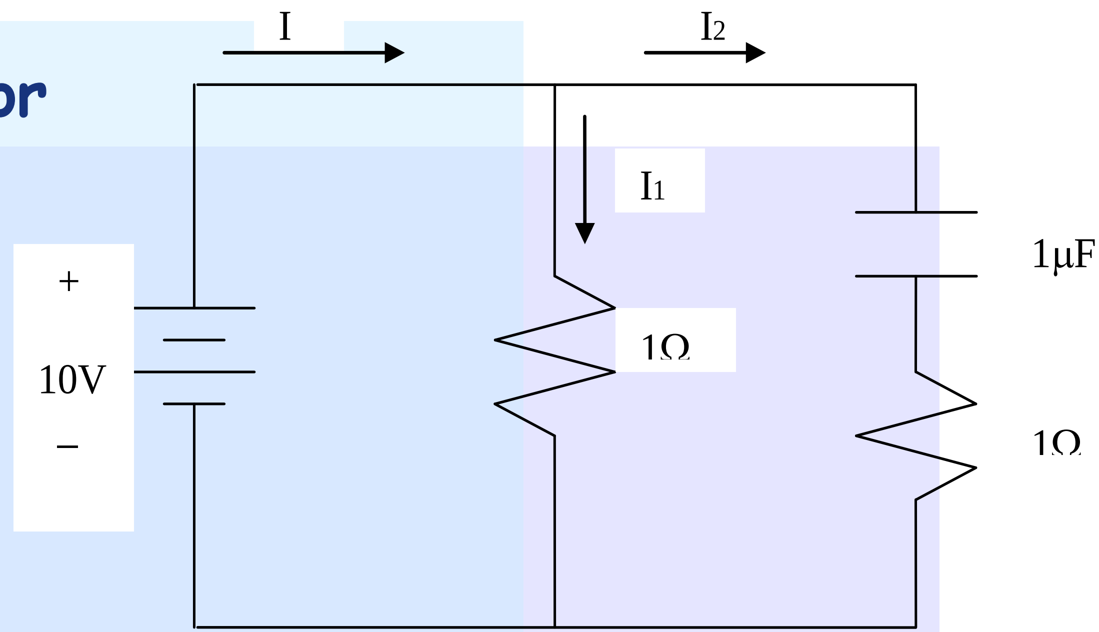
Ley de Corrientes de Kirchoff



$$\oiint_{S(\Omega)} \vec{J} \bullet d\vec{s} = 0 \Rightarrow \sum_{k=1}^{n} I_k = 0$$

Ejemplo

Encontrar el valor del potencial del condensador en función del tiempo si inicialmente éste se encuentra descargado



Sol^n

LCK $I = I_1 + I_2$

$$I_1 = \frac{V_{R1}}{R} = \frac{10V}{1\Omega} = 10\left[A\right]$$

$$I_2 = \frac{V_{R2}}{R} = \frac{10 - V_C}{1}$$

Además $I_2 = C\frac{dV_C}{dt}$

$$\Rightarrow C\frac{dV_c}{dt} + V_c = 10$$

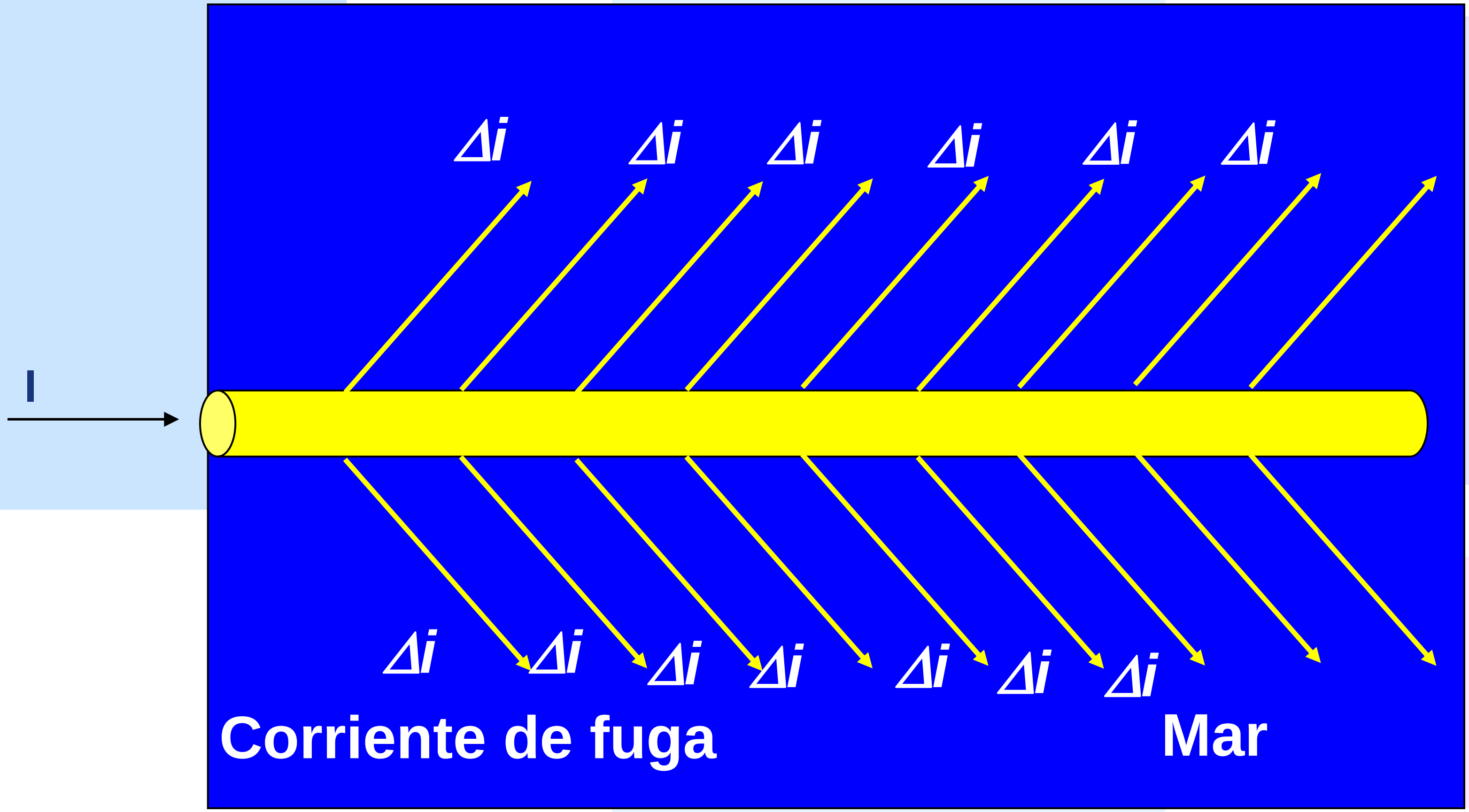
$$\Rightarrow V_c(t) = 10 + Ae^{-t/C}$$

CB $V_c(t=0) = \frac{Q_0}{C} = 0$

$$\therefore V_c(t) = 10(1 - e^{-t/C})$$

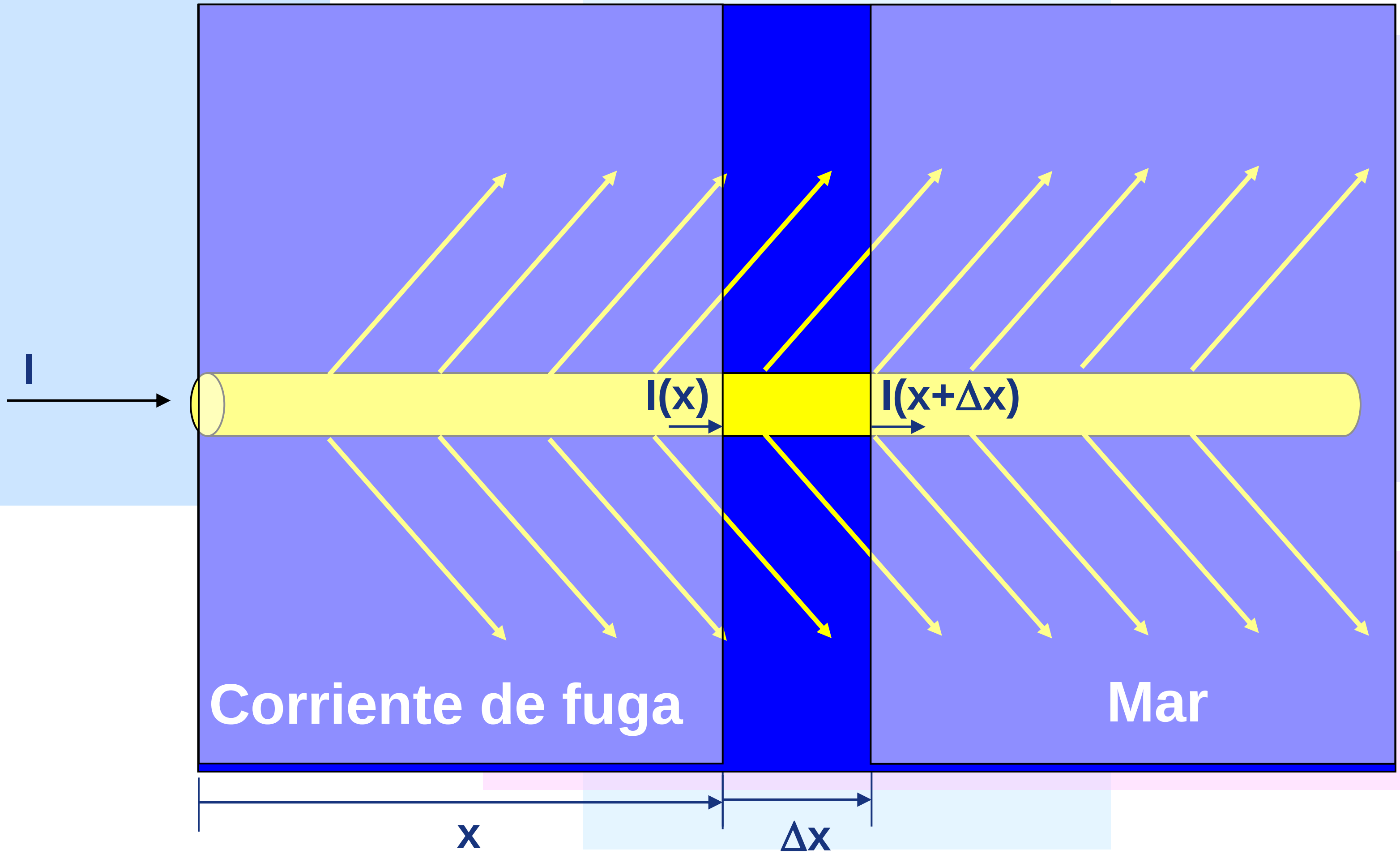
Ejemplo: Cable submarino

Se tiene un cable submarino con corriente de fuga

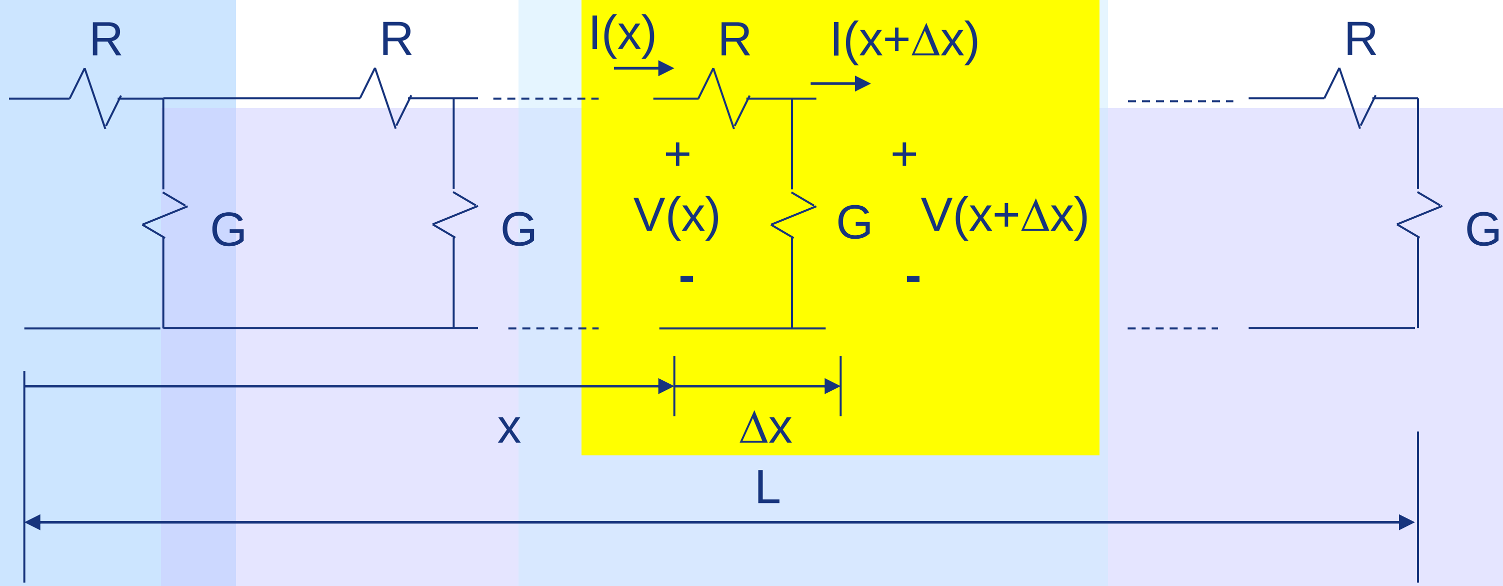


Ejemplo: Cable submarino

Modelamos un elemento diferencial Δx del cable

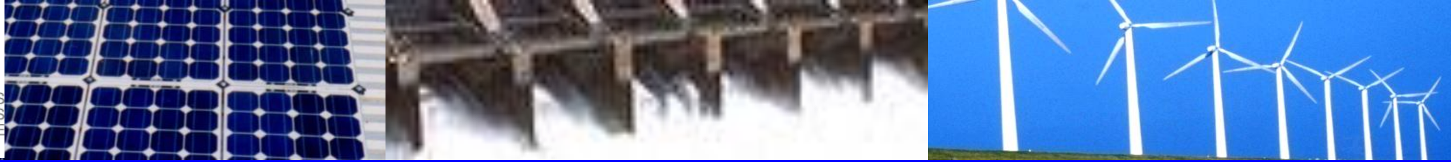


Ejemplo: Cable submarino

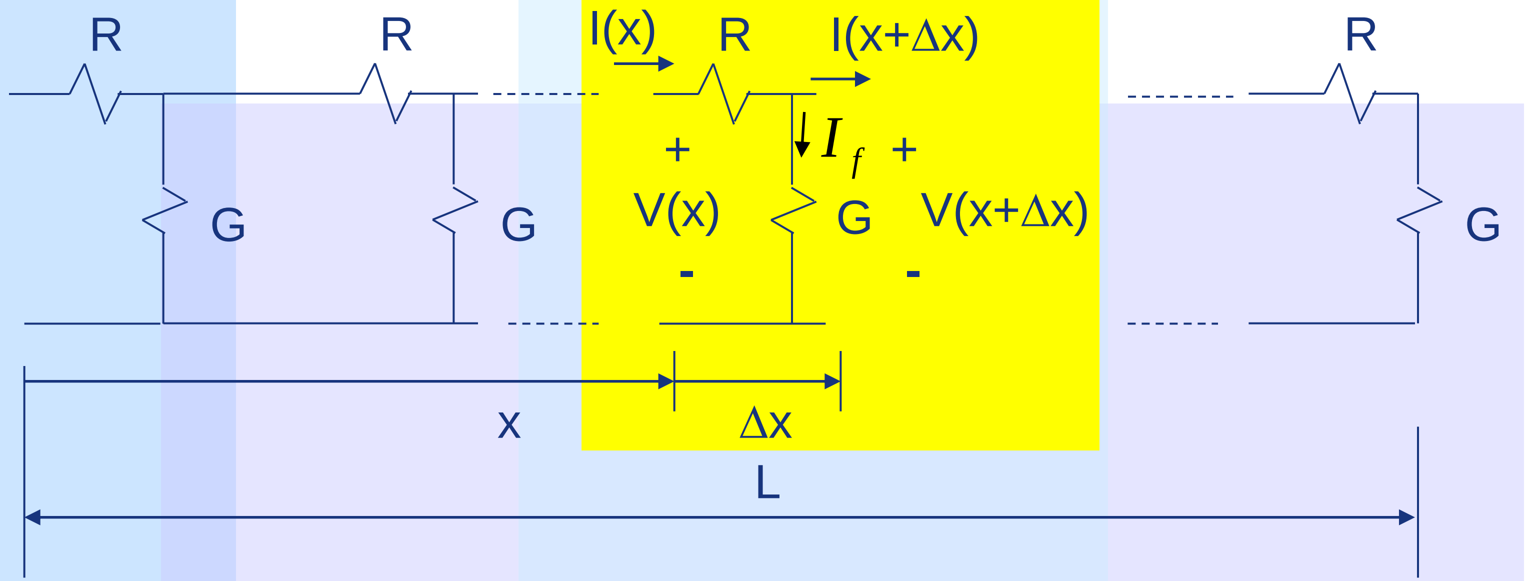


$$R = \text{Resistencia serie por unidad de largo} \left[\frac{\text{Ohm}}{\text{m}} \right]$$

$$G = \text{Conductancia de fuga por unidad de largo} \left[\frac{1}{\text{Ohm} \times \text{m}} \right]$$



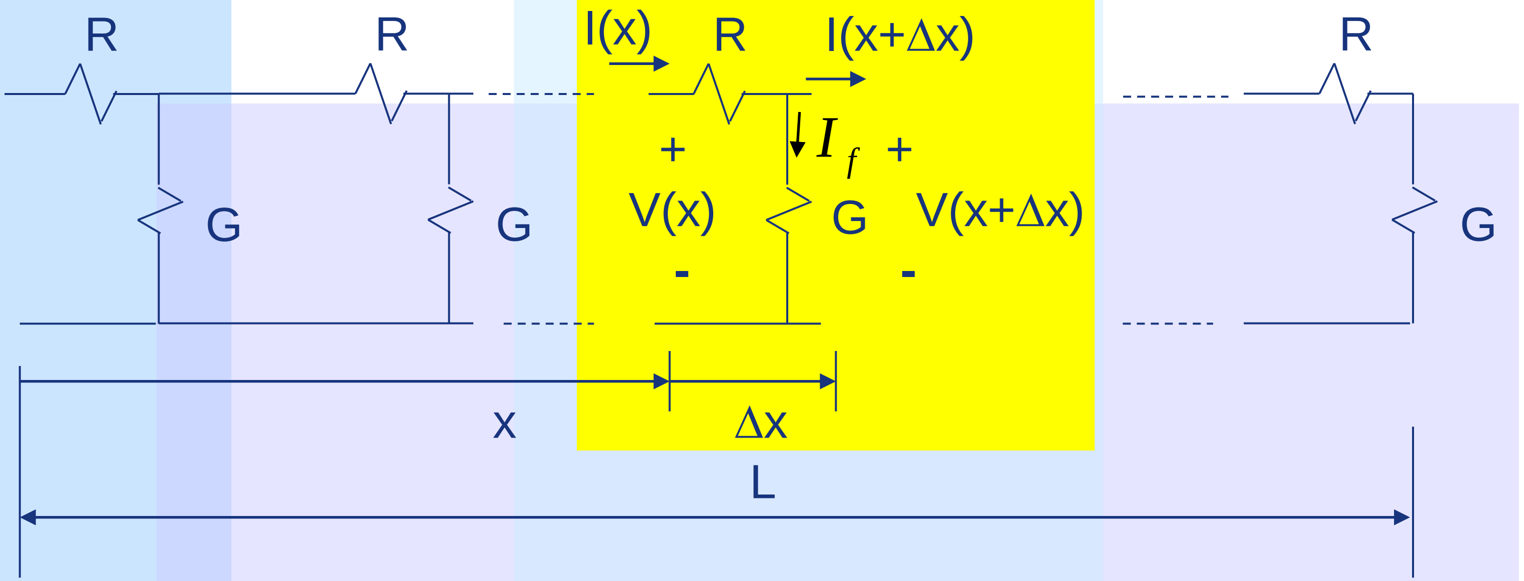
Ejemplo: Cable submarino



LCK:

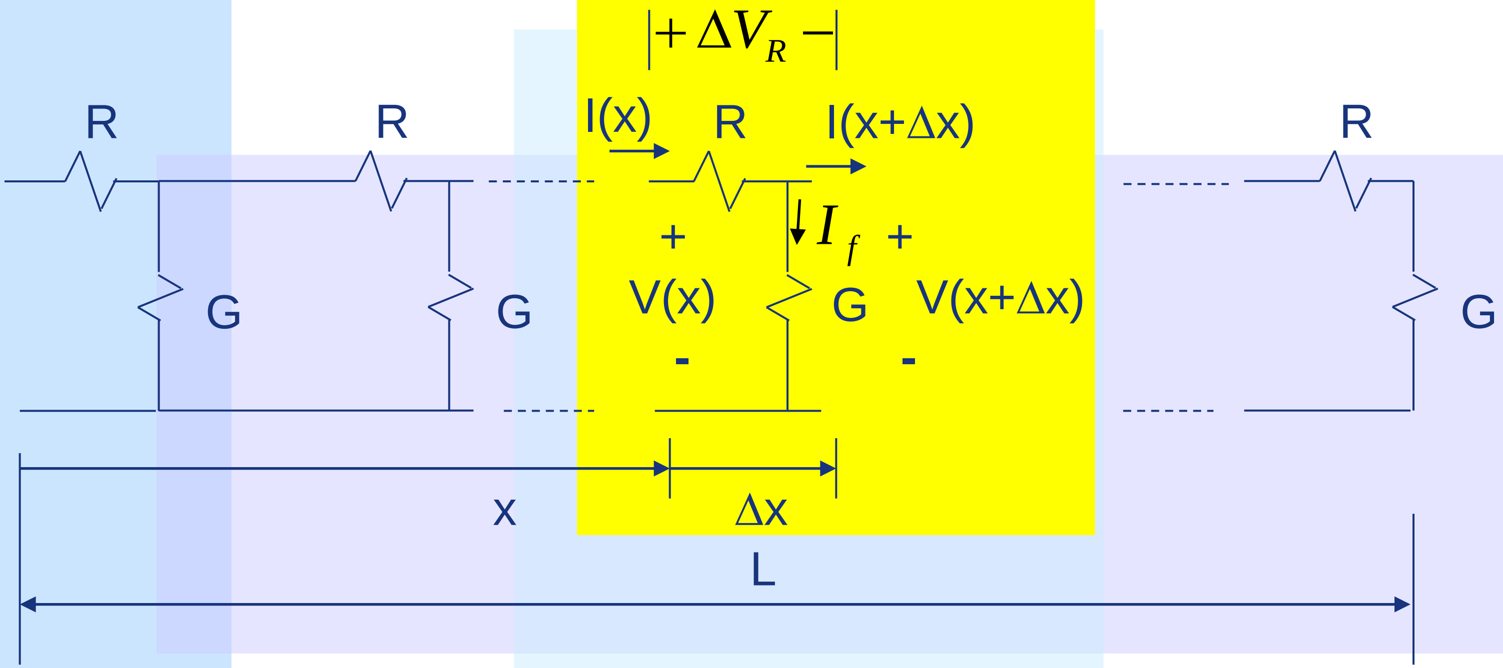
$$I(x) = I(x + \Delta x) + I_f$$

Ejemplo: Cable submarino



LCK: $$I(x) = I(x+\Delta x) + I_f$$

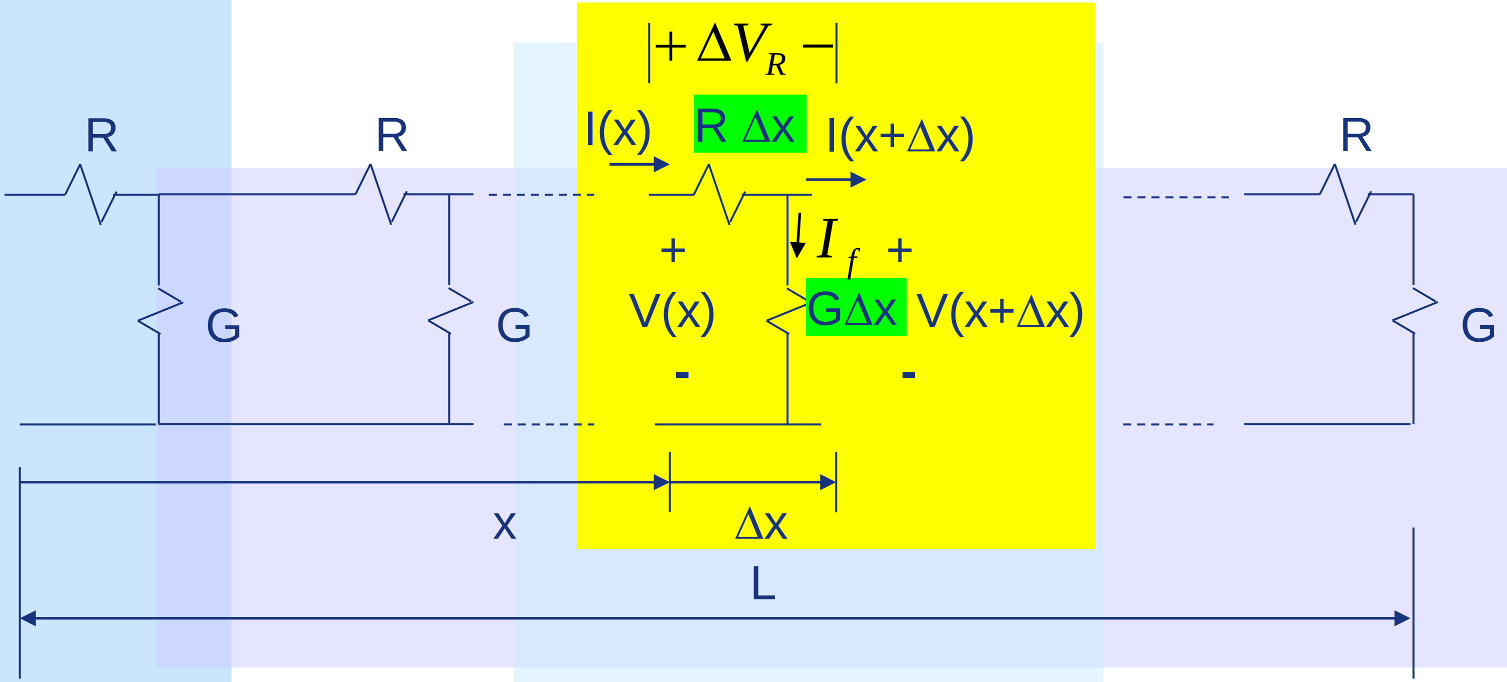
Ejemplo: Cable submarino



LCK: $$I(x) = I(x+\Delta x) + I_f$$

LVK: $$V(x) = \Delta V_R + V(x+\Delta x)$$

Ejemplo: Cable submarino

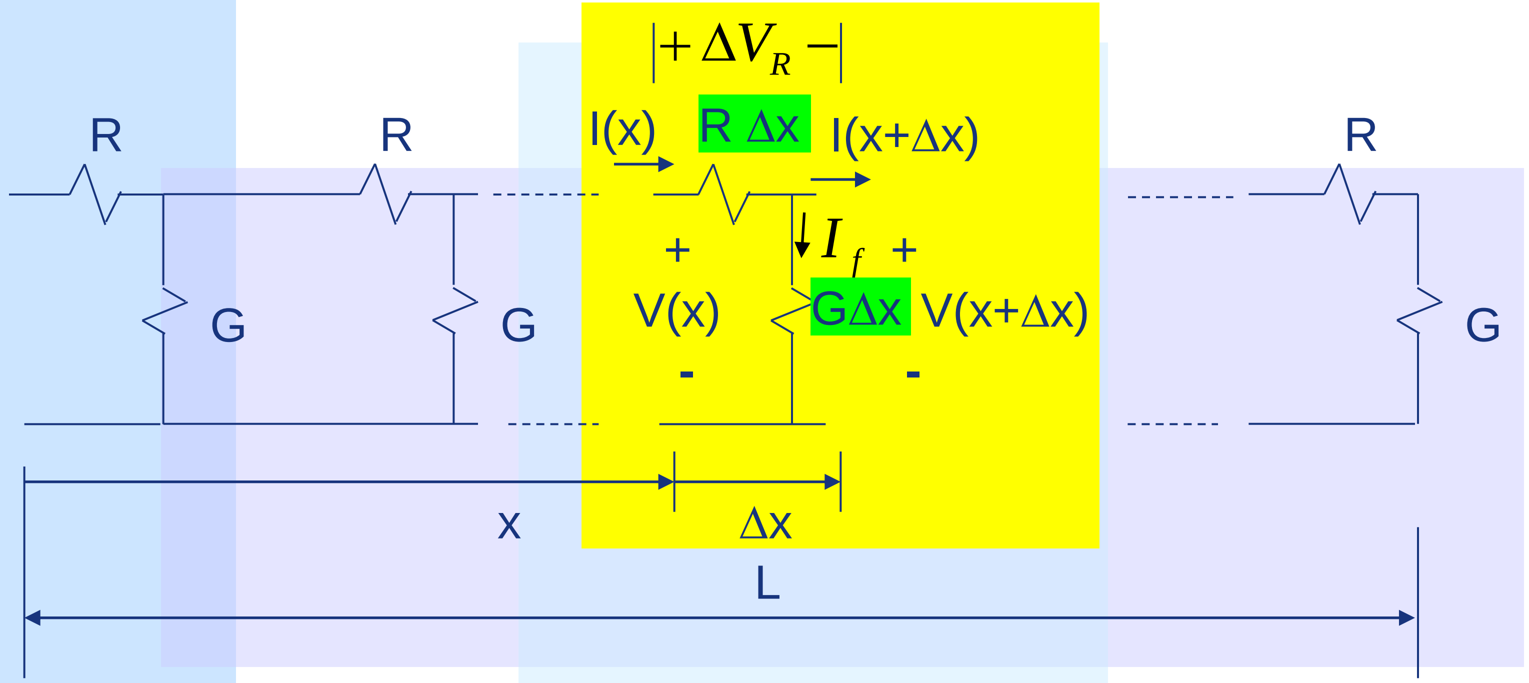


LCK:

$$I(x) = I(x+\Delta x) + I_f \qquad (G\Delta x)v(x+\Delta x) = I_f$$

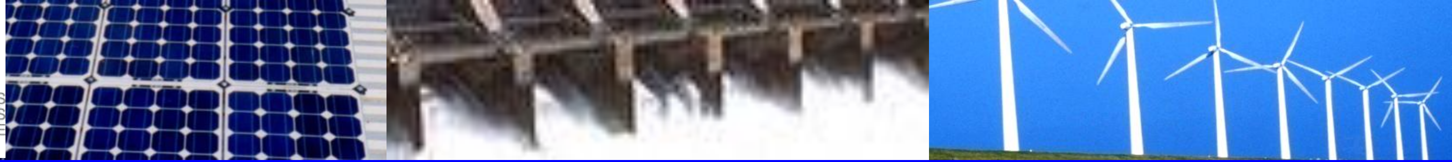
$$\Rightarrow I(x) = I(x+\Delta x) + G\Delta x V(x+\Delta x)$$

Ejemplo: Cable submarino

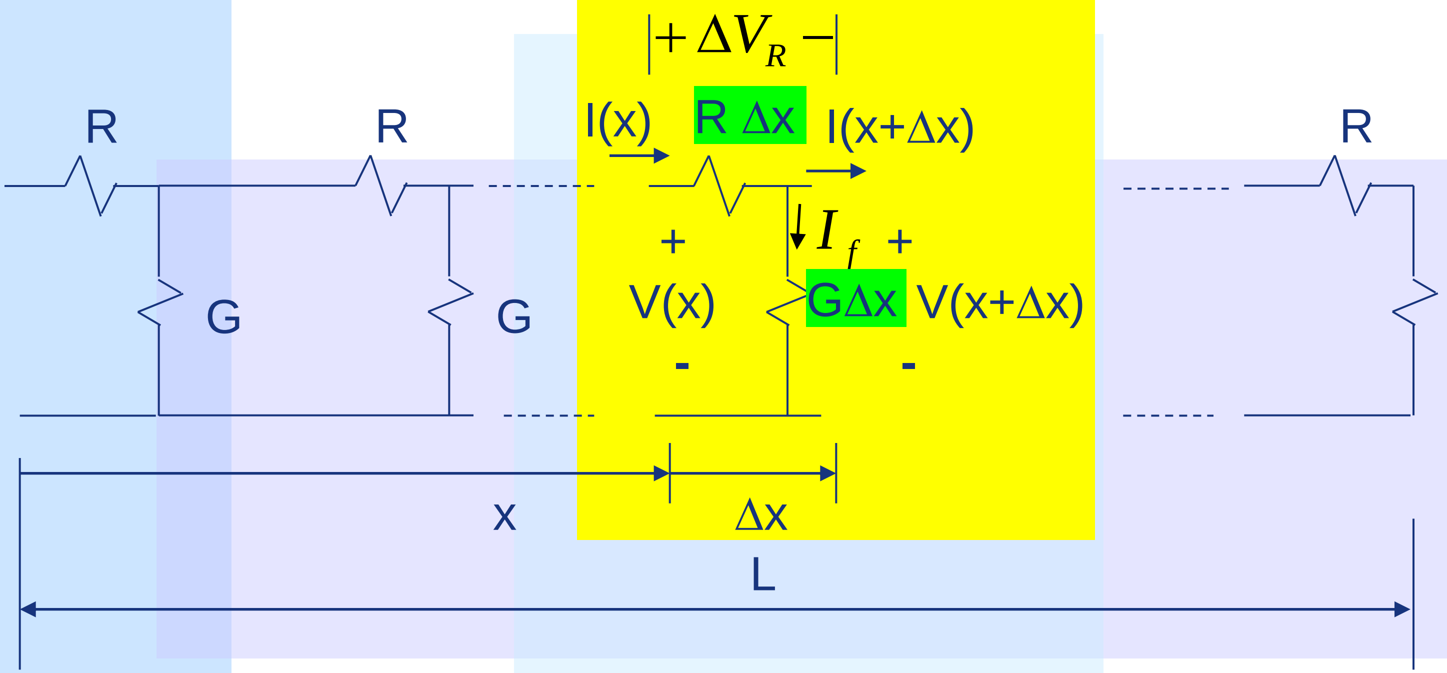


LVK: $$V(x) = \Delta V_R + V(x+\Delta x) \qquad \Delta V_R = (R\Delta x) I(x)$$

$$\Rightarrow V(x) = (R\Delta x) I(x) + V(x+\Delta x)$$

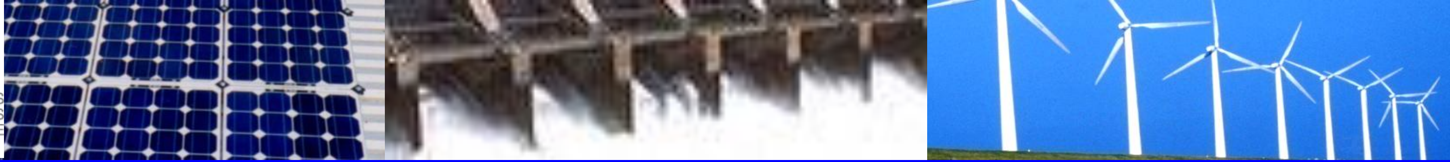


Ejemplo: Cable submarino

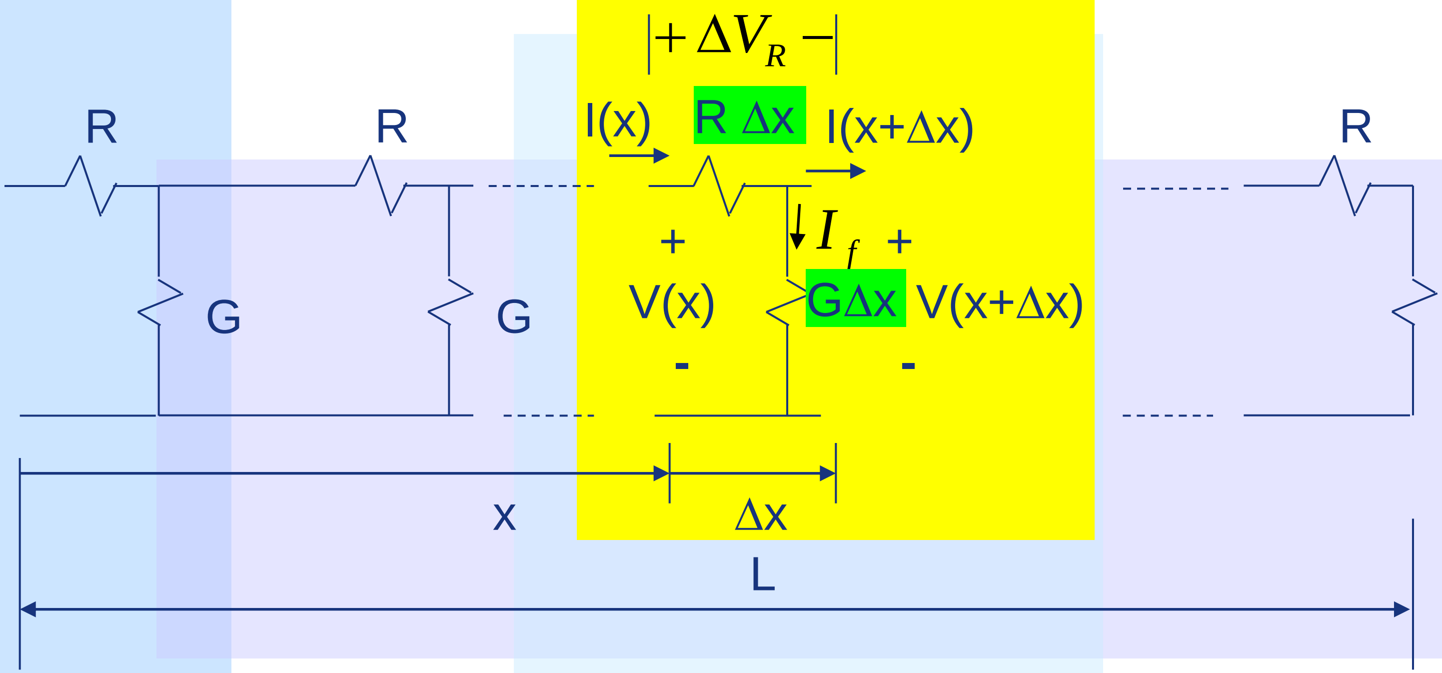


$$I(x)=I(x+\Delta x)+G\Delta xV(x+\Delta x)\Rightarrow\frac{I(x)-I(x+\Delta x)}{\Delta x}=GV(x+\Delta x)$$

$$V(x)=R\Delta xI(x)+V(x+\Delta x)\quad\Rightarrow\frac{V(x)-V(x+\Delta x)}{\Delta x}=RI(x)$$



Ejemplo: Cable submarino

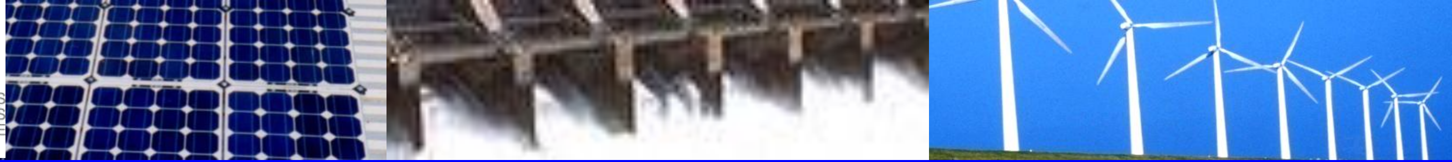


$$\Delta x \to 0 \Rightarrow \frac{\partial I(x)}{\partial x} = -GV(x)$$

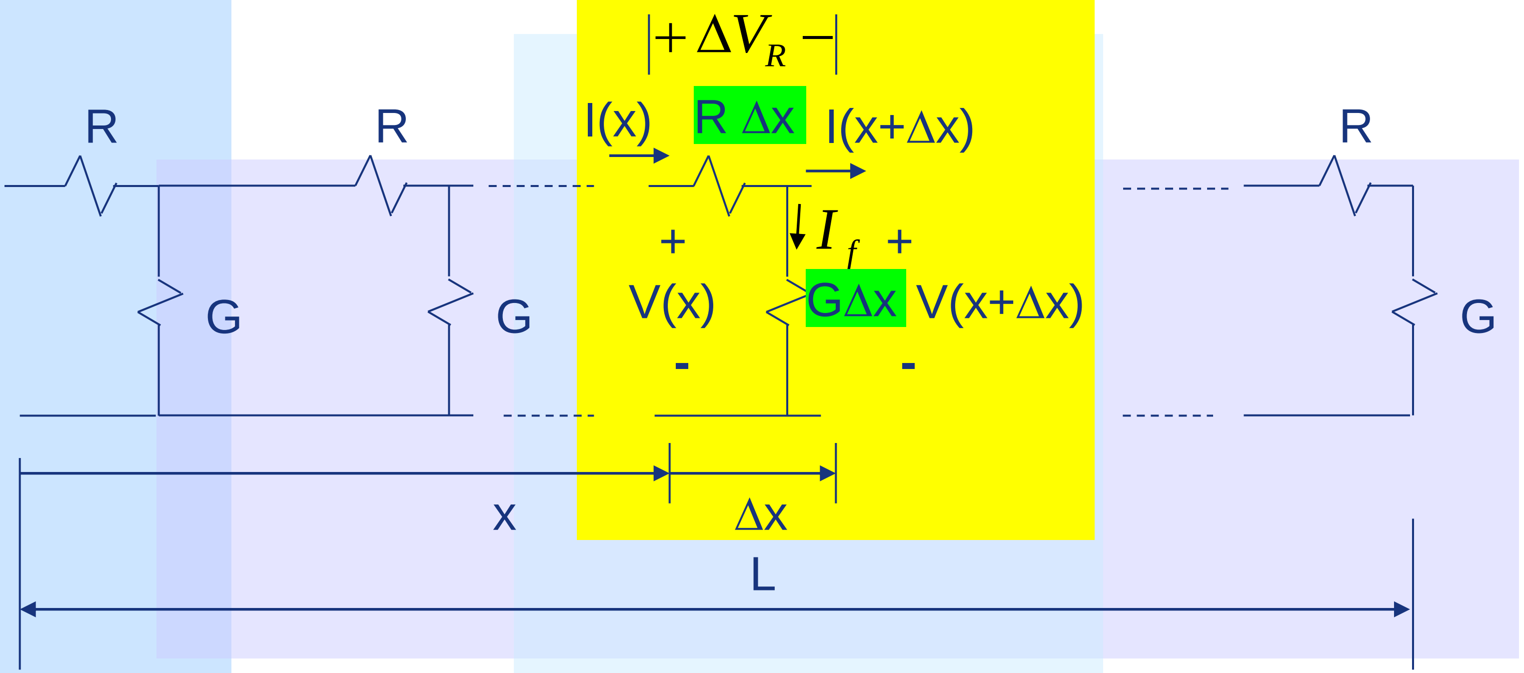
$$\frac{\partial V(x)}{\partial x} = -RI(x)$$

$$\frac{\partial^2 I(x)}{\partial x^2} = (GR)I(x)$$

$$\frac{\partial^2 V(x)}{\partial x^2} = (RG)V(x)$$



Ejemplo: Cable submarino



$$\frac{\partial^2 V(x)}{\partial x^2} = (RG)V(x) \Rightarrow V(x) = Ce^{-x/\lambda} + De^{x/\lambda} \text{ con } \lambda^2 = RG$$

CB:

$$V(x=0) = V_0 = C + D$$

$$V(x=L) = 0 = Ce^{-L/\lambda} + De^{L/\lambda}$$

$$\therefore V(x) = \frac{V_0}{e^{-L/\lambda} - e^{L/\lambda}} \left(-e^{-(x-L)/\lambda} + e^{(x-L)/\lambda}\right)$$