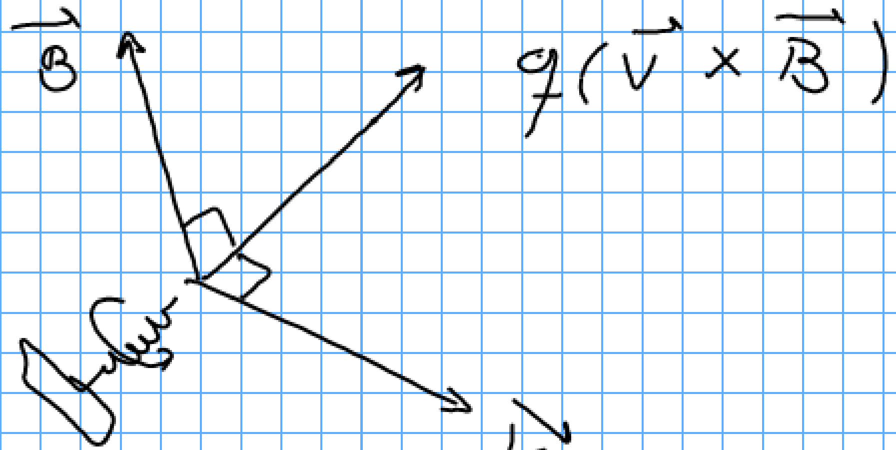
Campo magnético o inducción magnética

$$\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$ Fza. de Lorentz

$\vec{E} =$ Campo eléctrico , $\vec{B} =$ Campo Magnético



Unidades

$$\begin{array}{ccccccc} & & q & & \vec{v} & & \vec{B} \\ N & = & C & \times & \frac{m}{s} & \times & B \end{array} = \frac{C}{s} \times m \times B = A \times m \times B$$

$$[B] = \left[\frac{N}{A\,m}\right] = \text{Tesla } [T]$$

$$1\ T = 10^4 \text{ gauss. (cgs)}$$

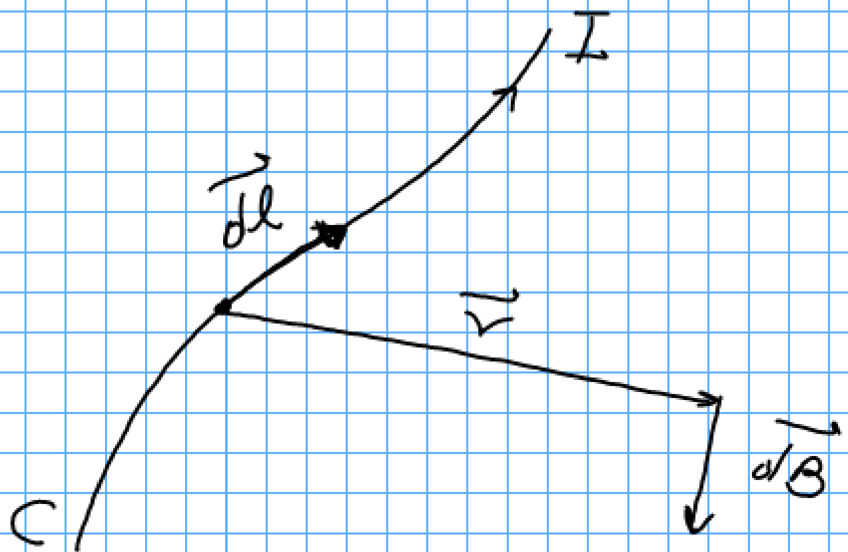
Campo magnético terrestre: 0.3 - 0.6 Gauss = 30000 - 60000 nT

$$1\ nT = 10^{-9}\ T = 1\gamma \text{ (gamma)}$$

$$1 \text{ Gauss} = 100000\ \gamma$$

Corrientes eléctricas generan C. Mag.

Ley de Biot-Savart

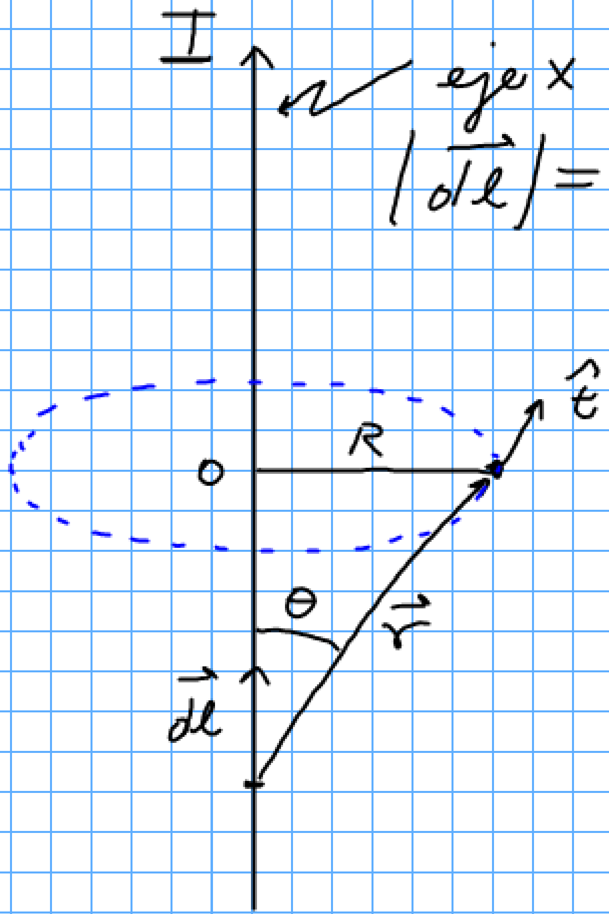


$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\,\vec{dl} \times \hat{r}}{r^2}$$

μ_0 = Permeabilidad del vacío

$$= 4\pi \times 10^{-7} \frac{N}{A^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int_C \frac{I\, \vec{d\ell} \times \hat{r}}{r^2}$$



$$|\vec{d\ell}| = dx$$

$$\vec{d\ell} \times \hat{r} = dx \,\text{sen}\,\theta\, \hat{t}$$

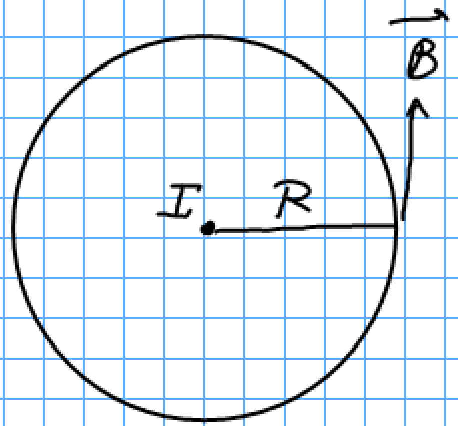
$$dB = \frac{\mu_0 I}{4\pi} \frac{\text{sen}\,\theta}{r^2} dx$$

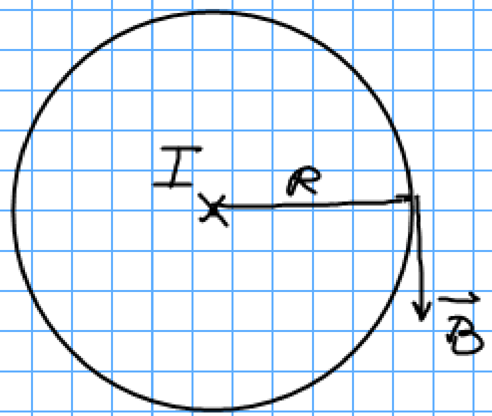
$$r = \sqrt{R^2 + x^2}, \quad \text{sen}\,\theta = \frac{R}{r}$$

$$B = \frac{\mu_0 I R}{4\pi} \int_{-\infty}^{+\infty} \frac{dx}{(R^2+x^2)^{3/2}}$$

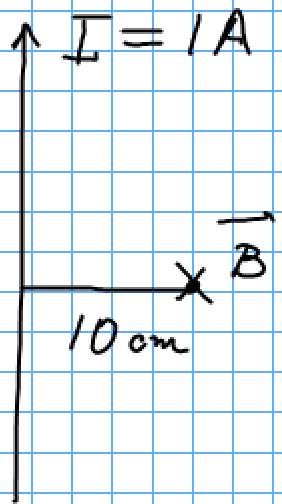
$$= \frac{\mu_0 I R}{4\pi} \left. \frac{x}{R^2\sqrt{R^2+x^2}} \right|_{-\infty}^{+\infty}$$

$$B = \frac{\mu_0 I}{4\pi R}(1+1) = \frac{\mu_0 I}{2\pi R}.$$





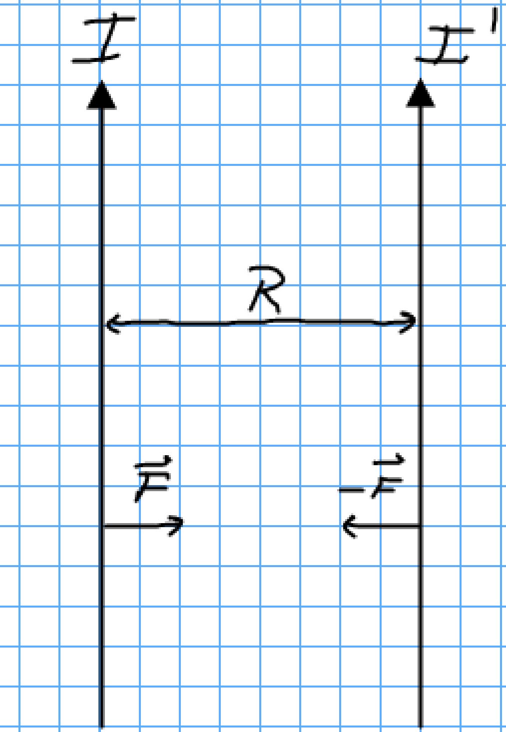
$$|\vec{B}| = \frac{\mu_0}{2\pi}\,\frac{I}{R}$$



$$B = 2\,\frac{\mu_0}{4\pi} \times \frac{1}{0.1} = 2\times 10^{-7} \times 10$$

$$= 2\times 10^{-6}\,T$$

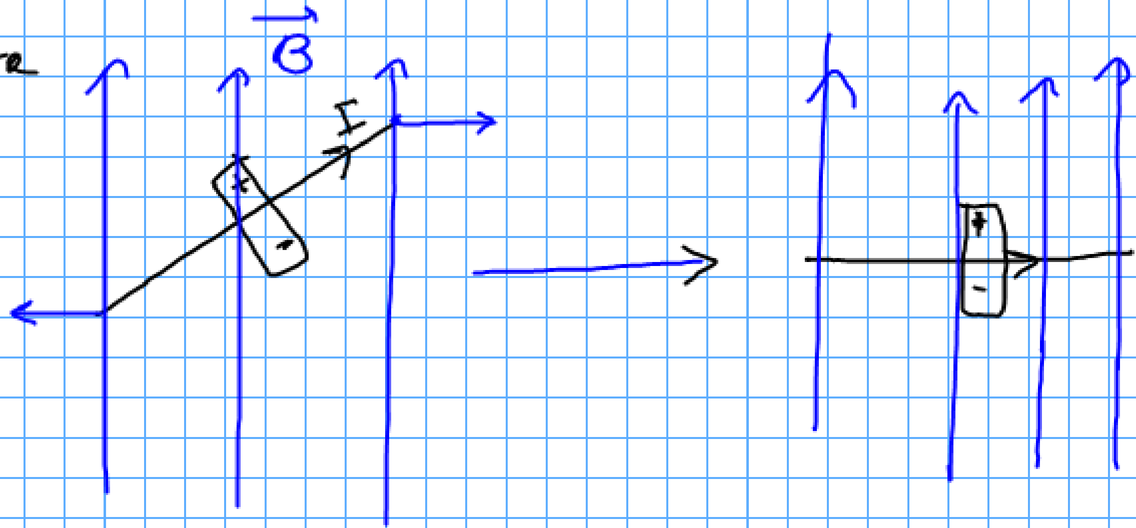
$$= 2000\,nT = 2000\,\gamma$$



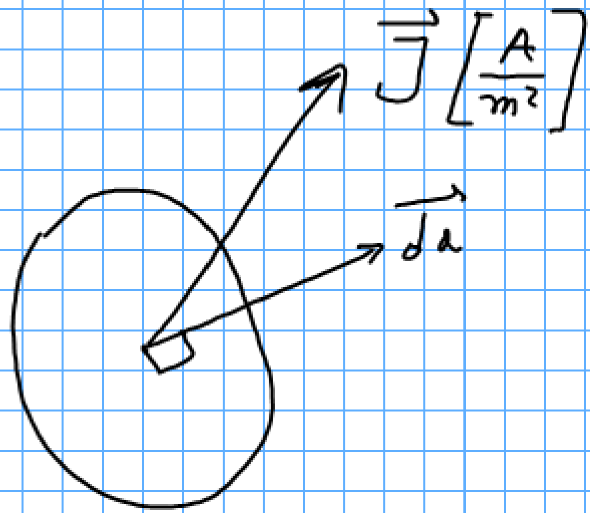
Fza por unidad de largo del conductor

$$|\vec{F}| = \frac{\mu_0}{2\pi} \frac{I\,I'}{R}$$

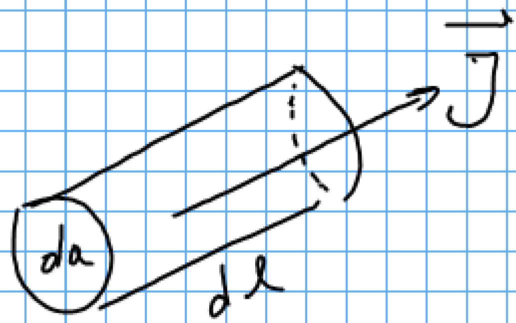
Espira



$\vec{J}$ = Vector corriente o flujo de Corriente.



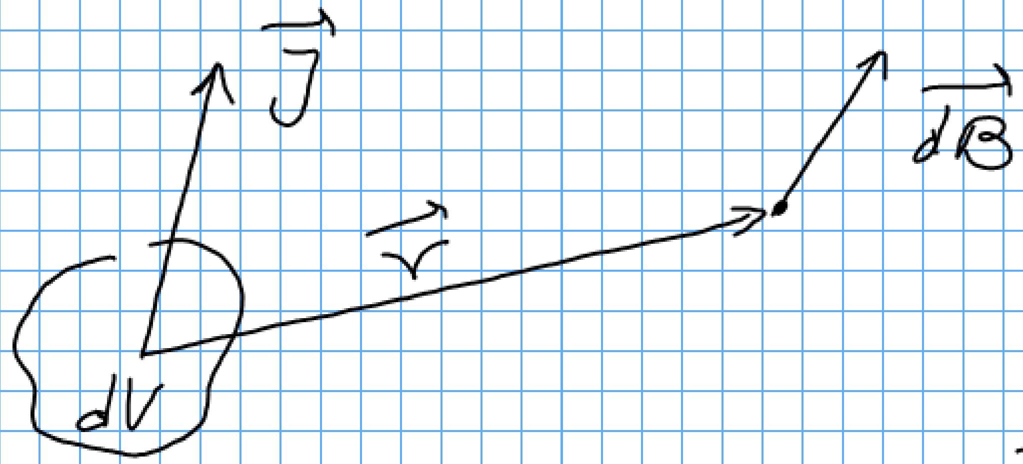
$\vec{J}\cdot\vec{da}$ = corriente (I) que atravieza el area da



$$\vec{J}\,dV = \vec{J}\,da\,dl = J\hat{J}\,da\,dl$$
$$= J\,da\,\hat{J}\,dl = I\,\vec{dl}$$

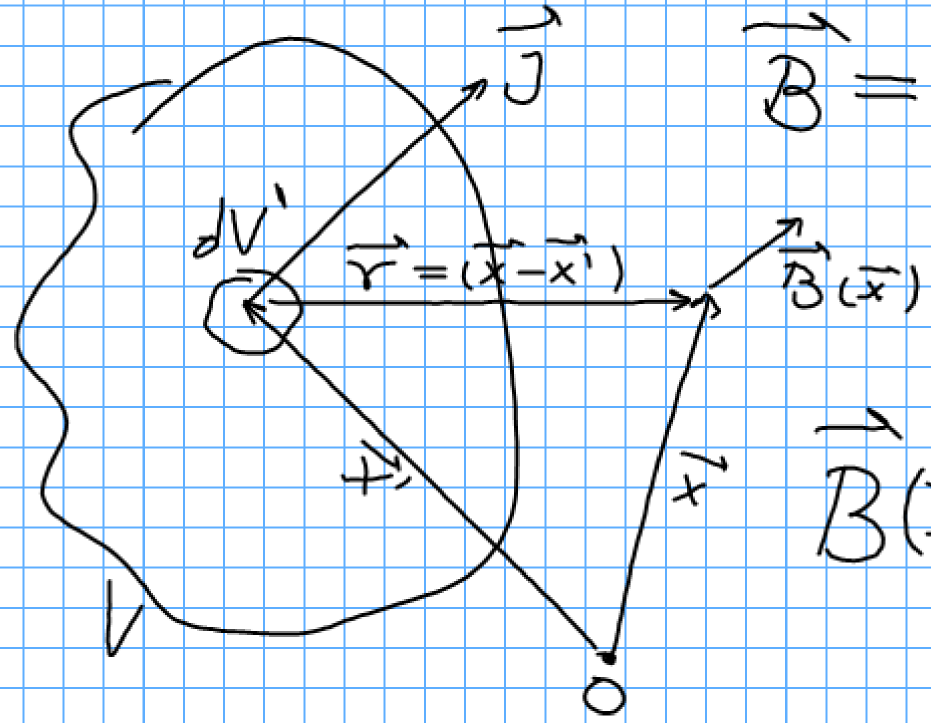
Para flujo de carga Volumetrico

$$I\,\vec{dl} \longleftrightarrow \vec{J}\,dV$$



$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl}\times\hat{r}}{r^2} \longrightarrow \frac{\mu_0}{4\pi}\frac{\vec{J}\,dV\times\hat{r}}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}\times\hat{r}}{r^2}\,dV$$

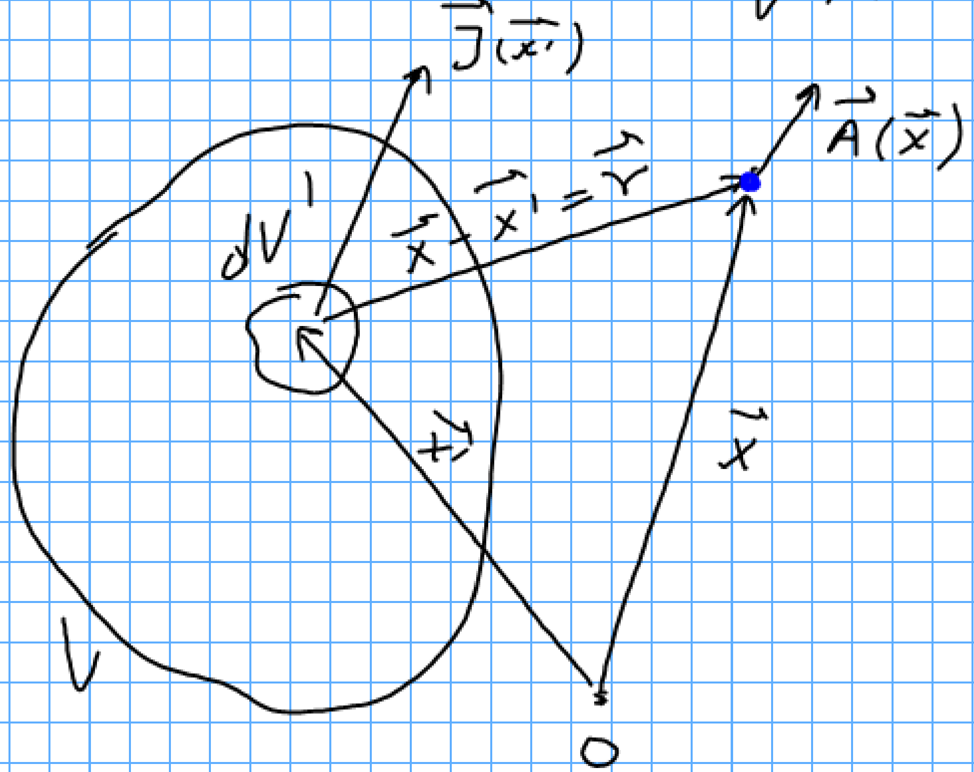


$$\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{x}')\times(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}\,dV'$$

Se puede demostrar que:

$$\vec{B} = \nabla \times \vec{A} \quad , \quad \vec{A} \text{ potencial vector}$$

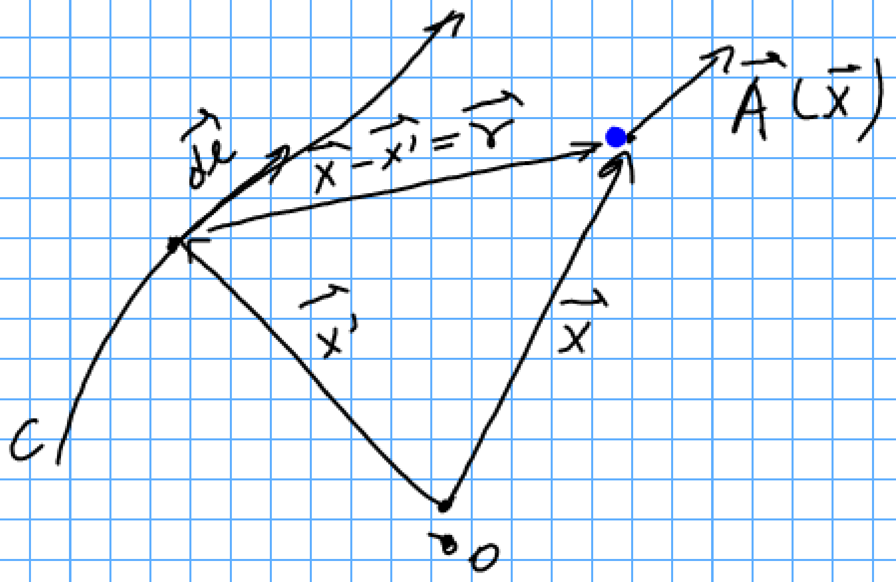
$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{x}\,')}{|\vec{x}-\vec{x}\,'|}\,dV' = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}}{r}\,dV'$$



Si $\vec{J}$ está concentrada en un circuito C, donde circula una corriente I cte.

$$\vec{J}\, dV' \longrightarrow I\, \vec{dl}$$

$$\vec{A}(\vec{x}) = \frac{\mu_0 I}{4\pi} \int_C \frac{\vec{dl}}{|\vec{x}-\vec{x}'|} = \frac{\mu_0 I}{4\pi} \int_C \frac{\vec{dl}}{r}$$



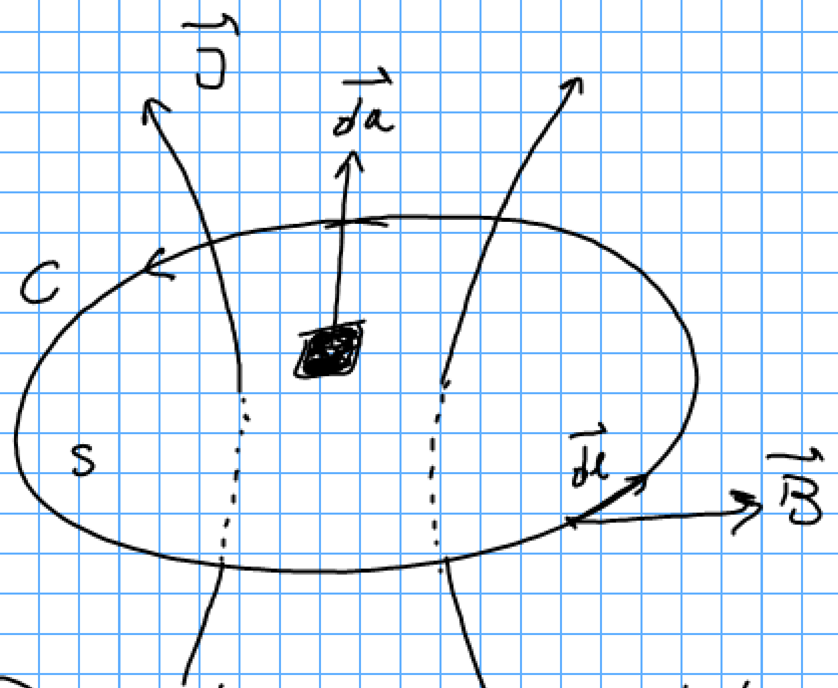
Se tiene entonces;

$$\nabla \cdot \vec{B} = \nabla \cdot (\nabla \times \vec{A}) = 0$$

$$\nabla \cdot \vec{B} = 0$$ No hay monopolos magnéticos

$$(\nabla \cdot \vec{g} = -4\pi G\rho)$$

Ley de Ampere



$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$I = \int_S \vec{J} \cdot \vec{da}$$

Por teorema de Stokes:

$$\oint_C \vec{B} \cdot \vec{dl} = \int_S (\nabla \times \vec{B}) \cdot \vec{da} = \mu_0 \int_S \vec{J} \cdot \vec{da}$$

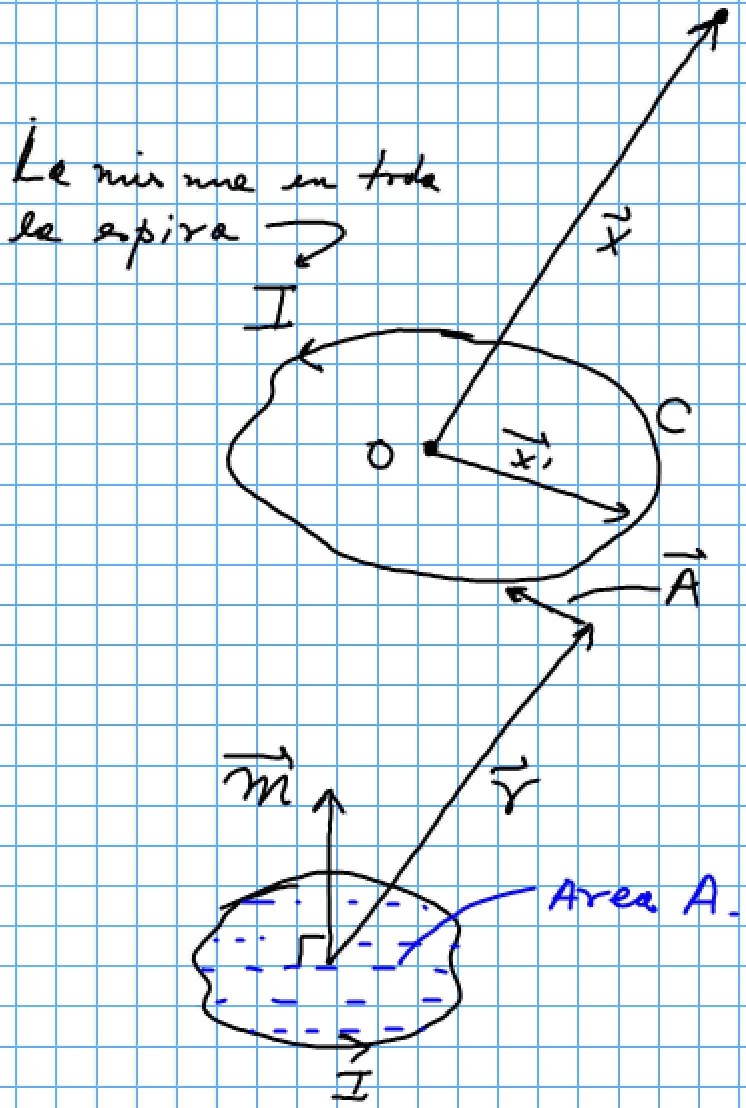
C es una curva arbitraria y ∴

$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad \text{(Ley de Ampere diferencial)}$$

En zonas donde $\vec{J}=0$, $\nabla \times \vec{B} = 0$, $\vec{B}$ es un campo que puede ser derivado de un potencial escalar (V)

$$\vec{B} = -\nabla V$$

Momento dipolar, Campo de un dipolo (espira) elemental: Imán elemental



$$\vec{A}(\vec{x}) = \frac{\mu_0 I}{4\pi} \oint_C \frac{\vec{dl}}{|\vec{x} - \vec{x}'|}$$

Para una espira elemental (infinitesimal) $|\vec{x}| \gg |\vec{x}'|$

se puede demostrar:

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3}$$

$$|\vec{m}| = I \times A = \text{momento dipolar magnético } [A\,m^2]$$

$$\vec{m} = I \times A \;\hat{m} \quad , \quad \hat{m} \perp \text{ a plano de espira.}$$

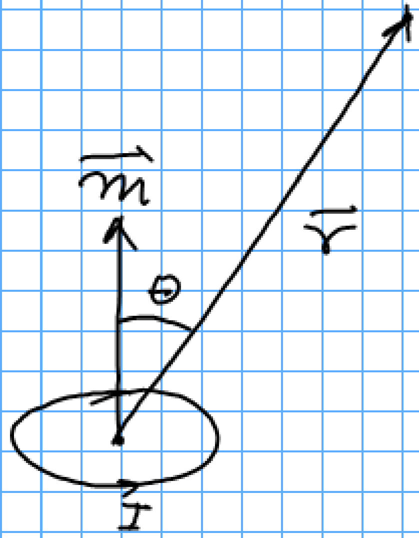
$$\vec{B} = \nabla \times \vec{A} = \frac{\mu_0}{4\pi} \nabla \times \left(\vec{m} \times \frac{\vec{r}}{r^3} \right)$$

Se puede demostrar sin mucha dificultad que:

$$\vec{B} = -\frac{\mu_0}{4\pi} \nabla \left(\vec{m} \cdot \frac{\vec{r}}{r^3} \right)$$

$$\vec{B} = -\nabla V \quad , \quad V = \text{Potencial escalar}$$

$$V = \frac{\mu_0}{4\pi} \, \vec{m} \cdot \frac{\vec{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{\vec{m} \cdot \hat{r}}{r^2}$$



$$\vec{B} = -\nabla V$$

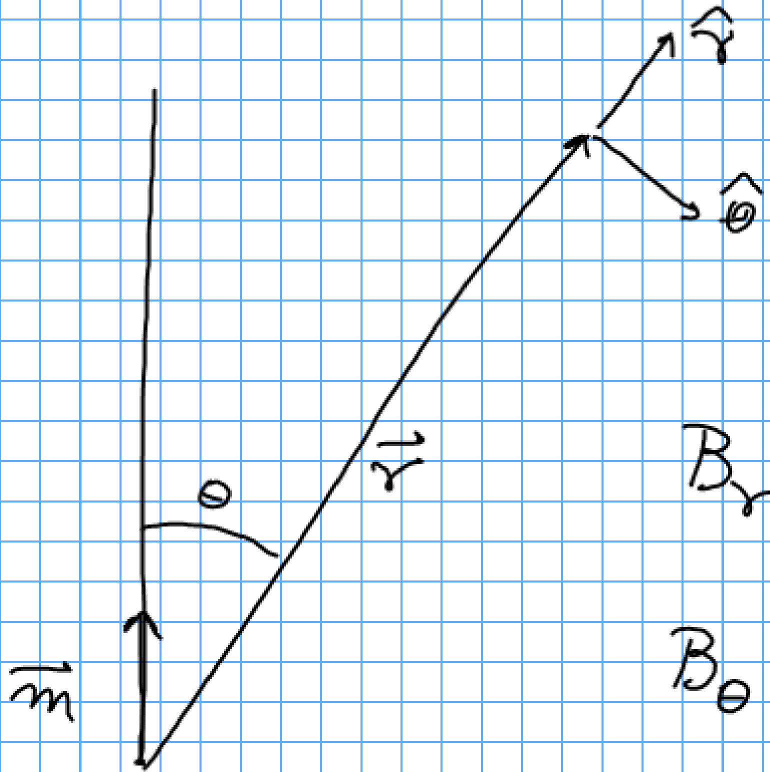
$$V = \frac{\mu_0}{4\pi} \frac{\vec{m} \cdot \hat{r}}{r^2} \quad , \quad \vec{m} \cdot \hat{r} = m\cos\theta$$

$$V = \frac{\mu_0}{4\pi} \frac{m\cos\theta}{r^2} = V(r,\theta)$$

Se tiene:

$$\vec{B} = \frac{\mu_0}{4\pi}\left[3(\hat{m}\cdot\hat{r})\hat{r} - \hat{m}\right]\frac{m}{r^3}$$

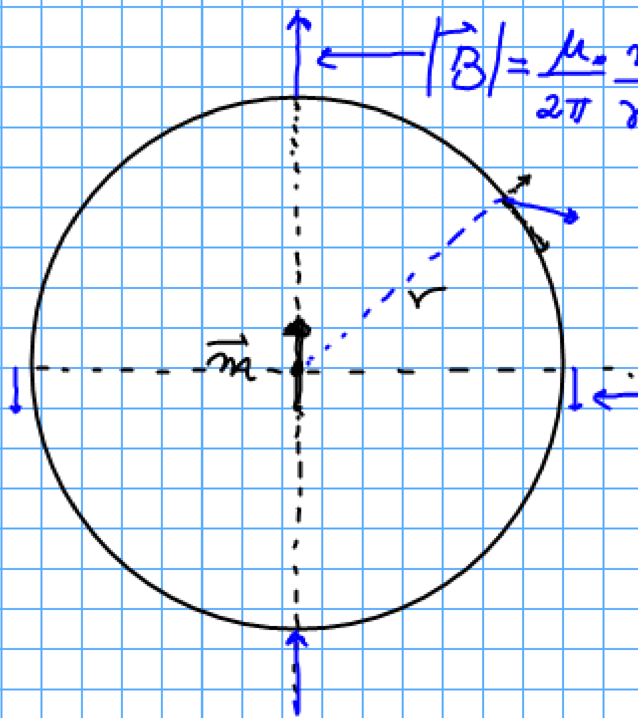
$$= \frac{\mu_0 m}{4\pi r^3}\left[3\cos\theta\,\hat{r} - \hat{m}\right]$$



$$V = \frac{\mu_0}{4\pi}\,\frac{m\cos\theta}{r^2} = V(r,\theta)$$

$$B_r = -\frac{\partial V}{\partial r} = 2\,\frac{\mu_0}{4\pi}\,\frac{m\cos\theta}{r^3}$$

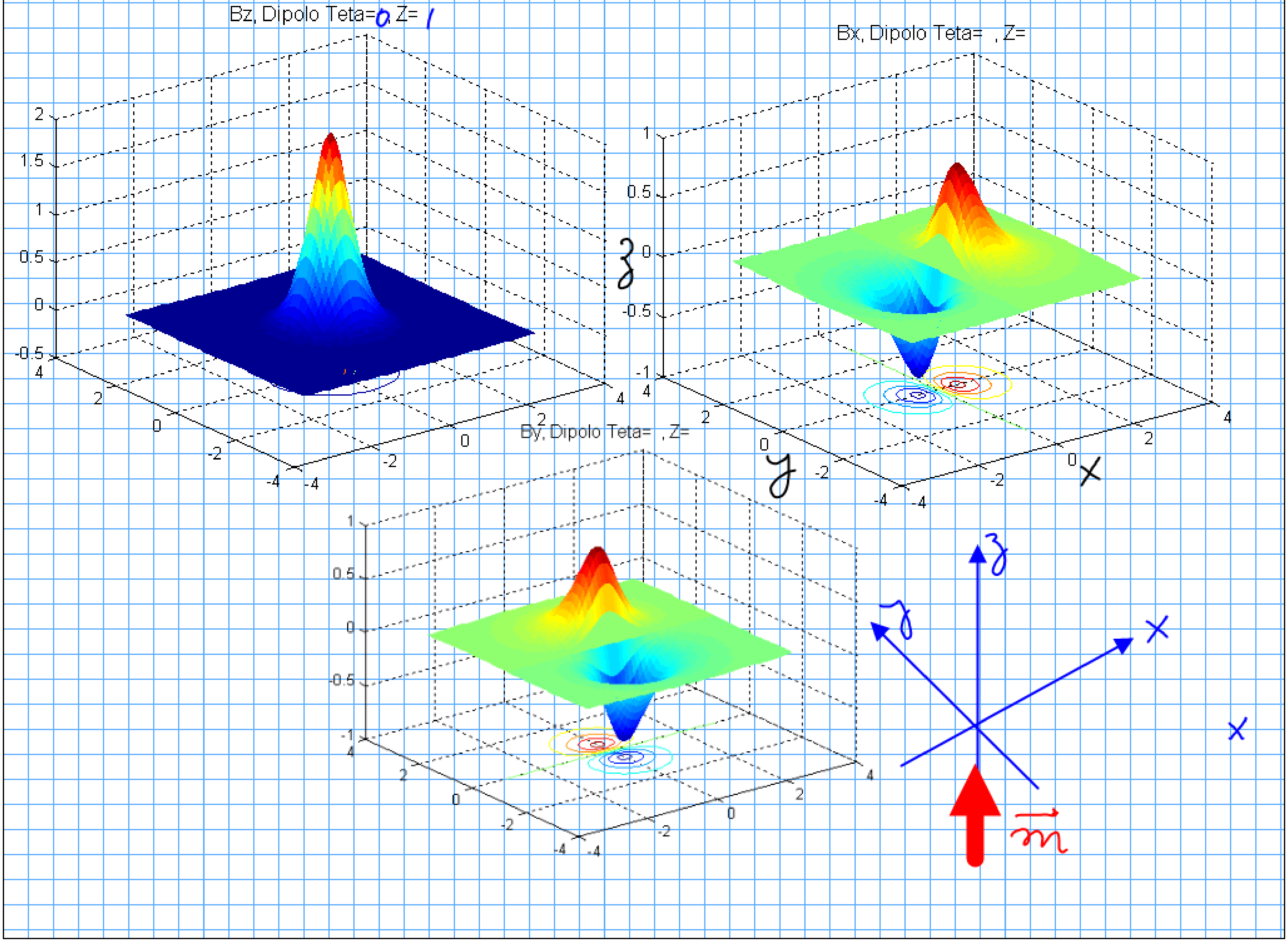
$$B_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{\mu_0}{4\pi}\,\frac{m\,\mathrm{sen}\,\theta}{r^3}$$



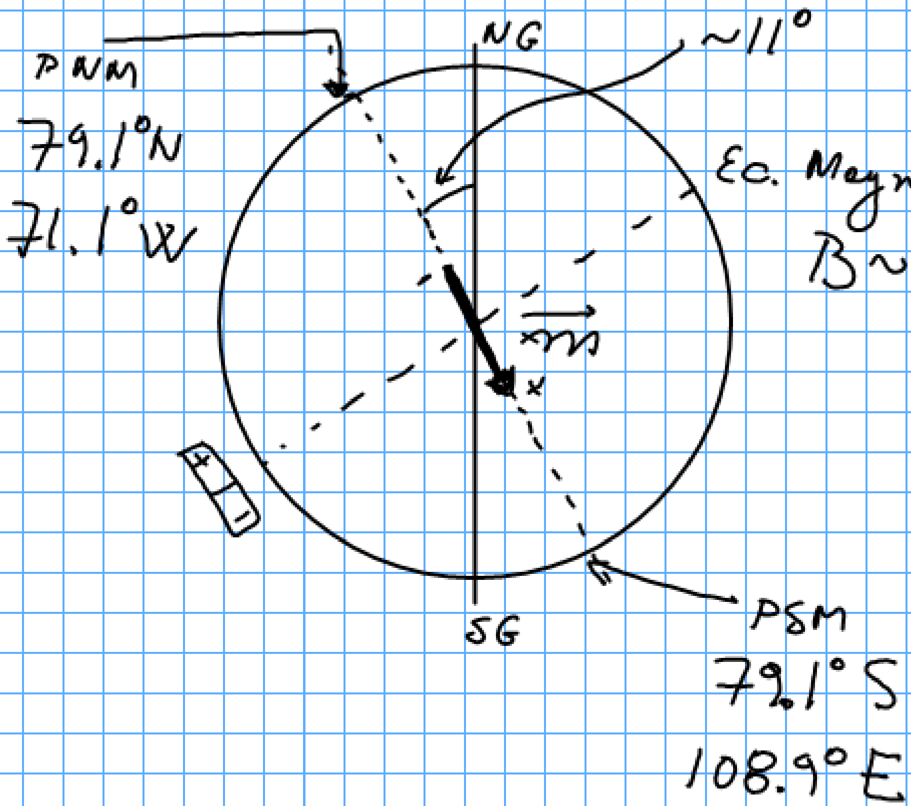
$$|\vec{B}| = \frac{\mu_0}{2\pi}\,\frac{m}{r^3}$$

$$|\vec{B}| = \sqrt{B_r^2 + B_\theta^2} = \frac{\mu_0}{4\pi}\,\frac{m}{r^3}\sqrt{3\cos^2\theta + 1}$$

$$|\vec{B}| = \frac{\mu_0}{4\pi}\,\frac{m}{r^3}$$


Bz, Dipolo Teta= 0, Z= 1
Bx, Dipolo Teta= , Z=
By, Dipolo Teta= , Z=
z
y
x
$\vec{m}$

Dipolo Magnético Terrestre



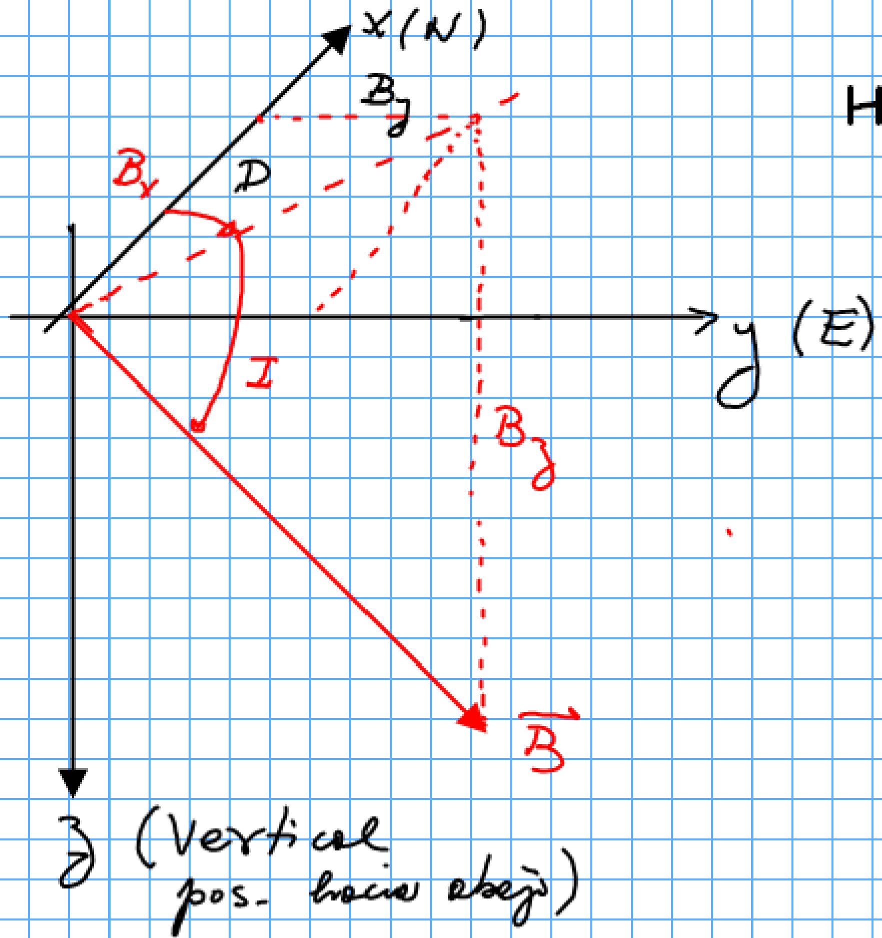
$$B \sim 31000\ nT = \frac{\mu_0}{4\pi}\,\frac{m}{R_T^3}\ ,\quad R_T \sim 6370\ km$$

$$m = B\,R_T^3\,\frac{4\pi}{\mu_0}$$

$$= 3.1\times 10^{-5} \times 6.37^3 \times 10^{18} \times 10^{7}$$

$$m \sim 8\times 10^{22}\ A\,m^2$$

Elementos del Campo Magnético Terrestre



H = Campo horizontal

$$= \sqrt{B_x^2 + B_y^2}$$

I = Inclinación

$$= \tan^{-1}\left(\frac{B_z}{H}\right)$$

D = Declinación

$$= \tan^{-1}\left(\frac{B_y}{B_x}\right)$$

T = Campo total = $\left|\vec{B}\right| = \sqrt{B_x^2 + B_y^2 + B_z^2}$

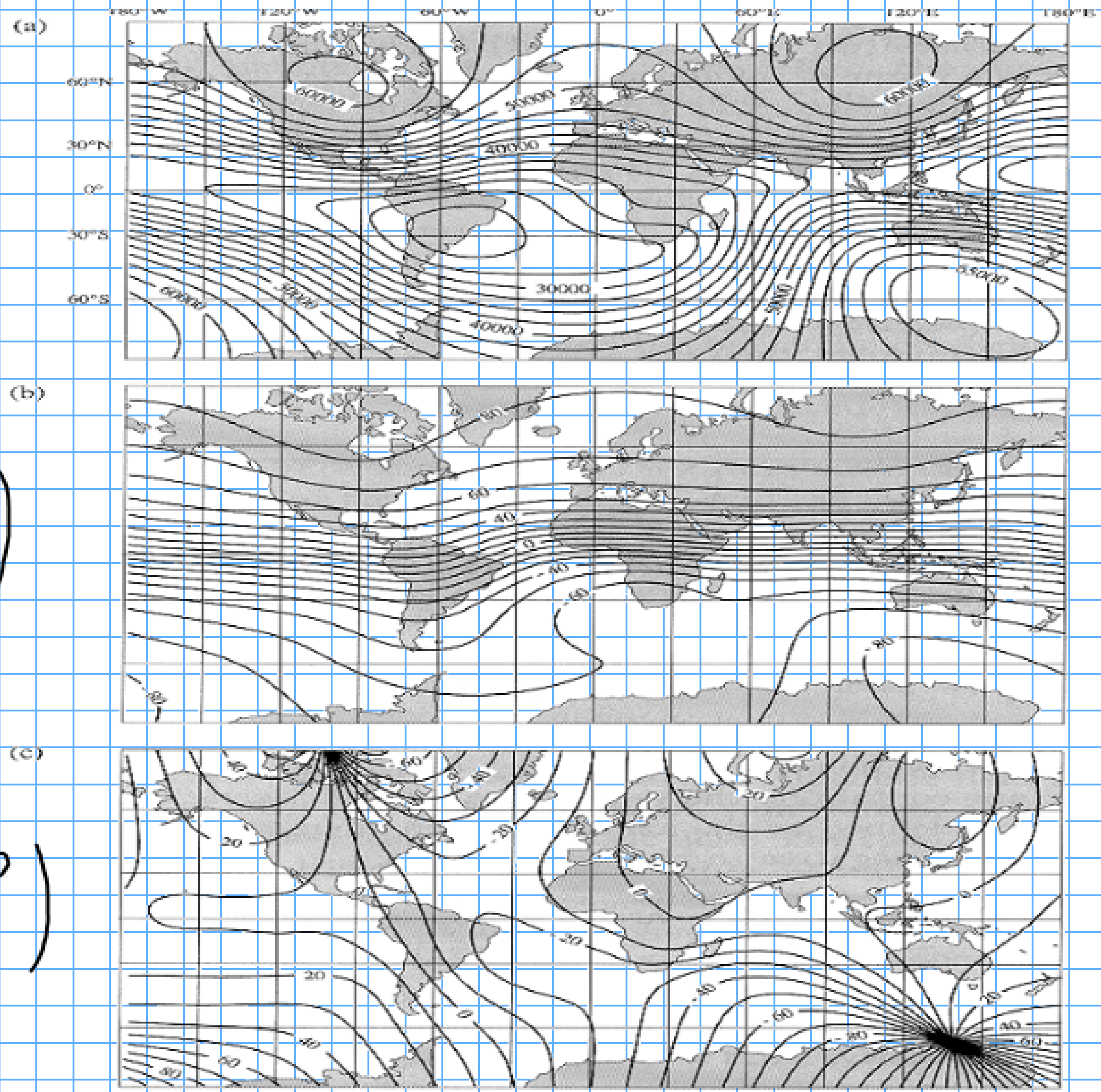
IGRF, International Geomagnetic Reference Field.

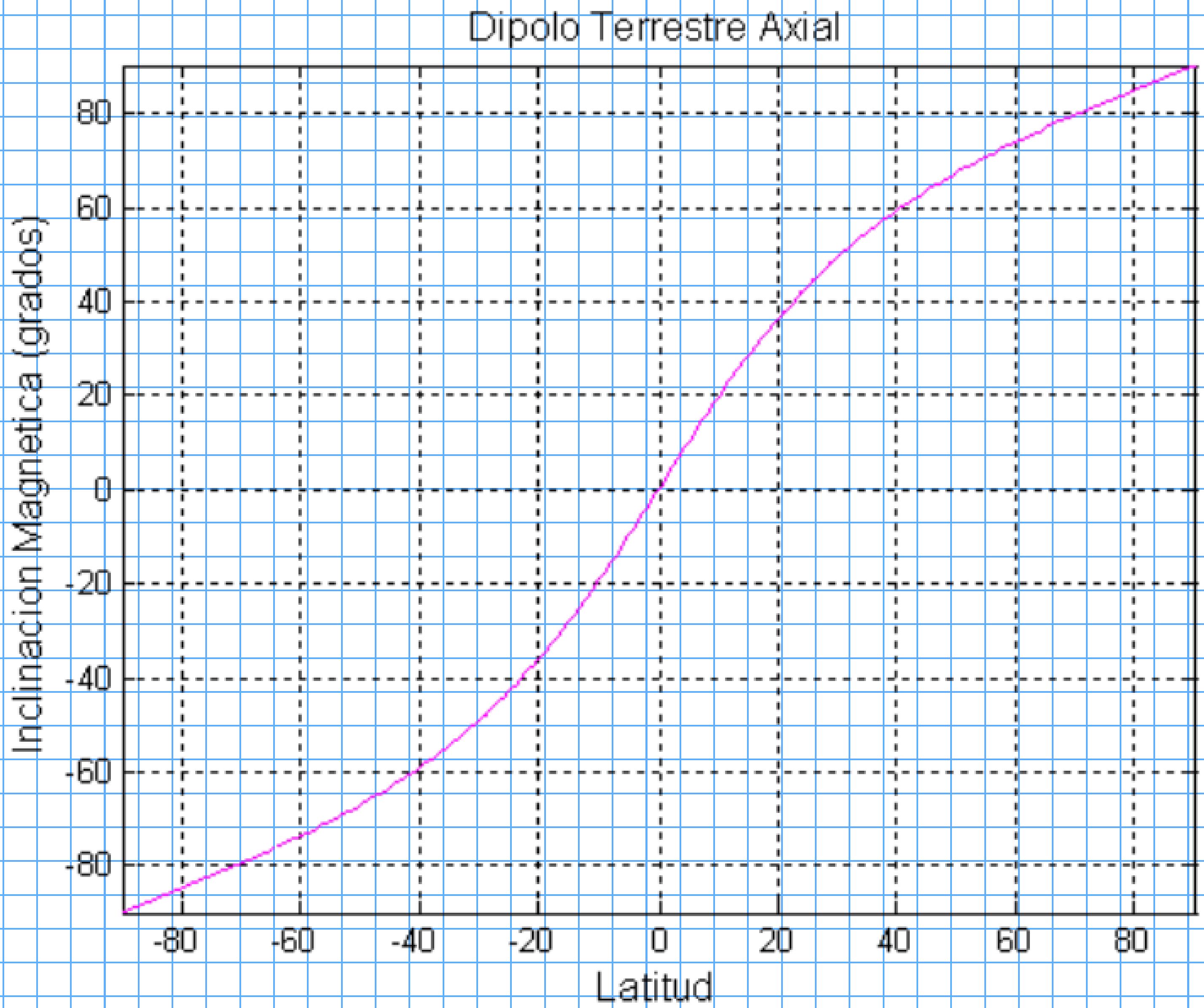
www.ngdc.noaa.gov/IAGA/vmod/igrf.html

T (nT)

I (°)

D (°)



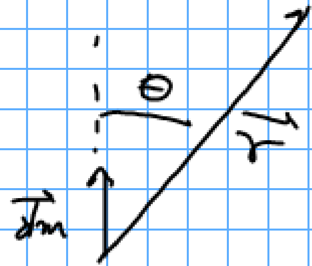

Dipolo Terrestre Axial
Inclinacion Magnetica (grados)
80
60
40
20
0
-20
-40
-60
-80
-80
-60
-40
-20
0
20
40
60
80
Latitud

Cuerpos Magnetizados

$\vec{M}$ = Magnetización: Momento dipolar por unidad de volumen

$d\vec{m} = \vec{M}\, dV$: Momento magnético en dV

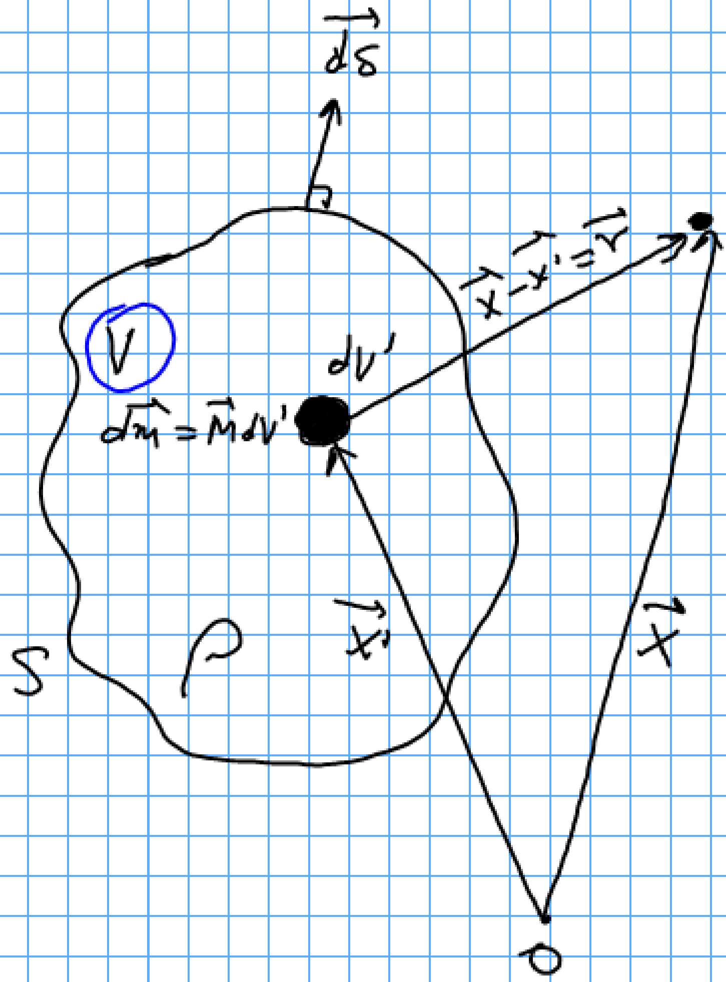
$[A\ m^2]$ $\left[\frac{A}{m}\right]$



$$, V = \frac{\mu_0}{4\pi} \frac{d\vec{m} \cdot \vec{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi} \frac{dm \cos\theta}{r^2}$$

$$d\vec{B} = -\nabla V$$



$$V(\vec{x}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{M} \cdot (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} \, dV'$$

Si $\vec{M}$ constante:

$$V(\vec{x}) = - \frac{\mu_0}{4\pi} \vec{M} \cdot \nabla \int_V \frac{dV'}{|\vec{x} - \vec{x}'|}$$

$$\nabla \left(\frac{1}{|\vec{x} - \vec{x}'|} \right) = - \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3}$$

$$V(\vec{x}) = - \frac{\mu_0}{4\pi} \vec{M} \cdot \nabla \left(- \frac{U}{G\rho} \right)$$

donde

$$U = -G\rho \int_V \frac{dv'}{|\vec{x} - \vec{x}'|}$$

es el potencial gravitatorio del volumen V, si este se considera lleno de material de densidad constante ρ

$$V = \frac{\mu_0}{4\pi G\rho} \vec{M} \cdot \nabla U = -\frac{\mu_0}{4\pi G\rho} \vec{M} \cdot \vec{g}$$

Relación de Poisson

Sea $\vec{M} = M\hat{\alpha} = M(l, m, n)$

$$\vec{M} \cdot \nabla U = M\hat{\alpha} \cdot \nabla U = M(l, m, n) \cdot \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right)$$

$$= M\left(l\frac{\partial U}{\partial x} + m\frac{\partial U}{\partial y} + n\frac{\partial U}{\partial z}\right)$$

$$= M\frac{\partial U}{\partial \alpha}$$

$$\frac{\partial}{\partial \alpha} = l\frac{\partial}{\partial x} + m\frac{\partial}{\partial y} + n\frac{\partial}{\partial z} = \text{derivada direccional en el sentido } \hat{\alpha}$$

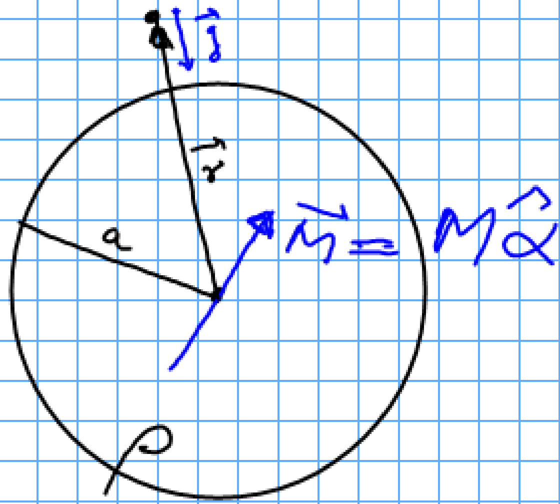
$$\therefore \quad V = \frac{\mu_0}{4\pi G\rho} M \frac{\partial U}{\partial \alpha} \quad (\text{Rel. Poisson})$$

Entonces

$$\vec{B} = -\nabla V = \frac{\mu_0}{4\pi\rho G} M \frac{\partial(-\nabla U)}{\partial\alpha} = \frac{\mu_0}{4\pi\rho G} \frac{\partial \vec{g}}{\partial \alpha}$$

Ejemplos de uso de relación de Poisson

1- Esfera uniformemente magnetizada

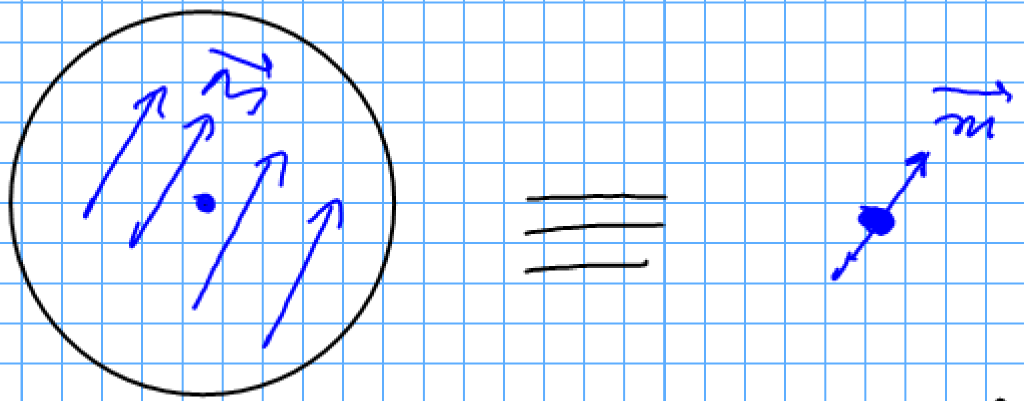


$$V = \frac{\mu_0}{4\pi} \frac{\vec{M} \cdot \nabla U}{\rho G} = -\frac{\mu_0}{4\pi} \frac{\vec{M} \cdot \vec{g}}{\rho G}$$

$$\vec{g} = -G\rho \frac{4}{3}\pi a^3 \frac{\hat{r}}{r^2}$$

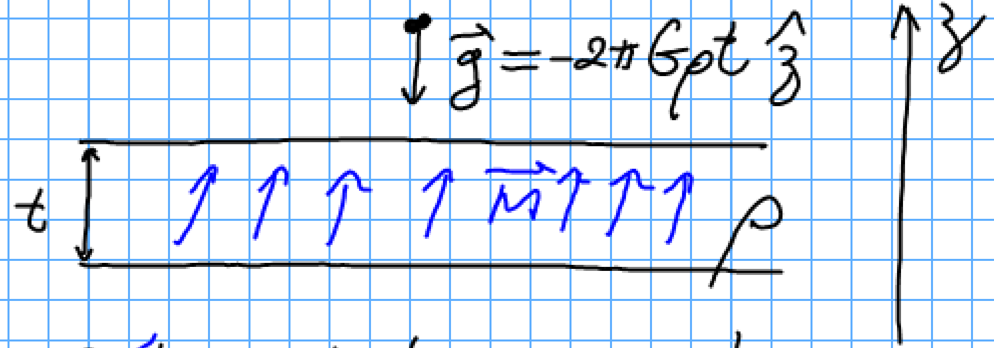
$$V = \frac{\mu_0}{4\pi}\,\frac{4}{3}\pi a^3 M\,\frac{\hat{a}\cdot\hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi}\,\frac{\vec{m}\cdot\hat{r}}{r^2}\,,\qquad \vec{m} = \frac{4}{3}\pi a^3 \vec{M}$$



2- Placa infinita

$$\vec{g} = -2\pi G\rho t\,\hat{z}$$

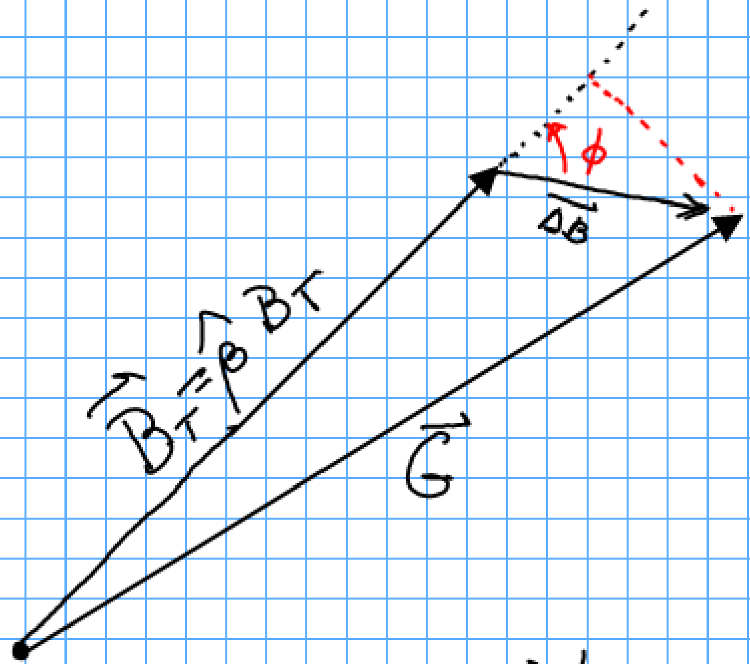


$$V = \frac{\mu_0}{\cancel{4\pi}_{\,2}}\,\frac{\vec{M}\cdot\hat{z}}{\cancel{\rho G}}\,\cancel{2\pi G\rho}\, t = \frac{\mu_0 t}{2} M_z = \text{Cte.}$$

$$\vec{B} = -\nabla V = 0$$

El campo magnético de una placa infinita uniformemente magnetizada es nulo.

Anomalía de Campo Total (A_T)



$\vec{B}_T$ = Campo Mag. de referencia (IGRF)

$\vec{G}$ = Campo medido

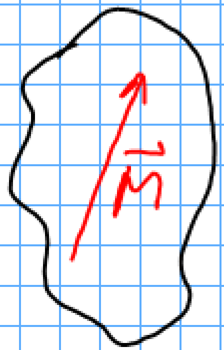
$\Delta\vec{B}$ = Campo anómalo

$$A_T = |\vec{G}| - |\vec{B}_T|$$

En general $|\vec{\Delta B}| << |\vec{B}_T|$ $\therefore$ $|\vec{G}| \sim |\vec{B}_T| + |\vec{\Delta B}| \cos\phi$

$$A_T \simeq |\vec{\Delta B}| \cos\phi = \hat{\beta} \cdot \vec{\Delta B} \quad , \hat{\beta} = (l', m', n')$$

Suponiendo $\vec{\Delta B}$ lo produce un cuerpo uniforme-mente magnetizado: $\vec{M} = \hat{\alpha} M$, $\hat{\alpha} = (l, m, n)$



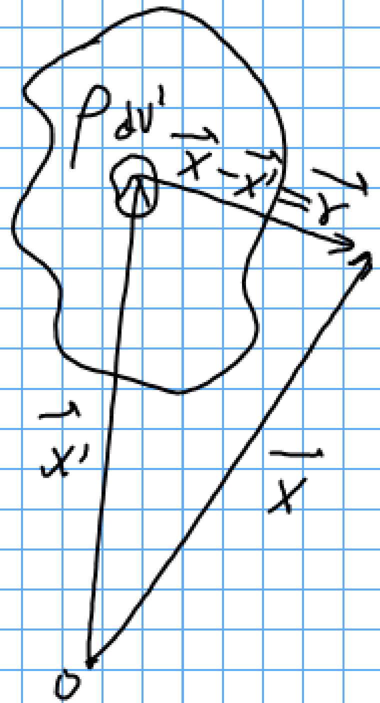
$$V = \frac{\mu_0 M}{4\pi\rho G}\frac{\partial U}{\partial \alpha} = \frac{\mu_0 M}{4\pi\rho G}\left[l\frac{\partial U}{\partial x} + m\frac{\partial U}{\partial y} + n\frac{\partial U}{\partial z}\right]$$

$$\vec{\Delta B} = -\nabla V = -\frac{\mu_0 M}{4\pi\rho G}\nabla\left(\frac{\partial U}{\partial \alpha}\right)$$

$$\therefore \quad A_T = -\frac{\mu_0 M}{4\pi\rho G}\,\hat{\beta}\cdot\nabla\left(\frac{\partial U}{\partial \alpha}\right)$$

$$= -\frac{\mu_0 M}{4\pi\rho G}\frac{\partial^2 U}{\partial \beta\,\partial \alpha}, \quad \frac{\partial}{\partial \beta} = l'\frac{\partial}{\partial x} + m'\frac{\partial}{\partial y} + n'\frac{\partial}{\partial z}$$

$$A_T = -\frac{\mu_0 M}{4\pi\rho G}\left(l'\frac{\partial}{\partial x} + m'\frac{\partial}{\partial y} + n'\frac{\partial}{\partial z}\right)\left(l\frac{\partial}{\partial x} + m\frac{\partial}{\partial y} + n\frac{\partial}{\partial z}\right)U$$



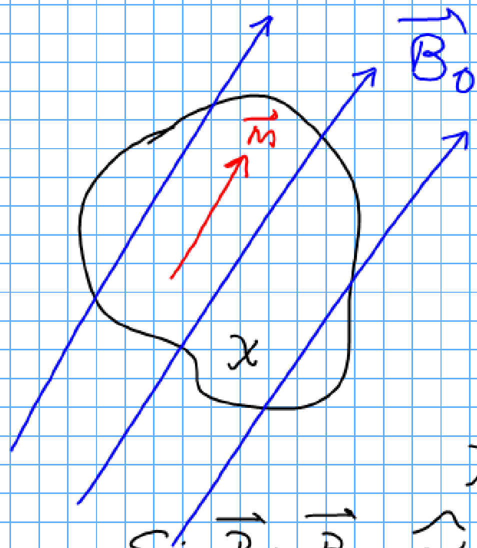
$$-\frac{U}{\rho G} = \int_V \frac{dV'}{|\vec{x}-\vec{x}'|} = \int_V \frac{dV'}{r} = \phi$$

$$A_T = \frac{\mu_0}{4\pi} M\left(l'\frac{\partial}{\partial x} + m'\frac{\partial}{\partial y} + n'\frac{\partial}{\partial z}\right)\left(l\frac{\partial}{\partial x} + m\frac{\partial}{\partial y} + n\frac{\partial}{\partial z}\right)\phi$$

$\hat{\beta}$ CAMPO DE REFERENCIA.

$\hat{\alpha}$ MAGNETIZACIÓN

Magnetización Inducida



$$\vec{B}_0 = B_0 \hat{\alpha}$$

$$\vec{M} \sim \chi \frac{\vec{B}_0}{\mu_0} = \frac{\chi B_0}{\mu_0} \hat{\alpha}$$

χ = Suceptibilidad magnética (SI)

$$\chi \ll 1$$

Si $\vec{B}_0 = \vec{B}_T \quad \hat{\alpha} = \hat{\beta}, \quad M = \frac{\chi B_0}{\mu_0}$

$$A_T = \frac{\mu_0}{4\pi} \frac{\chi B_0}{\mu_0} \frac{\partial^2 \phi}{\partial \alpha^2} = \frac{\chi}{4\pi} B_0 \frac{\partial^2 \phi}{\partial \alpha^2}$$

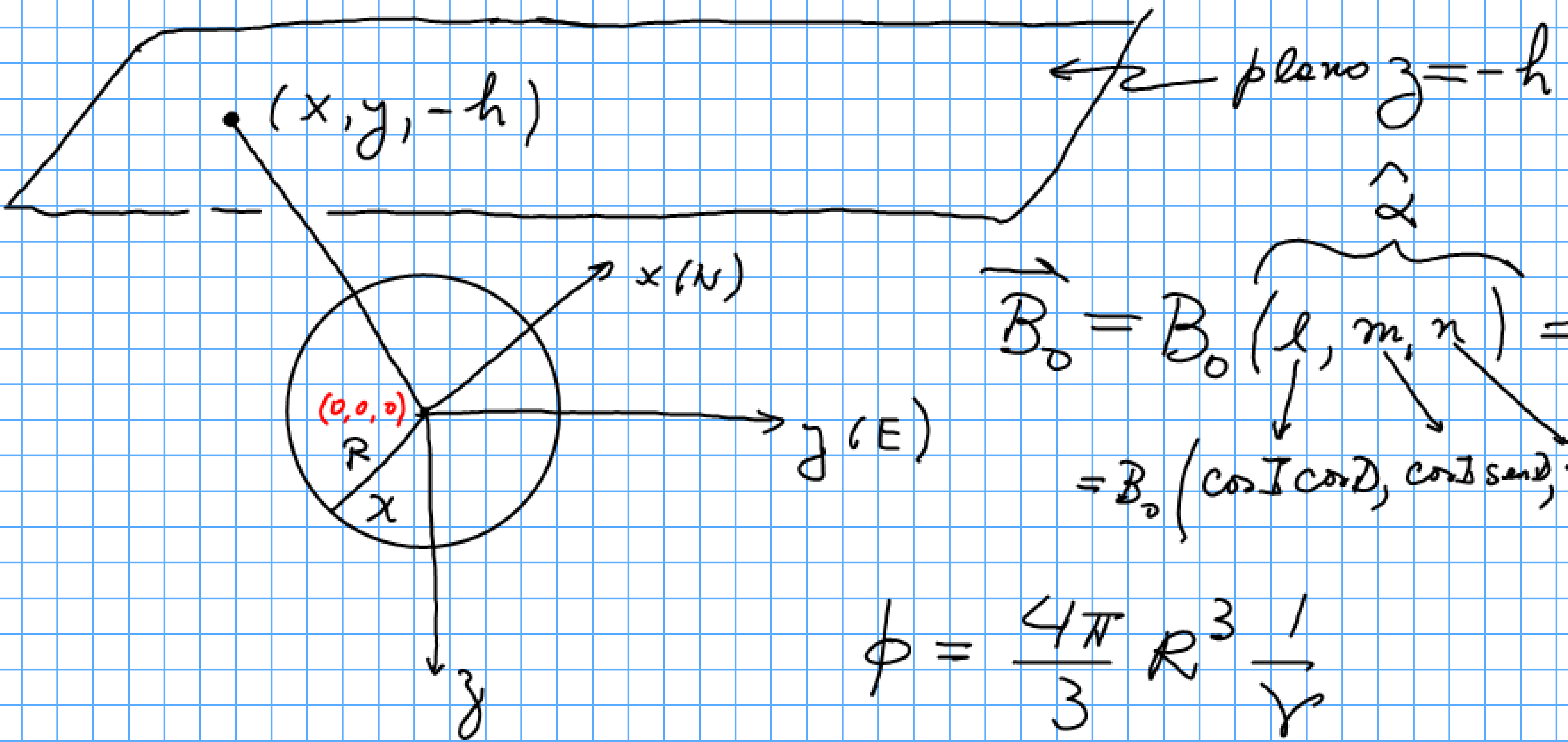
$$= \chi_{cgs} B_0 \frac{\partial^2 \phi}{\partial \alpha^2}, \quad \chi_{cgs} = \frac{\chi}{4\pi} = \chi_{emu}$$

Mediciones de susceptibilidad magnética en laboratorio

ROCA	$\chi_{cgs} \times 10^6$
Dolomita	8
Caliza	23
Arenisca	32
Pizarra	52
Metamórfica	350
Ignea ácida	650
Ignea básica	2600

$$\boxed{\chi = \chi_{SI} = 4\pi\,\chi_{cgs}}$$

Esfera Magnetizada en Campo Terrestre



$$\vec{B_0} = B_0 \overbrace{(l, m, n)}^{\hat{\alpha}} = \vec{B_T}$$

$$= B_0 (\cos I \cos D, \cos I \operatorname{sen} D, \operatorname{sen} I)$$

$$\phi = \frac{4\pi}{3} R^3 \frac{1}{r}$$

$$A_T = \chi_{cgs} B_0 \frac{4\pi}{3} R^3 \frac{\partial^2}{\partial \alpha^2} \left(\frac{1}{r}\right), \quad r = \sqrt{x^2+y^2+z^2}$$

$$\frac{\partial}{\partial \alpha}\left(\frac{1}{r}\right) \doteq - \frac{lx + my + nz}{r^3}$$

$$A_T = -\chi_{emu} B_0 \frac{4\pi}{3} R^3 \frac{\partial}{\partial \alpha}\left(\frac{lx + my + nz}{r^3} \right)$$

$$= -\chi_{emu} B_0 \frac{4\pi}{3} R^3 \left[(lx + my + nz) \frac{\partial r^{-3}}{\partial \alpha} + r^3 \frac{\partial}{\partial \alpha}(lx + my + nz) \right]$$

$$= -\chi_{emu} B_0 \frac{4\pi}{3} R^3 \left[-\frac{3(xl + my + nz)^2}{r^5} + \frac{l^2 + m^2 + n^2}{r^3} \right]$$

$$l^2 + m^2 + n^2 = 1, \quad xl + my + nz = \vec{r} \cdot \hat{\alpha}$$

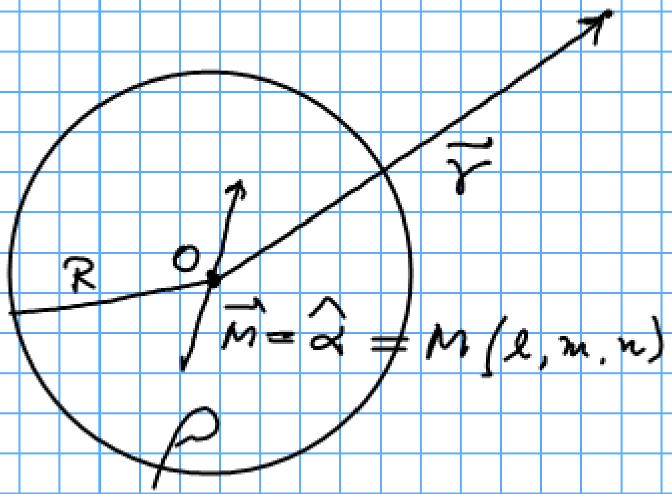
$$A_T = \chi_{emu} B_0 \frac{4\pi}{3} \left(\frac{R}{r}\right)^3 \left[3(\hat{r} \cdot \hat{\alpha})^2 - 1 \right]$$

Sobre la superficie $(z = -h)$

$$\vec{r} = (x, y, -h) \quad , \quad r = \sqrt{x^2 + y^2 + h^2}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(x, y, -h)}{\sqrt{x^2 + y^2 + h^2}}$$

Esfera Magnetizada : Caso general



$$\vec{B}_T = B_T\,\hat{\beta} = B_T\,(l', m', n')$$

Relación de Poisson: $V = -\dfrac{\mu_0}{4\pi}\dfrac{M}{\rho G}\,\hat{\alpha}\cdot\vec{g}$, $\vec{B} = -\nabla V$

$$\vec{g} = -\frac{G\rho V}{r^3}\,\hat{r} \;,\quad V = \frac{\mu_0}{4\pi} M V \frac{\hat{\alpha}\cdot\vec{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi} M V \left[\frac{lx + my + nz}{r^3}\right]$$

$$\vec{B} = -\nabla V = -\frac{\mu_0}{4\pi} M V \left[\hat{\alpha}\cdot\vec{r}\, \nabla\left(\frac{1}{r^3}\right) + \nabla(\hat{\alpha}\cdot\hat{r}) \frac{1}{r^3} \right]$$

$$= \frac{\mu_0}{4\pi} M V \frac{1}{r^3} \left[3(\hat{\alpha}\cdot\hat{r})\hat{r} - \hat{\alpha} \right]$$

$$\vec{B} = \frac{\mu_0}{\cancel{4\pi}} M \frac{\cancel{4\pi}}{3} \left(\frac{R}{r}\right)^3 \left[3(\hat{\alpha}\cdot\hat{r})\hat{r} - \hat{\alpha} \right]$$

$$A_T = \hat{\beta}\cdot\vec{B} = \frac{\mu_0 M}{3} \left(\frac{R}{r}\right)^3 \left[3(\hat{\alpha}\cdot\hat{r})(\hat{\beta}\cdot\hat{r}) - \hat{\beta}\cdot\hat{\alpha} \right]$$

Si la magnetización de la esfera es, o fue producida por un campo externo $\vec{B}_0 = B_0\,\hat{\alpha}$:

$$M = \chi \frac{B_0}{\mu_0} = 4\pi \chi_{emu} \frac{B_0}{\mu_0}$$

$$A_T = \frac{4\pi}{3}\chi_{emu} B_0 \left(\frac{R}{r}\right)^3 \left[3(\hat{\alpha}\cdot\hat{r})(\hat{\beta}\cdot\hat{r}) - \hat{\alpha}\cdot\hat{\beta}\right]$$

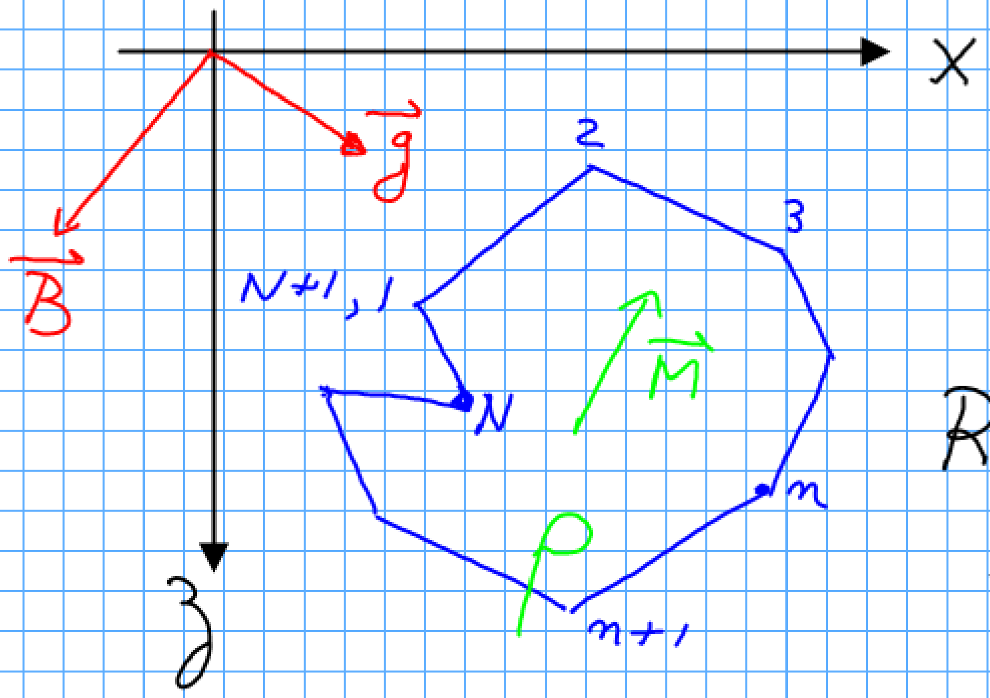
En el caso común de magnetización inducida por el campo de referencia:

$$\vec{B}_0 = \vec{B}_T \ , \quad \hat{\alpha} = \hat{\beta} \ , \quad B_0 = B_T$$

$$\therefore A_T = \frac{4\pi}{3}\chi_{emu} B_0 \left(\frac{R}{r}\right)^3 \left[3(\hat{\alpha}\cdot\hat{r})^2 - 1\right]$$

Modelación magnética 2D

I.J. Won & M. Bevis, Computing the gravitational and magnetic anomalies due to a polygon: Algorithms and Fortran subroutines, Geophysics 52, pp 232-238, 1987.



$$\vec{g} = (g_x, g_z)$$

$$\vec{B} = (B_x, B_z)$$

Relación de Poisson:

$$V = \frac{\mu_0}{4\pi\rho G} M \frac{\partial U}{\partial \alpha}$$

$$\vec{M} = M\hat{\alpha} = M(\ell, m, n), \quad \frac{\partial}{\partial \alpha} = \ell\frac{\partial}{\partial x} + m\frac{\partial}{\partial y} + n\frac{\partial}{\partial z}$$

$$\vec{B} = -\nabla V \; , \quad \vec{g} = -\nabla U$$

$$\vec{B} = -\nabla V = \frac{\mu_0}{4\pi\rho G} M \frac{\partial}{\partial \alpha}(-\nabla U)$$

$$= \frac{\mu_0}{4\pi\rho G} M \frac{\partial \vec{g}}{\partial \alpha}$$

$$\therefore \quad B_x = \frac{\mu_0}{4\pi\rho G} M \frac{\partial g_x}{\partial \alpha}$$

$$B_z = \frac{\mu_0}{4\pi\rho G} M \frac{\partial g_z}{\partial \alpha}$$

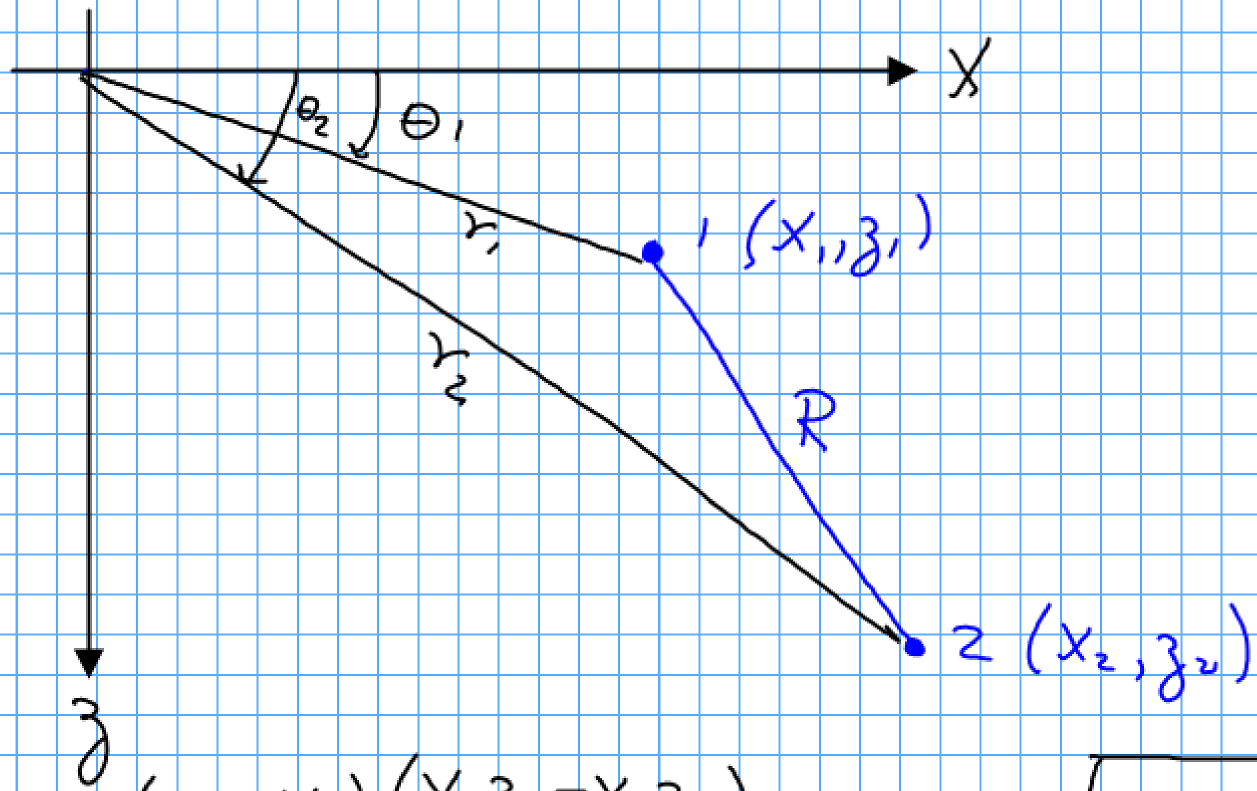
$$g_z = 2G\rho \sum_{n=1}^{N} Z_n$$

$$g_x = 2G\rho \sum_{n=1}^{N} X_n$$

Z_n , X_n dependen de los vertices $n, n+1$.
Sin perdida de generalidad para estos vertices sucesivos usamos indices 1 y 2:

$$Z = A\left[(\theta_1 - \theta_2) + B \ln \frac{r_2}{r_1}\right]$$

$$X = A\left[-(\theta_1 - \theta_2)B + \ln \frac{r_2}{r_1}\right]$$

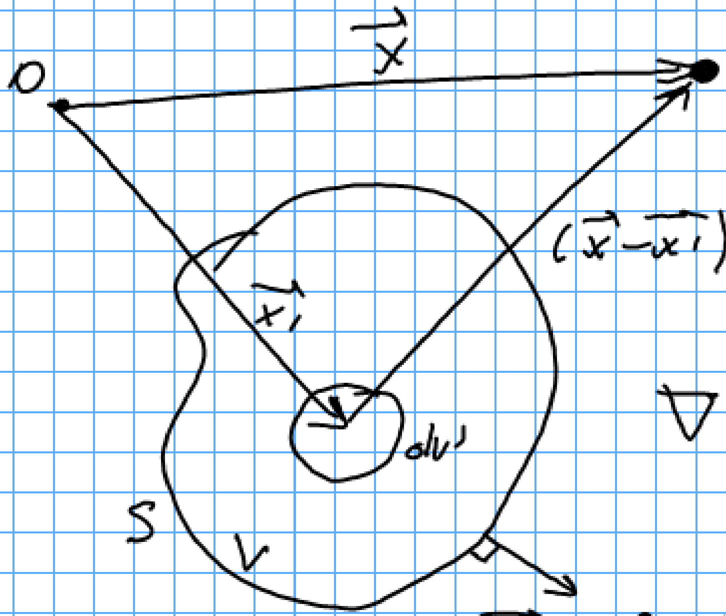


$$A = \frac{(x_2 - x_1)(x_1 z_2 - x_2 z_1)}{R^2}, \quad R = \sqrt{(x_2 - x_1)^2 + (z_2 - z_1)^2}$$

$$B = \frac{z_2 - z_1}{x_2 - x_1}, \quad \theta_1 = \operatorname{atan}\left(\frac{z_1}{x_1}\right), \quad \theta_2 = \operatorname{atan}\left(\frac{z_2}{x_2}\right)$$

$$r_1 = \sqrt{x_1^2 + z_1^2}, \quad r_2 = \sqrt{x_2^2 + z_2^2}$$

Modelación magnética volumen magnetizado



$$V(\vec{x}) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{M}(\vec{x}')\cdot(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}\,dV'$$

$(\vec{x}-\vec{x}') = \vec{r}$

$\vec{dS}' = \hat{n}\,dS'$

$$\nabla\left(\frac{1}{r}\right) = \nabla\left[\frac{1}{|\vec{x}-\vec{x}'|}\right] = -\frac{\vec{r}}{r^3} = -\frac{\vec{x}-\vec{x}'}{|\vec{x}-\vec{x}'|^3}$$

$$= -\nabla'\left[\frac{1}{|\vec{x}-\vec{x}'|}\right] = -\nabla'\left(\frac{1}{r}\right)$$

$$V(\vec{x}) = \frac{\mu_0}{4\pi}\int_V \vec{M}\cdot\nabla'\left(\frac{1}{r}\right)dV'$$

Utilizando la identidad vectorial:

$$\nabla\cdot(\phi\vec{A}) = \nabla\phi\cdot\vec{A} + \phi\nabla\cdot\vec{A}$$

$$\phi = \frac{1}{r}, \quad \vec{A} = \vec{M}, \text{ se tiene:}$$

$$V(\vec{x}) = \frac{\mu_0}{4\pi}\int_V \nabla'\cdot\left(\frac{\vec{M}}{r}\right)dV' - \frac{\mu_0}{4\pi}\int_V \frac{\nabla\cdot\vec{M}}{r}\,dV'$$

Utilizando el teorema de la divergencia: $\left(\int_V \nabla\cdot\vec{A}\,dV = \int_S \vec{A}\cdot\vec{dS}\right)$

$$V(\vec{x}) = \frac{\mu_0}{4\pi}\int_S \frac{\vec{M}\cdot\vec{dS}'}{r} - \frac{\mu_0}{4\pi}\int_V \frac{\nabla\cdot\vec{M}}{r}\,dV'$$

$$V(\vec{x}) = \frac{\mu_0}{4\pi}\int_S \frac{Q_S}{r}\, ds' + \frac{\mu_0}{4\pi}\int_V \frac{Q_V}{r}\, dv'$$

$$Q_S = \vec{M}\cdot\hat{n} \quad , \quad Q_V = -\nabla\cdot\vec{M}$$

Q_S , Q_V se interpretan como "cargas magnéticas" superficiales y volumétricas respectivamente (Blakely, ec. 9.16)

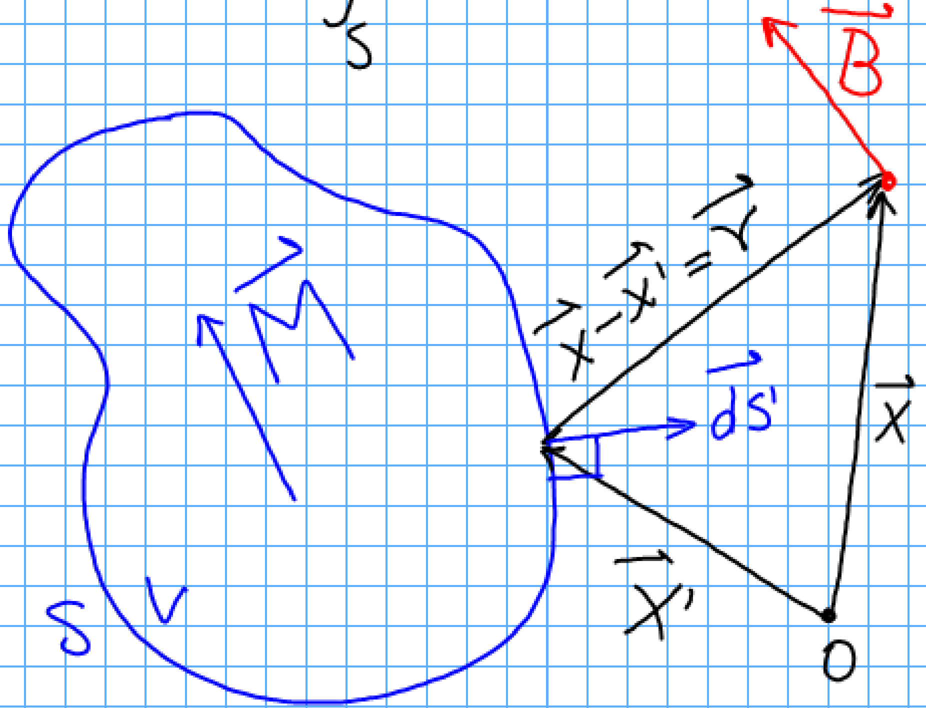
Magnetización constante ($\nabla\cdot\vec{M} = 0$)

$$V(\vec{x}) = \frac{\mu_0}{4\pi}\int_S \frac{\vec{M}\cdot\vec{ds'}}{r} = \frac{\mu_0}{4\pi}\vec{M}\cdot\int_S \frac{\vec{ds'}}{r}$$

$$\therefore \quad \vec{B} = -\nabla V = -\frac{\mu_0}{4\pi}\int_S \nabla\left(\frac{\vec{M}\cdot\hat{n}}{r}\right) ds'$$

$\vec{M} = \vec{cte}$, $\hat{n} = \hat{n}(\vec{x}\,')$ entonces:

$$\vec{B}(\vec{x}) = -\frac{\mu_0}{4\pi}\int_S \vec{M}\cdot\hat{n}\;\nabla\left(\frac{1}{r}\right) ds' = \frac{\mu_0}{4\pi}\int_S \vec{M}\cdot\hat{n}\,\frac{\vec{r}}{r^3}\,ds'$$



$$\vec{ds'} = \hat{n}\,ds'$$

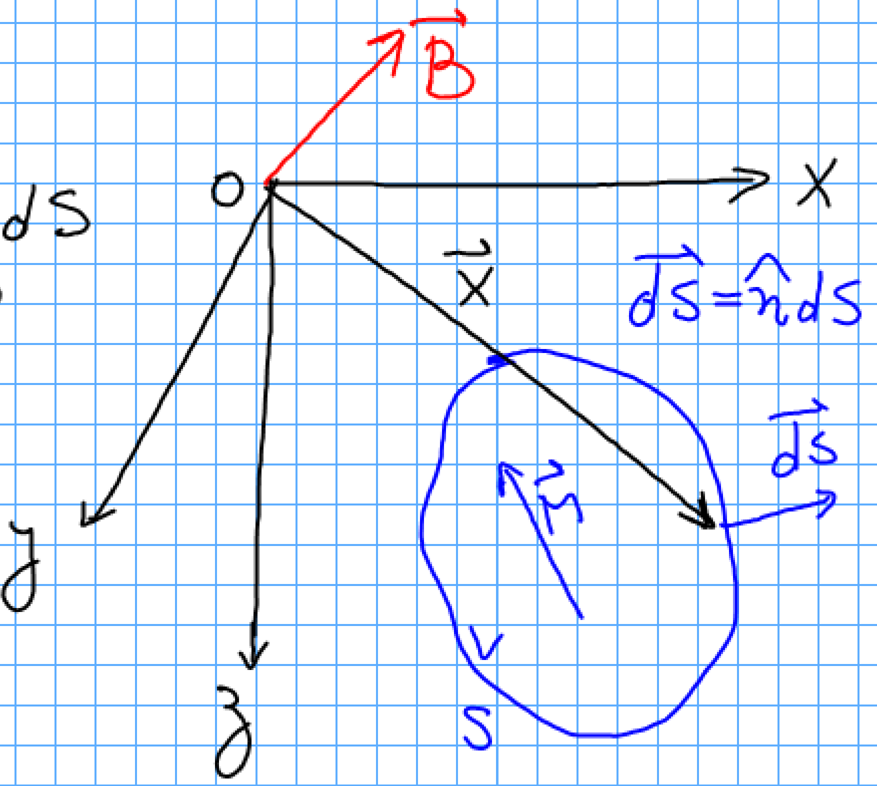
$$\hat{n} = \hat{n}(\vec{x}\,')$$

Sin perdida de generalidad, eligiendo $\vec{x} = 0$, es decir evaluando $\vec{B}$ en el origen, se tiene:

$$\vec{B}(\vec{0}) = -\frac{\mu_0}{4\pi} \int_S \vec{M} \cdot \hat{n} \frac{\vec{x}'}{|\vec{x}'|^3} dS'$$

prescindiendo de las primas $\vec{x}' \to \vec{x}$, $dS' \to dS$, para $\vec{B}$ en el origen se tiene:

$$\vec{B}(\vec{0}) = -\frac{\mu_0}{4\pi} \int_S \vec{M} \cdot \hat{n} \frac{\vec{x}}{|\vec{x}|^3} dS$$



La forma mas simple de aproximar un volumen cualquiera, es mediante un poliédrico de N caras planas S_i donde en cada una de ellas el vector unitario normal $\hat{n}_i$ es constante. En estas condiciones la última ecuación se transforma en:

$$\vec{B} = -\frac{\mu_0}{4\pi}\sum_{i=1}^{N}\left(\vec{M}\cdot\hat{n}_i\right)\int_{S_i}\frac{\vec{x}}{|\vec{x}|^3}\,dS$$