SÍSMICA

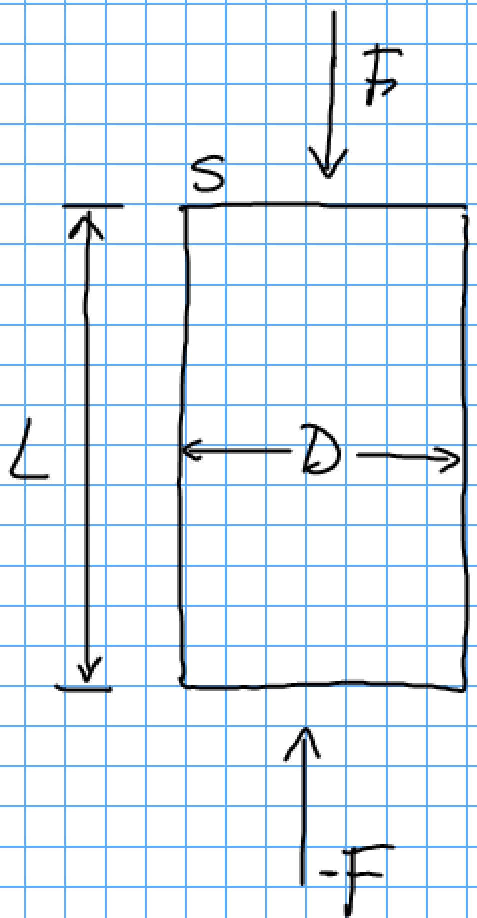
Medios Elásticos

Un medio elástico isótropo, en general inhomogéneo, se describe por 2 parámetros elásticos independientes. En simología se usan los parámetros de Lamé λ, μ . Cualquier otro parámetro elástico se puede escribir en función de ellos. Por ejemplo E, σ Modulo de Young y coeficiente de Poisson respectivamente, son:

$$\lambda = \frac{E\sigma}{(1+\sigma)(1-2\sigma)} \,, \quad \mu = \frac{E}{2(1+\sigma)}$$

Incompresibilidad: $K = -V\dfrac{\partial P}{\partial V}$

$$K = \lambda + \frac{2}{3}\mu$$



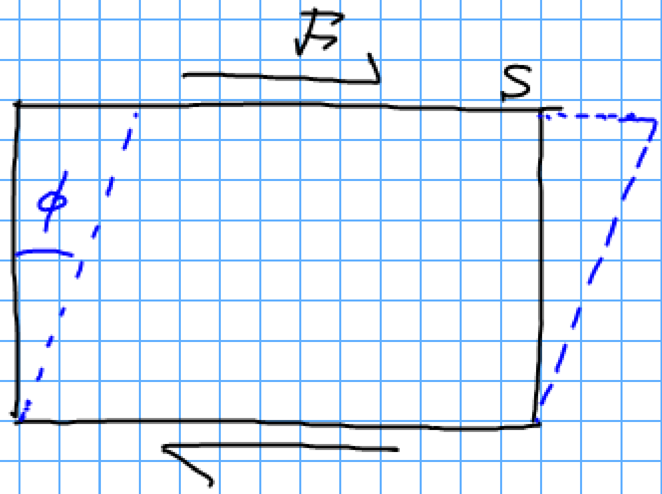
$$L \rightarrow L - \Delta L$$

$$D \rightarrow D + \Delta D$$

$$\frac{F}{S} = E\,\frac{\Delta L}{L}$$

$$E\left[\frac{Fza}{Area}\right]$$, Presión

$$\sigma = \frac{\Delta D / D}{\Delta L / L} \sim \frac{1}{4}$$

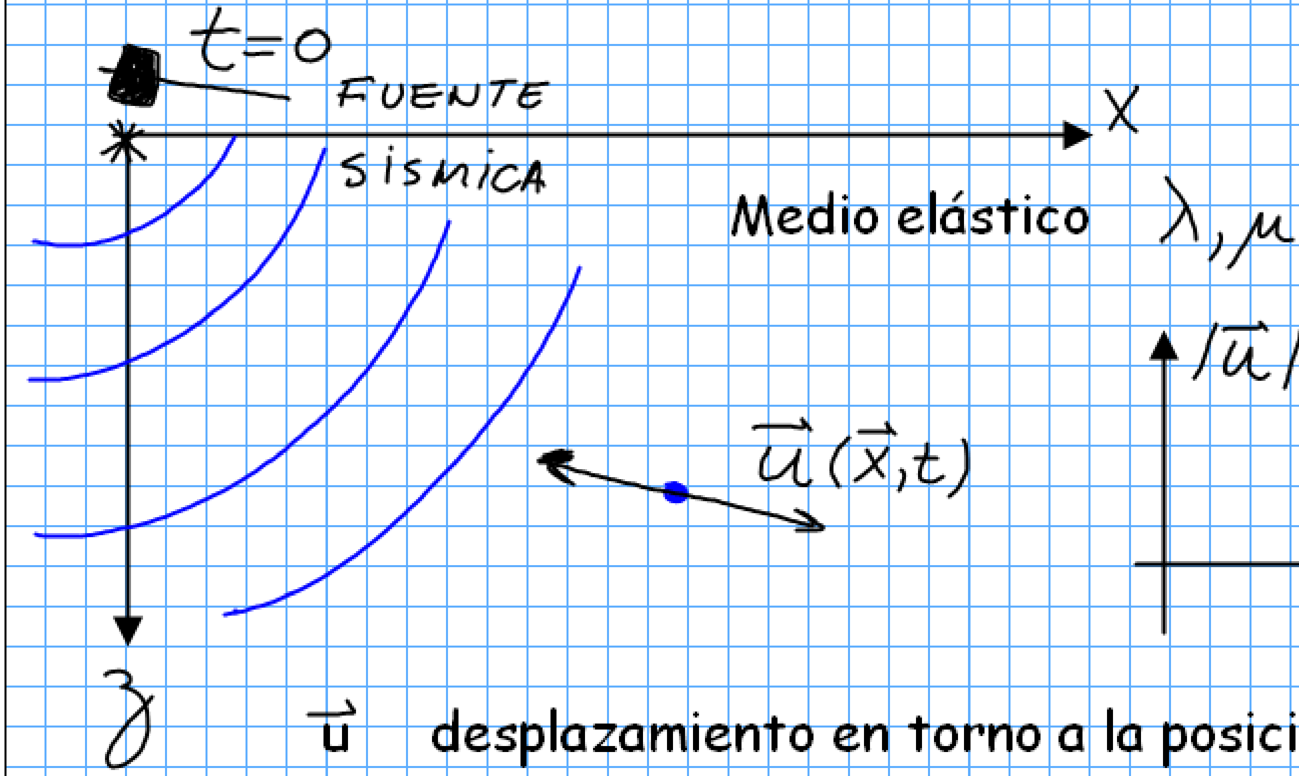


$$\frac{F}{S} = 2\mu\phi$$

μ parametro de lame (rigidez)

	$\rho\left[\frac{g}{cc}\right]$	E (Pa)	μ (Pa)	σ
Acero	7.85	2.139×10^{11}	8.19×10^{10}	0.31
Granito	2.67	$0.4 - 0.7 \times 10^{11}$	$0.2 - 0.3 \times 10^{11}$	0.1 - 0.25
Basalto	2.95	$0.6 - 0.8 \times 10^{11}$	0.3×10^{11}	0.25

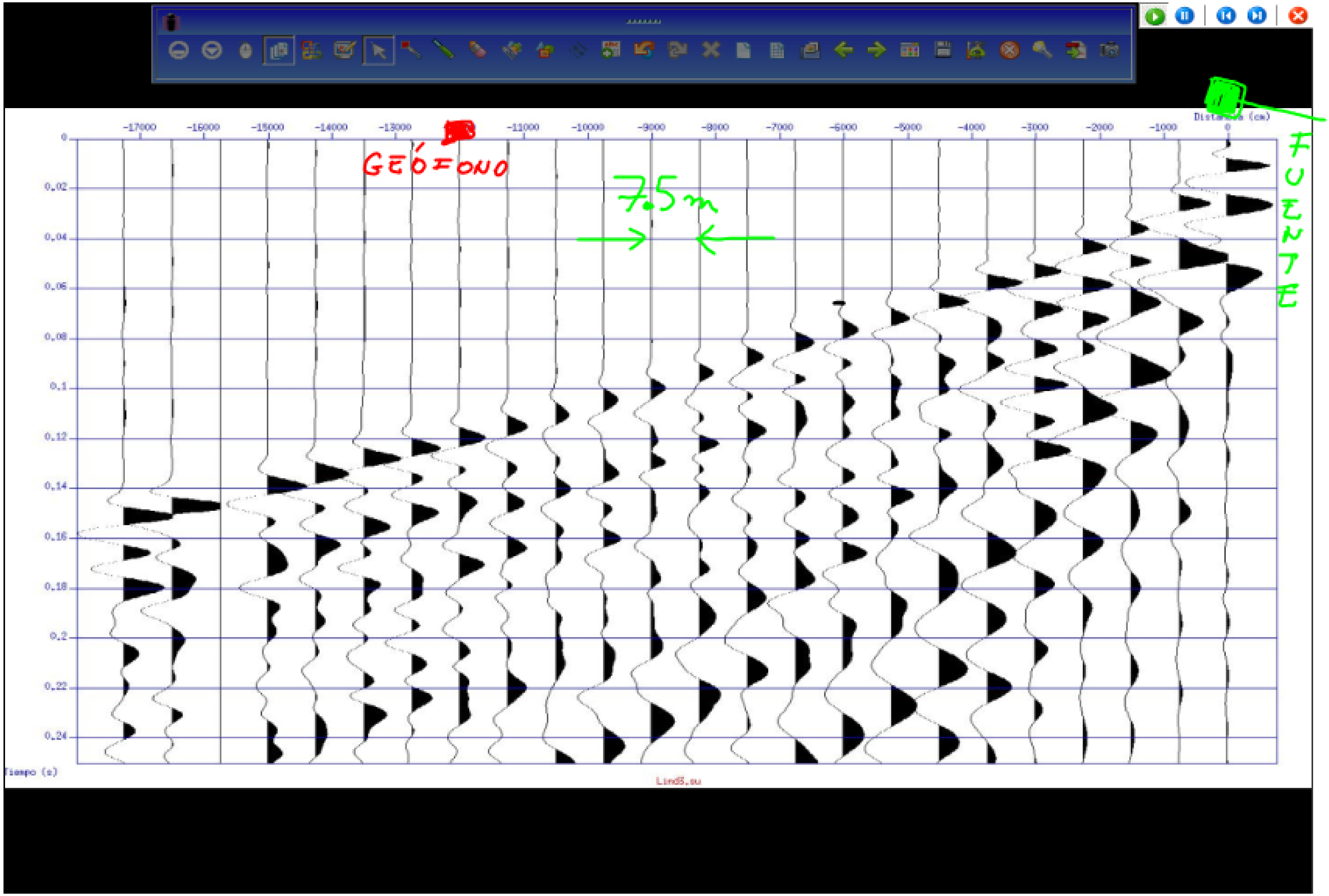
$$1\ \text{Pa} = 1\ \frac{N}{m^2}, \quad 1\ \text{bar} = 0.987\ \text{Atm} = 10^5\ \text{Pa}$$



$\vec{u}$ desplazamiento en torno a la posición de equilibrio.

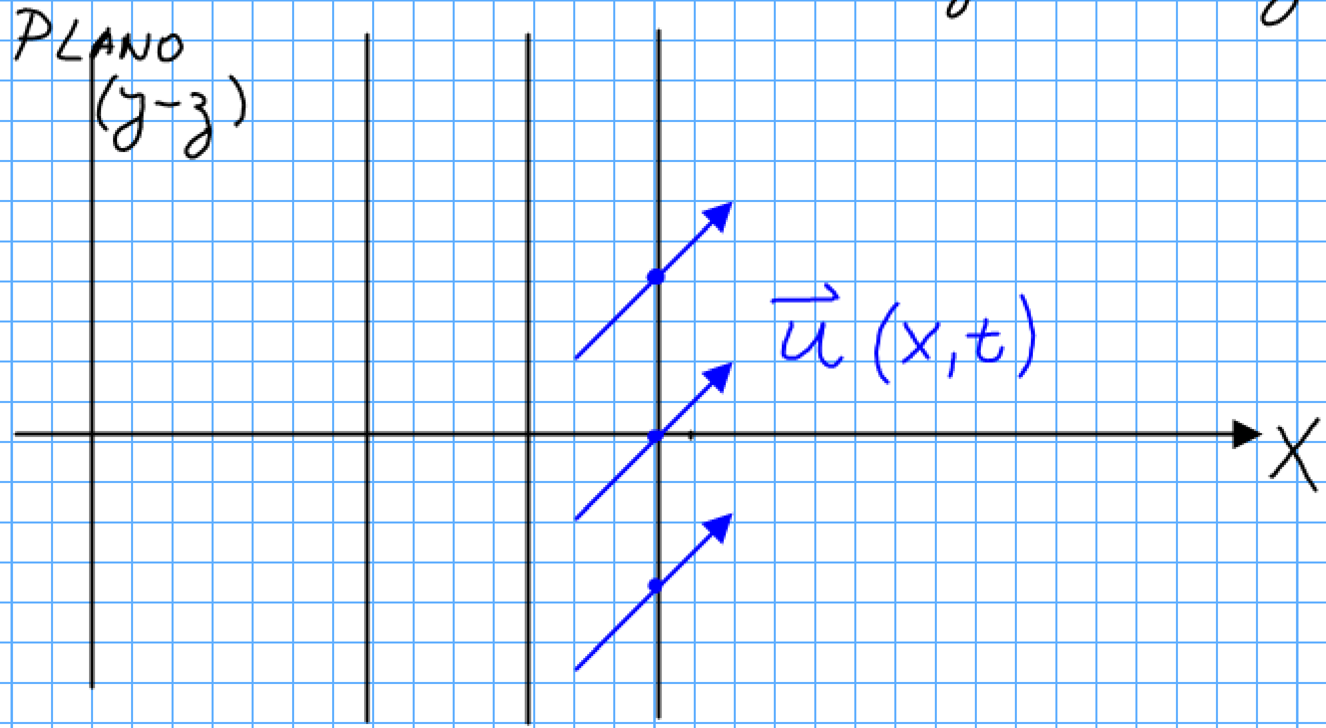
Para un medio homogéneo, la ecuación dinámica o de ondas es:

$$\rho \frac{\partial^2 \vec{u}}{\partial t^2} = (\lambda+\mu)\nabla(\nabla\cdot\vec{u}) + \mu \nabla^2 \vec{u}$$


Distancia (cm)
-17000 -16000 -15000 -14000 -13000 -11000 -10000 -9000 -8000 -7000 -6000 -5000 -4000 -3000 -2000 -1000 0
0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18 0.2 0.22 0.24
Tiempo (s)
GEÓFONO
7.5 m
FUENTE

Asumamos una solución de onda plana propagandose en el sentido x:

$$\vec{u}(\vec{x},t) = u_x(x,t)\hat{x} + u_y(x,t)\hat{y} + u_z(x,t)\hat{z}$$



Introduciendo esta solución en la Ec. de Ondas se tiene:

$$\rho \frac{\partial^2 u_x}{\partial t^3} = (\lambda + 2\mu) \frac{\partial^2 u_x}{\partial x^3}$$

$$\rho \frac{\partial^2 u_y}{\partial t^2} = \mu \frac{\partial^2 u_y}{\partial x^2} \quad , \quad \rho \frac{\partial^2 u_z}{\partial t^2} = \mu \frac{\partial^2 u_z}{\partial x^2}$$

ó

$$\frac{1}{\alpha^2} \frac{\partial^2 u_x}{\partial t^2} = \frac{\partial^2 u_x}{\partial x^2} \quad , \quad \alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} = V_p$$

$$\frac{1}{\beta^2} \frac{\partial^2 u_y}{\partial t^2} = \frac{\partial^2 u_y}{\partial x^2} \quad , \quad \beta = \sqrt{\frac{\mu}{\rho}} = V_s$$

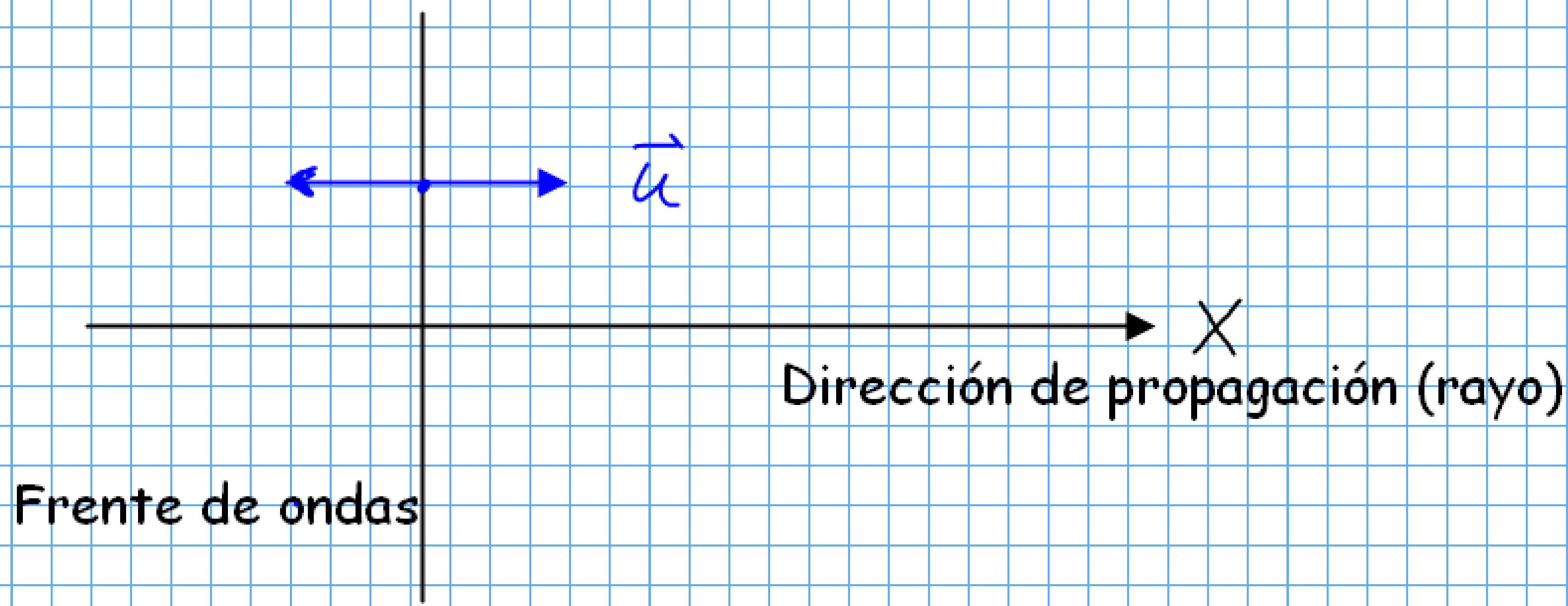
$\beta < \alpha$ Siempre

En un medio elástico se propagan 2 tipos de onda :

1) Ondas P (Primarias) con velocidad

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} = V_p$$

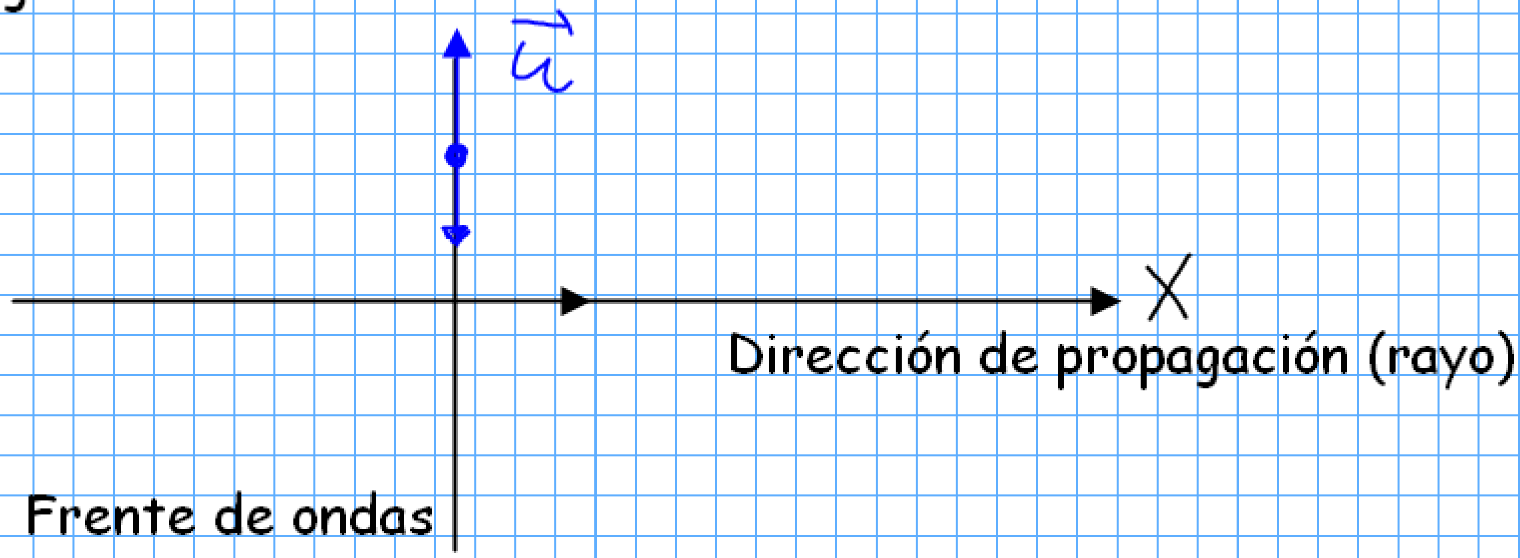
Al paso de la onda, el movimiento de las partículas del medio elástico es paralelo a la dirección de propagación:



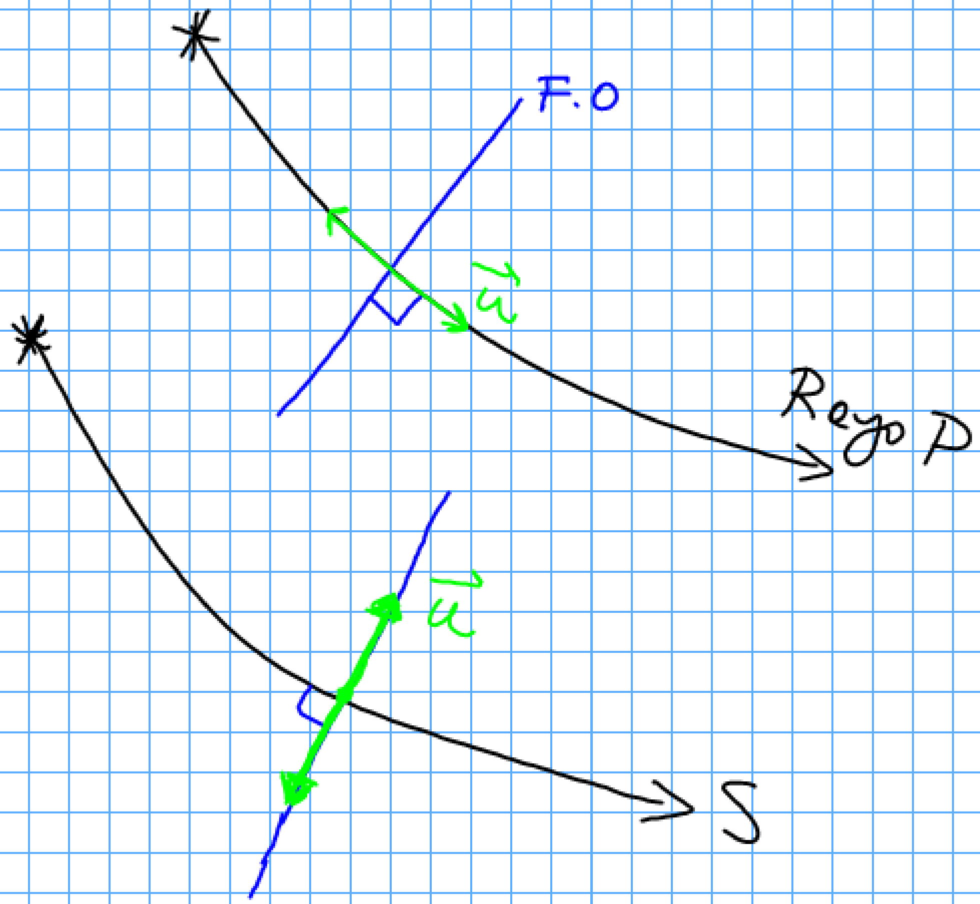
2) Ondas S (Secundaria) con velocidad

$$\beta = \sqrt{\frac{\mu}{\rho}} = V_s < \alpha$$

El movimiento de partículas es perpendicular a dirección de propagación



El concepto se genaraliza a medios inhomogéneos complejos:



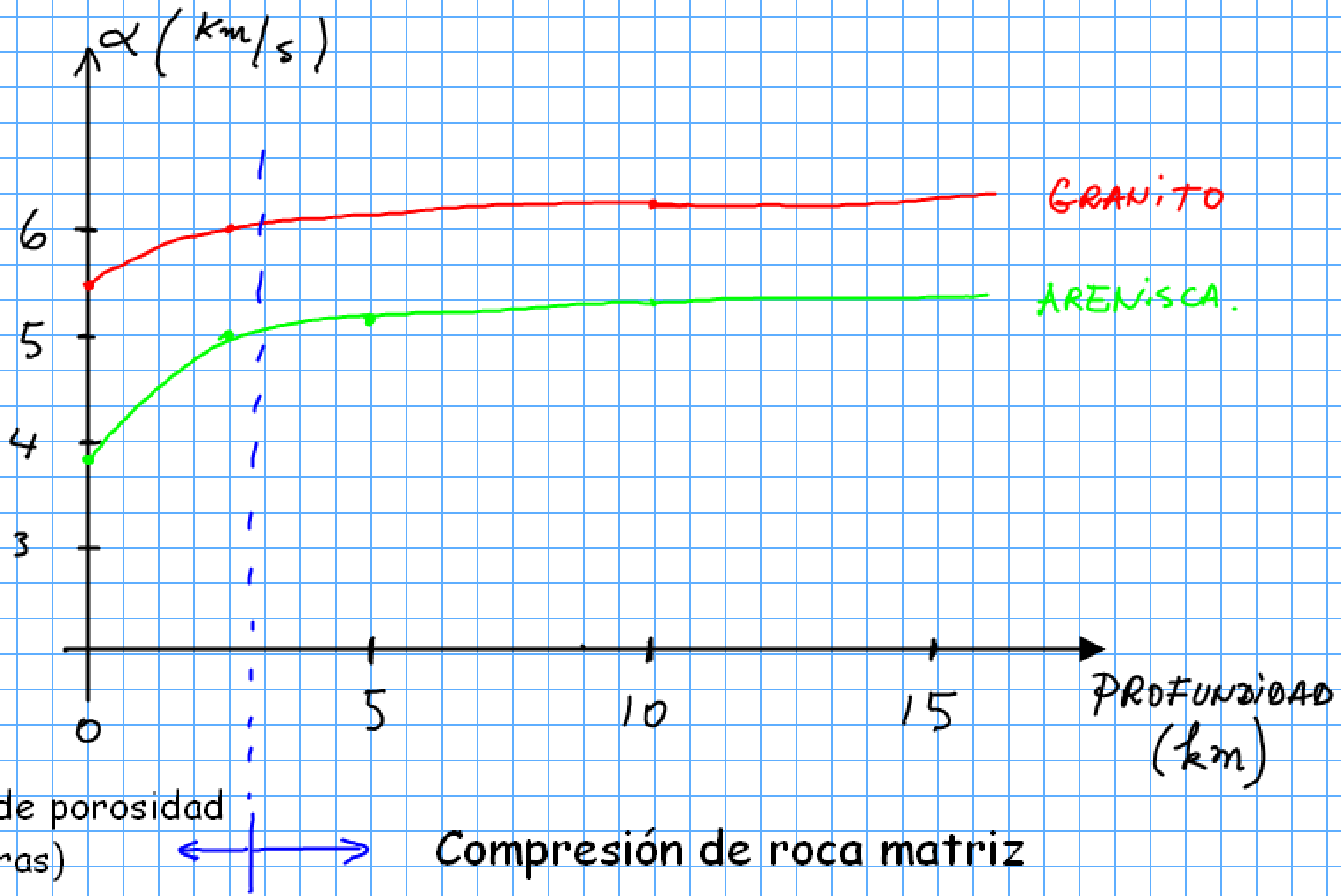
ROCA	$\alpha\,[m/s]$	$\beta\,[m/s]$
GRANITO	5640	2870
GABBRO	6450	3420
BASALTO	6400	3200
DUNITA	7400 − 8600	3790 − 4370
ARENISCA	1400 − 4300	
ARENA	1800	500
CALIZA	5970	2880
AGUA	1500	0 $(\mu=0)$

EN MUCHOS CASOS, UNA BUENA APROXIMACIÓN ES CONSIDERAR

$$\lambda = \mu$$

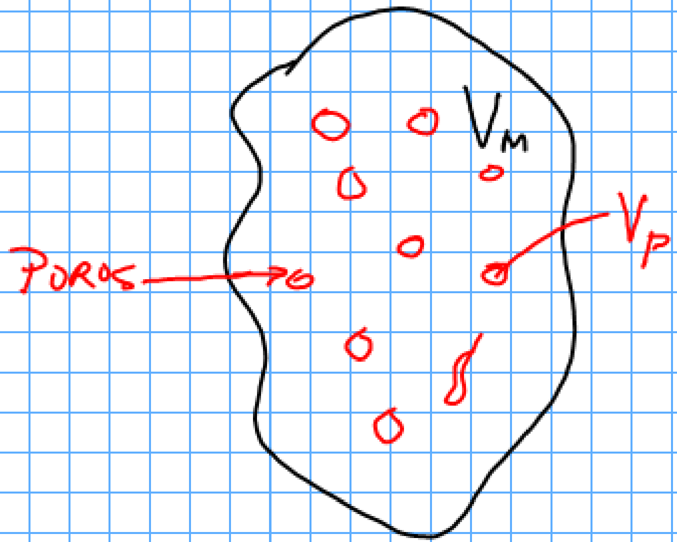
$$\frac{\alpha}{\beta} = \sqrt{3}$$

Efecto de la presión sobre velocidad de propagación



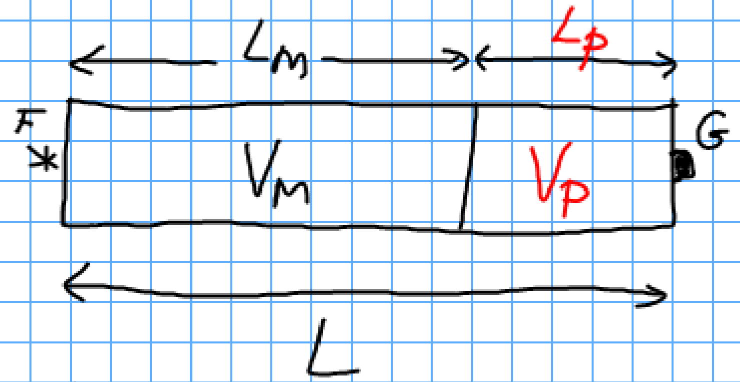
Cierre de porosidad (fracturas) ⟵|⟶ Compresión de roca matriz

Velocidad versus porosidad



Porosidad $= \phi = \dfrac{\text{Vol. Poros}}{\text{Vol Total.}}$

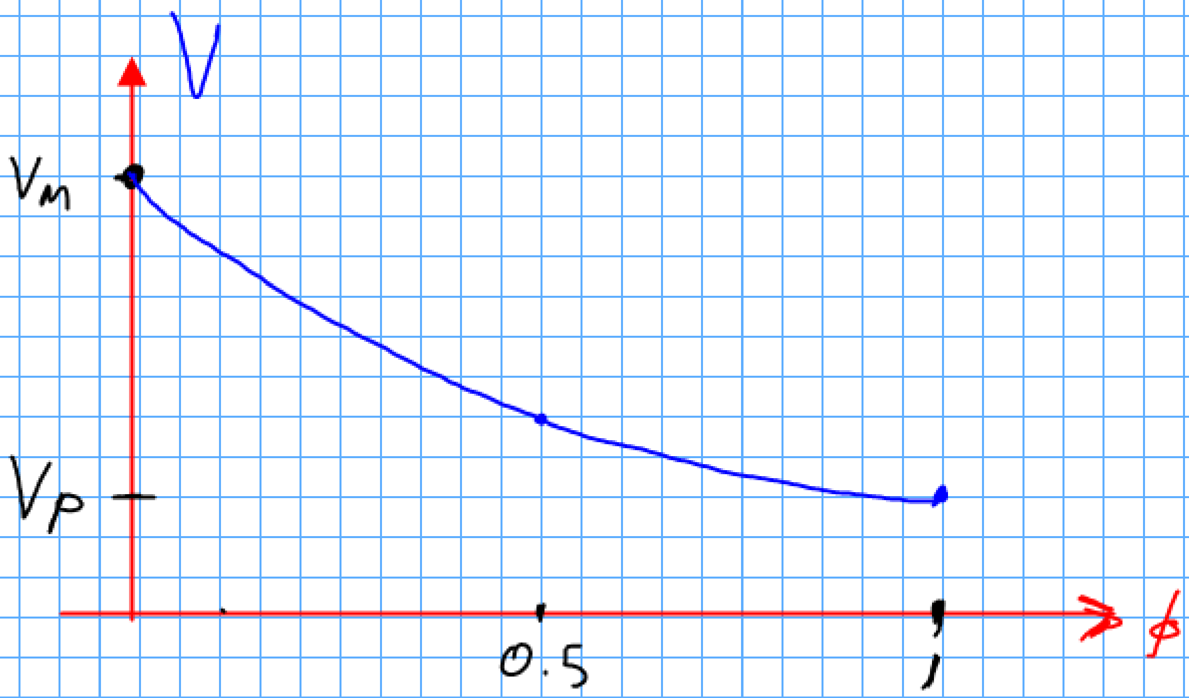
Modelo Lineal



Se tiene $L_M = (1-\phi)L$, $L_p = \phi L$

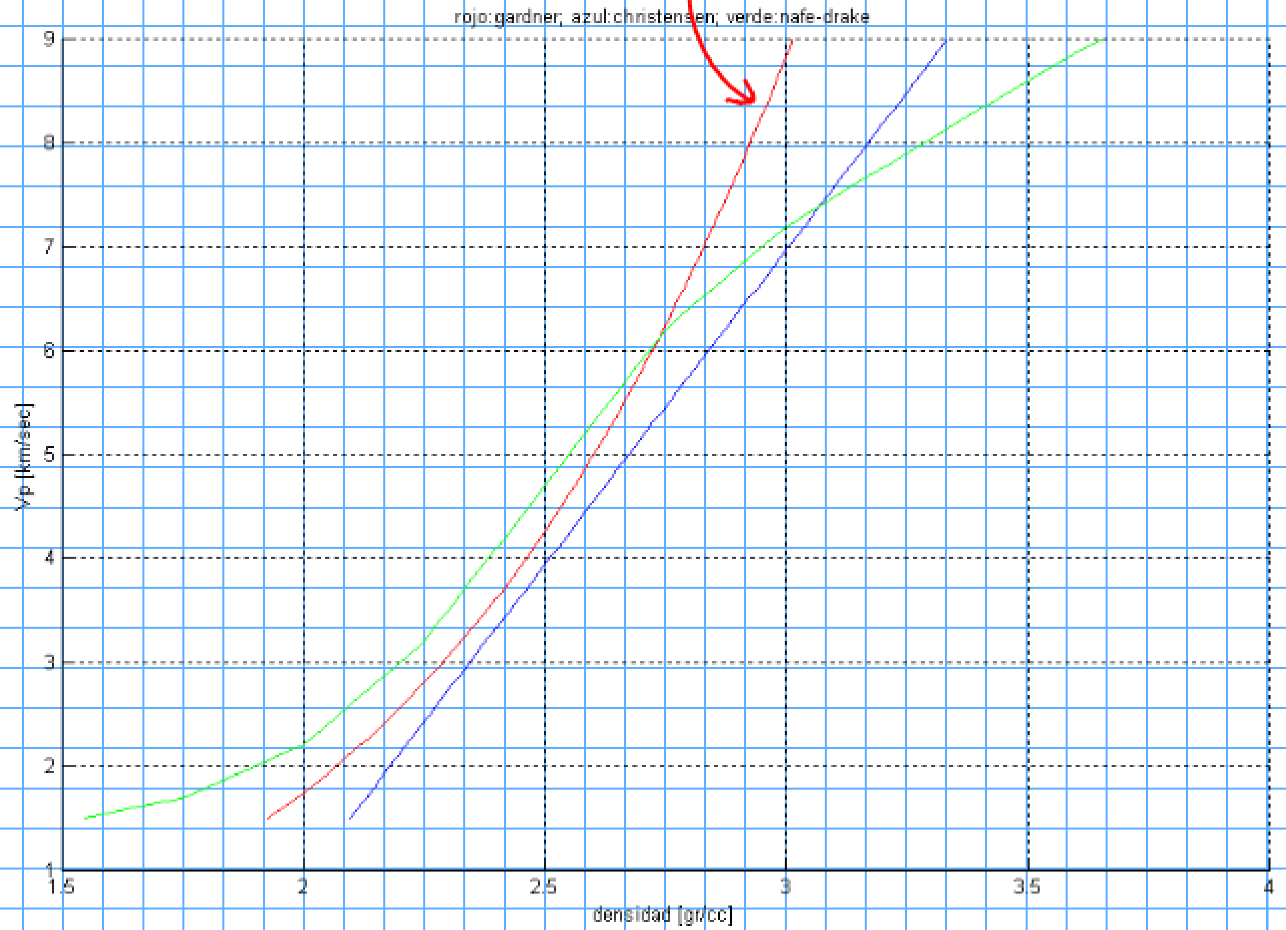
$$T = \frac{L}{V} = \frac{L_M}{V_M} + \frac{L_p}{V_p} = \frac{(1-\phi)L}{V_M} + \frac{\phi L}{V_p}$$

$$\therefore \boxed{\frac{1}{V} = \frac{1-\phi}{V_M} + \frac{\phi}{V_p}}$$

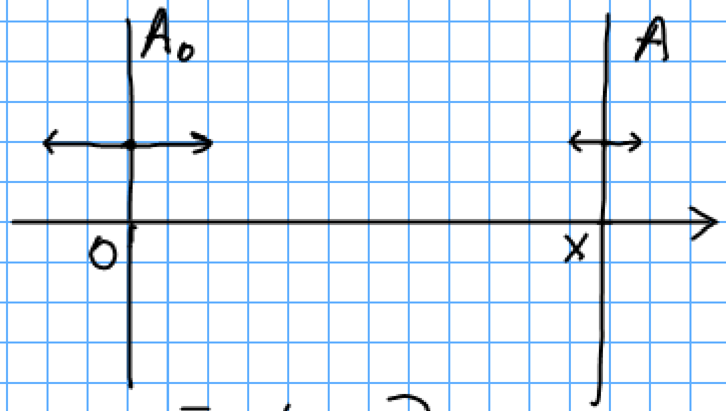

V
Vm
Vp
0.5
1
ϕ

Relación entre densidad y velocidad de propagación (Vp)

$$\rho = 1.74\, V_p^{1/4}$$



Atenuación Intrinseca (Roce interno)



$$A = A_0 e^{-\alpha x}$$

$\alpha \left[\frac{1}{\text{distancia}}\right]$ = coeficiente de absorción, $\alpha = \alpha(f)$

Por ejemplo para un cierto granito: (50 hz)

$$\alpha = 0.384 \text{ km}^{-1}$$

Para ciertos tipos de Areniscas:

$$\alpha \sim 1.0 - 2.0 \text{ km}^{-1}$$

En sismología se usa el factor de calidad Q, que es aproximadamente cte. Se tiene:

$$\alpha = \frac{\pi f}{V Q}$$

f, frecuencia

V, Velocidad.

$$\therefore \quad Q = \frac{\pi f}{\alpha V}$$

Para el granito anterior:

$$Q = \frac{3.14 \times 50}{0.384 \times \underset{(km/s)}{5.0}} = 82. \quad \text{(Adimensional)}$$

Usando Q:

$$A = A_0 e^{-\frac{\pi f}{VQ} x} = A_0 e^{-\frac{\pi f}{Q} t}, \quad t = \frac{x}{V}$$

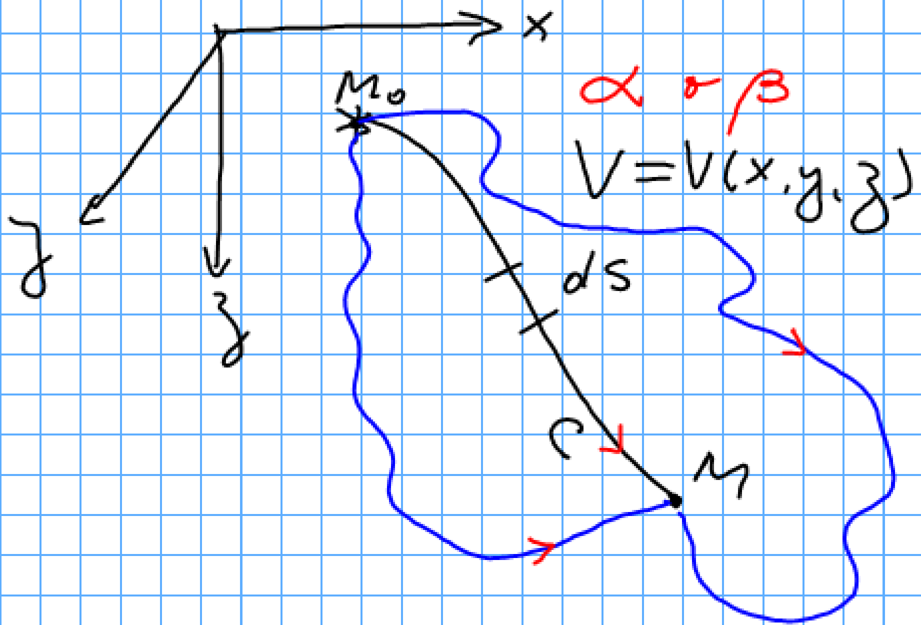
$$= A_0 e^{-\frac{t}{t_*}}, \quad t_* = \frac{Q}{\pi f}$$

$$A = A_0 e^{-1} = \frac{A_0}{e}, \quad \frac{\pi f}{VQ} x = 1 = \frac{\pi f}{f \lambda Q} x$$

$$\frac{x}{\lambda} = \frac{Q}{\pi}$$

= # de longitudes de onda en que por atenuación intrinseca, la amplitud decae en un factor 1/e.

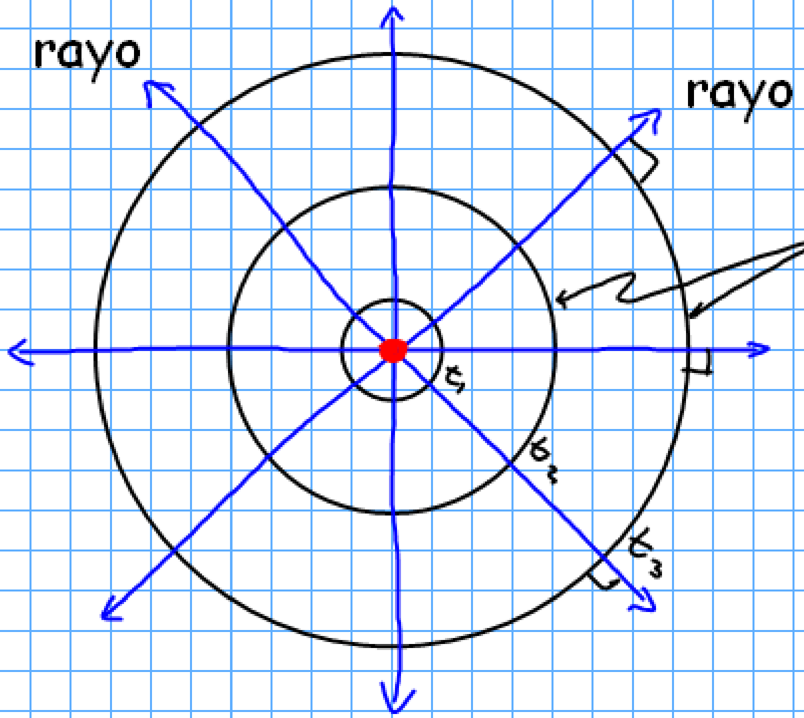
Principio de FERMAT: Teoría de rayos



$$T = \int_{M_0,\,C}^{M} \frac{dS}{V}$$

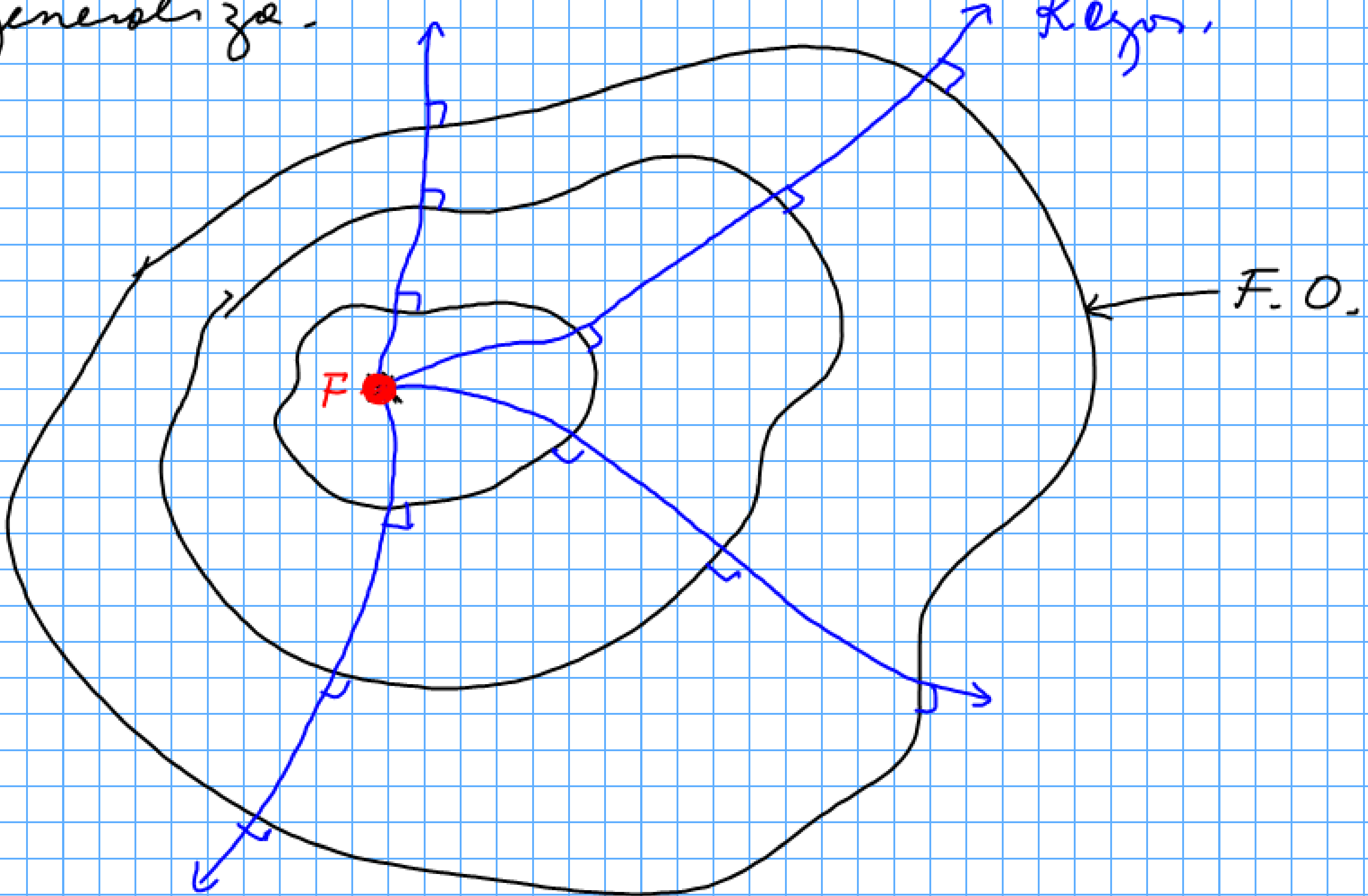
La señal sísmica se propaga desde MO a M a lo largo de la curva (C) que hace la integral de tiempo estacionaria (un mínimo). La curva para la cual T es estacionaria es lo que denominamos "rayo".

Si V es constante, los rayos son rectos.



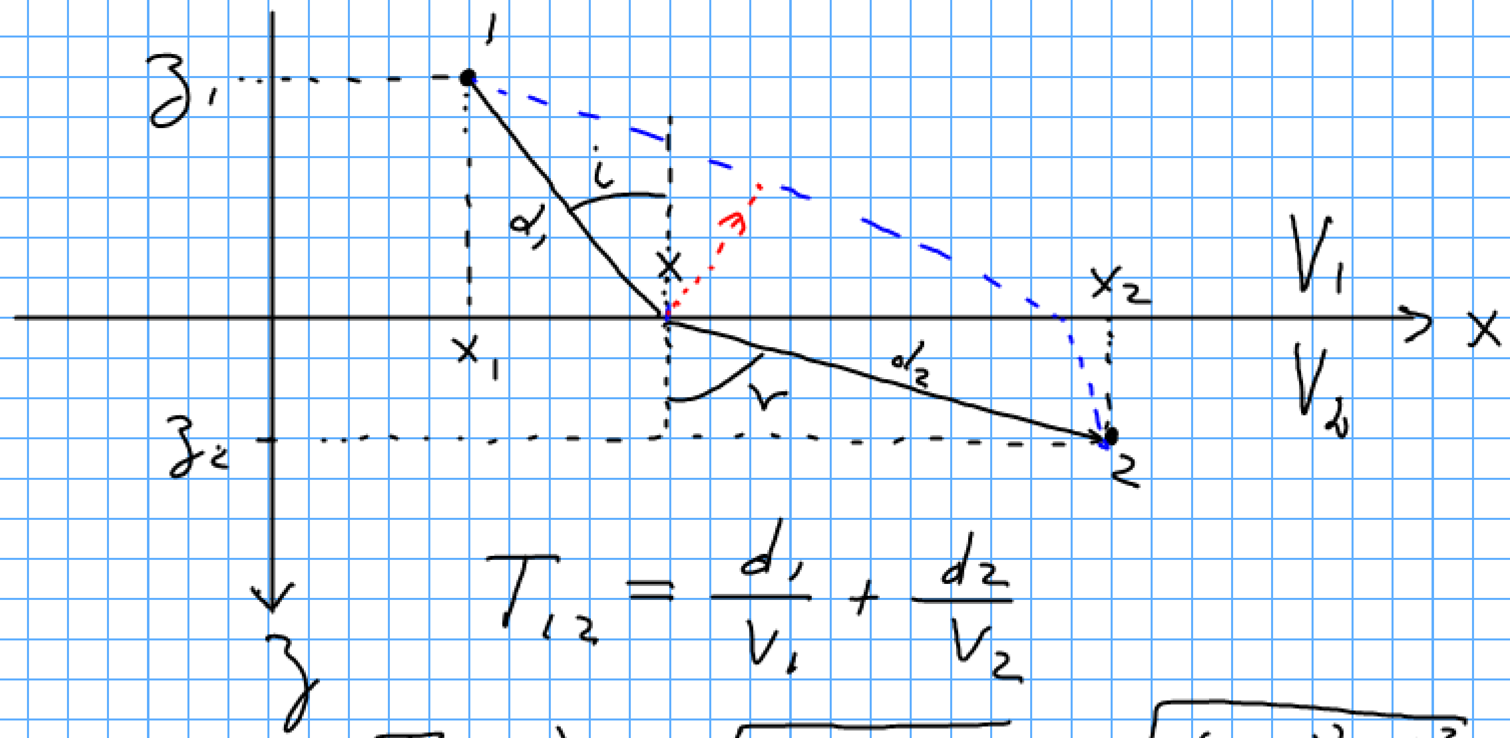
Frentes de onda esféricos,
superficies en el espacio
perpendiculares a rayos

Si V varíe punto a punto este concepto se generaliza.



Leyes básicas para determinar la geometría de los rayos: Reflexión y Refracción

REFRACCIÓN



$$T_{12} = \frac{d_1}{V_1} + \frac{d_2}{V_2}$$

$$T_{12}(x) = \frac{\sqrt{(x-x_1)^2+z_1^2}}{V_1} + \frac{\sqrt{(x-x_2)^2+z_2^2}}{V_2}$$

Condición de mínimo para T_{12} :

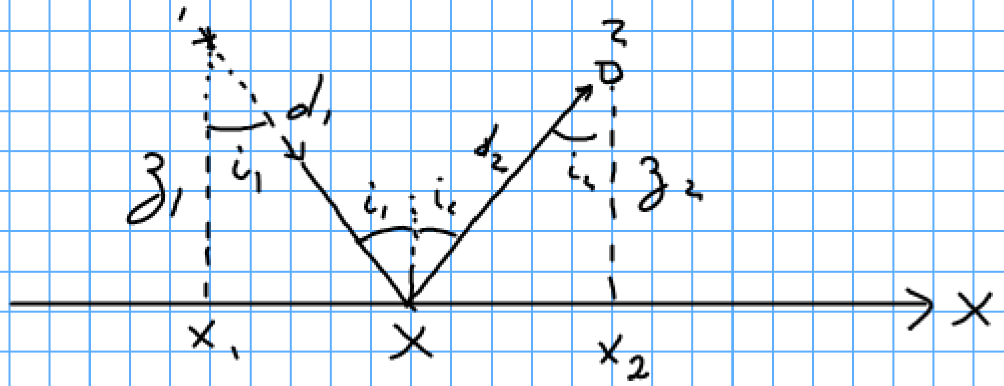
$$\frac{dT_{12}}{dx} = 0 \implies$$

$$\frac{x - x_1}{V_1 d_1} = \frac{x_2 - x}{V_2 d_2} \implies$$

$$\frac{\operatorname{sen} i}{V_1} = \frac{\operatorname{sen} r}{V_2}$$ Ley de Snell !!

$$= p$$ = Parámetro de rayo. Se mantiene constante en la propagación

REFLEXIÓN



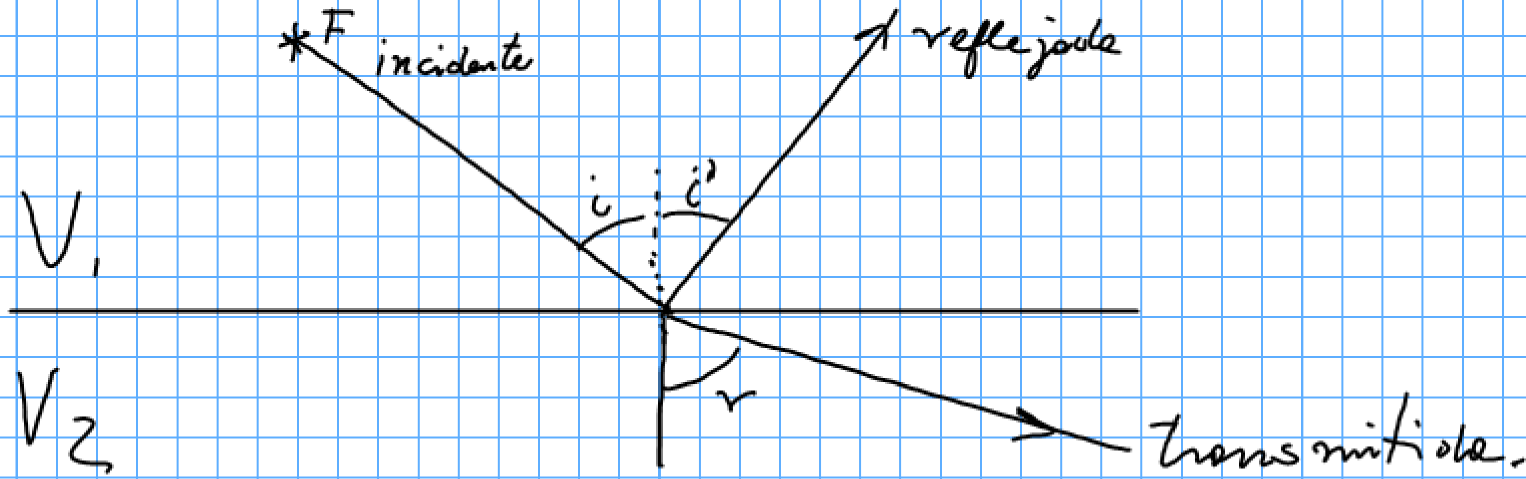
$$T_{12} = \frac{d_1}{V_1} + \frac{d_2}{V_2}$$

P. FERMAT: $\frac{dT_{12}}{dx} = 0 \Longrightarrow$

$$\frac{\operatorname{Sen} i_1}{V_1} = \frac{\operatorname{Sen} i_2}{V_2} = p$$, parámetro de rayo

Sí $V_1 = V_2 \Longrightarrow i_1 = i_2$, Ley clásica de reflexión

RESUMEN

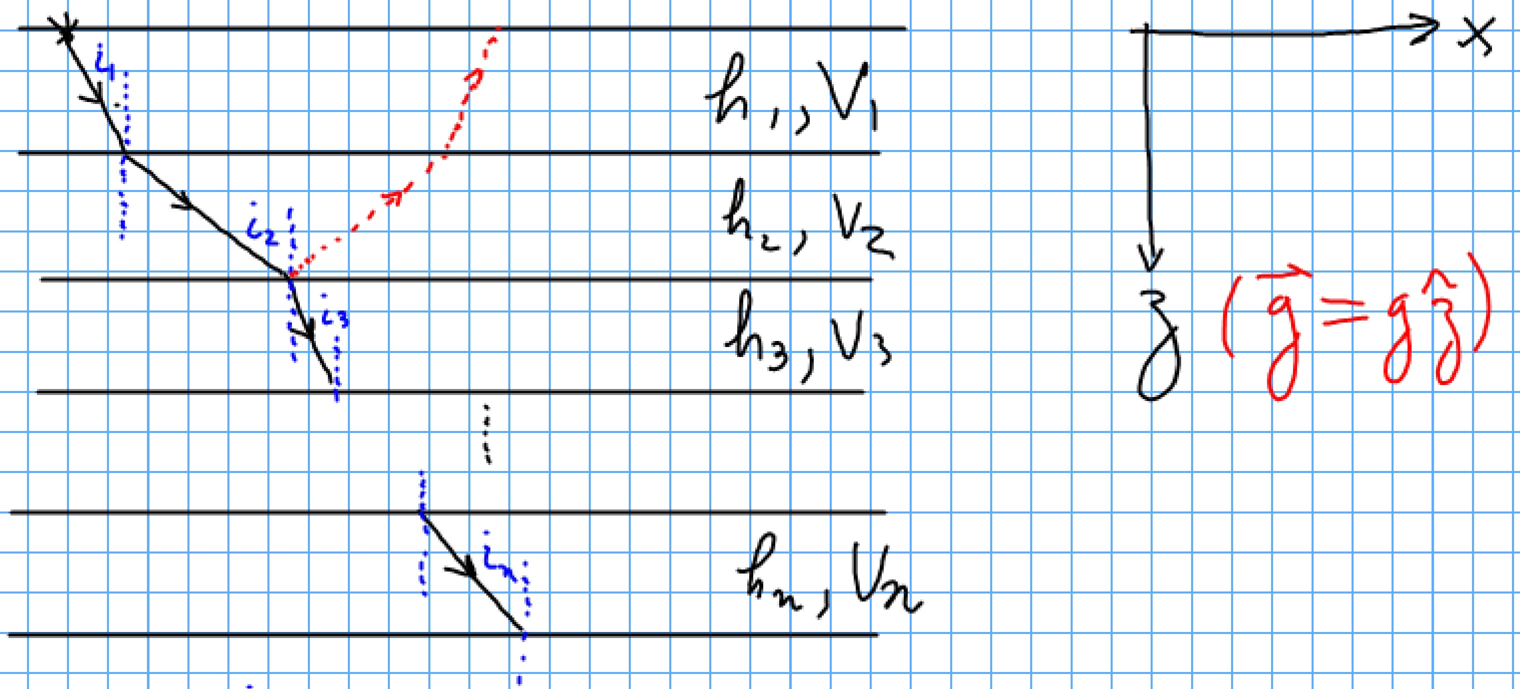


$i = i'$ si la reflejada es del mismo tipo que la incidente.

$$\frac{\operatorname{sen} i}{V_1} = \frac{\operatorname{sen} r}{V_2}$$, Siempre.

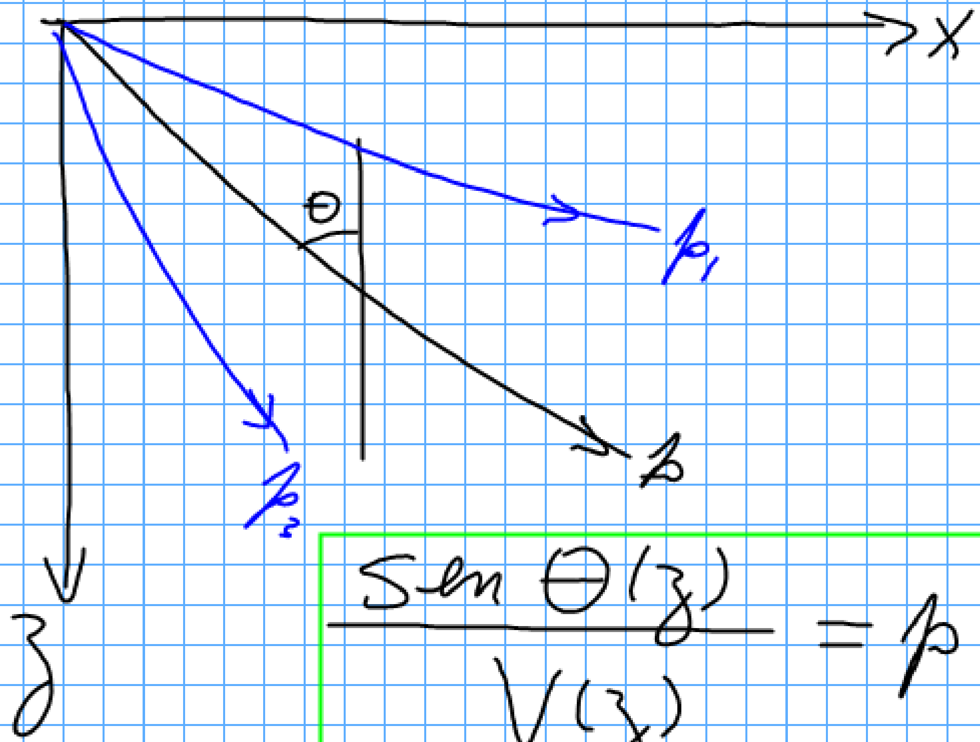
Propagación de ondas en medios donde V depende solo de la profundidad (z)

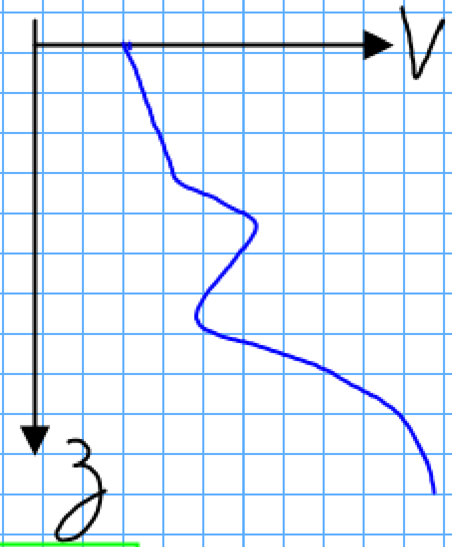
Capas planas homogéneas horizontales



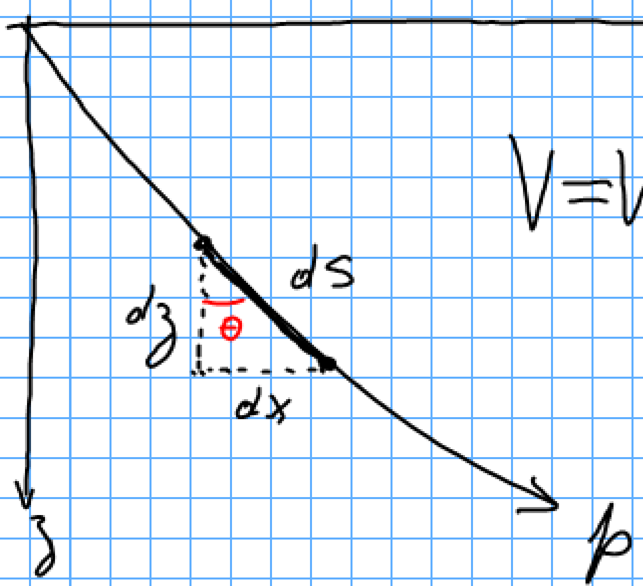
$$\frac{\operatorname{sen} i_1}{V_1} = \frac{\operatorname{sen} i_2}{V_2} = \frac{\operatorname{sen} i_3}{V_3} = \cdots = \frac{\operatorname{sen} i_n}{V_n} = p = \text{Cte.}$$

Variación general de V con z: V = V(z)





$$\boxed{\frac{\operatorname{sen}\theta(z)}{V(z)} = p = \text{cte.}}$$



$$V = V(z)$$

$$p = \frac{\operatorname{sen}\theta}{V}, \quad \operatorname{sen}\theta = pV$$

$$\cos\theta = \sqrt{1 - \operatorname{sen}^2\theta}$$

$$= \sqrt{1 - (pV)^2}$$

$$dx = \tan\theta \, dz$$

$$ds = \frac{dz}{\cos\theta}, \quad dT = \frac{ds}{V} = \frac{dz}{V\cos\theta}.$$

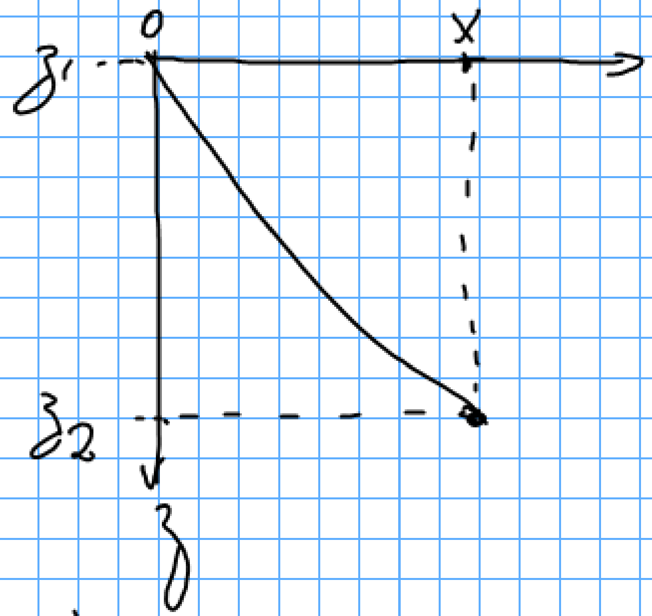
$$\therefore \quad dx = \tan\theta \, dz = \frac{\operatorname{sen}\theta}{\cos\theta} dz = \frac{pV}{\sqrt{1-(pV)^2}} dz$$

$$dT = \frac{dz}{V\cos\theta} = \frac{dz}{V\sqrt{1-(pV)^2}}$$

Por integración obtenemos X, T:

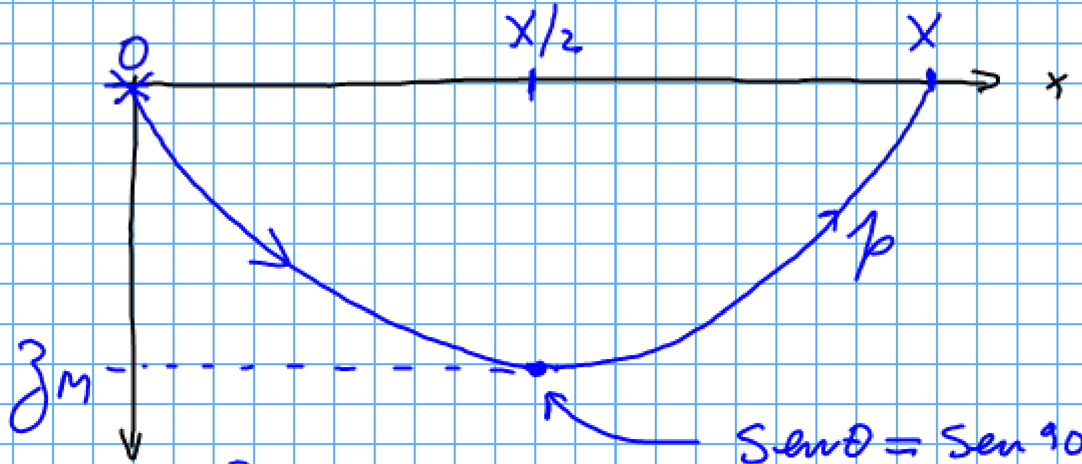
$$X = \int_{z_1}^{z_2} \frac{pV}{\sqrt{1-(pV)^2}}\, dz$$

$$T = \int_{z_1}^{z_2} \frac{dz}{V\sqrt{1-(pV)^2}}$$



$$V = V(z)$$

Rayo que se refracta continuamente, que tiene un punto de retorno, y vuelve luego a la superficie



$\operatorname{sen}\theta = \operatorname{sen} 90^\circ = 1$

$$X = 2\int_0^{z_M} \frac{pV}{\sqrt{1-(pV)^2}}\,dz \qquad \frac{\operatorname{sen}\theta}{V_M} = \frac{1}{V_M} = p, \; V_M = \frac{1}{p}$$

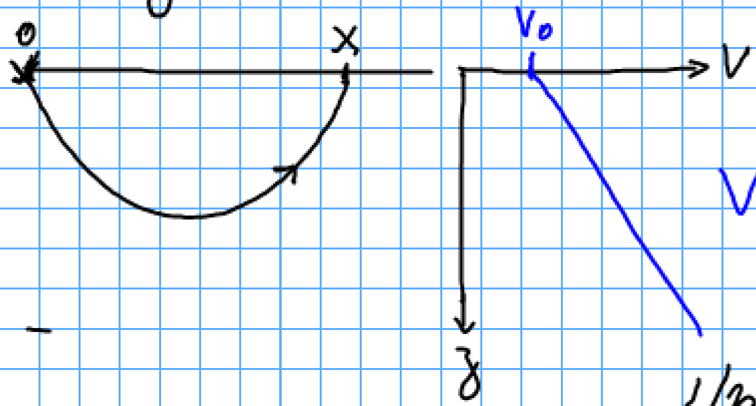
Normalmente en estos casos se cambia de $z \rightarrow V$

$$dz = \frac{dz}{dV}\,dV \qquad X(p) = 2\int_{V_0}^{1/p} \frac{pV}{\sqrt{1-(pV)^2}}\,\frac{dz}{dV}\,dV$$

Similarmente:

$$T(p) = 2\int_{V_0}^{1/p} \frac{1}{V\sqrt{1-(pV)^2}}\,\frac{dz}{dV}\,dV$$

Ej:



$$V = V_0 + kz \longrightarrow \frac{dz}{dV} = \frac{1}{k}.$$

$$X = \frac{2}{k}\int_{V_0}^{1/p} \frac{pV}{\sqrt{1-(pV)^2}}\,dV, \quad T = \frac{2}{k}\int_{V_0}^{1/p} \frac{dV}{V\sqrt{1-(pV)^2}}$$

$$X = X(p), \qquad T = T(p)$$

Integrando:

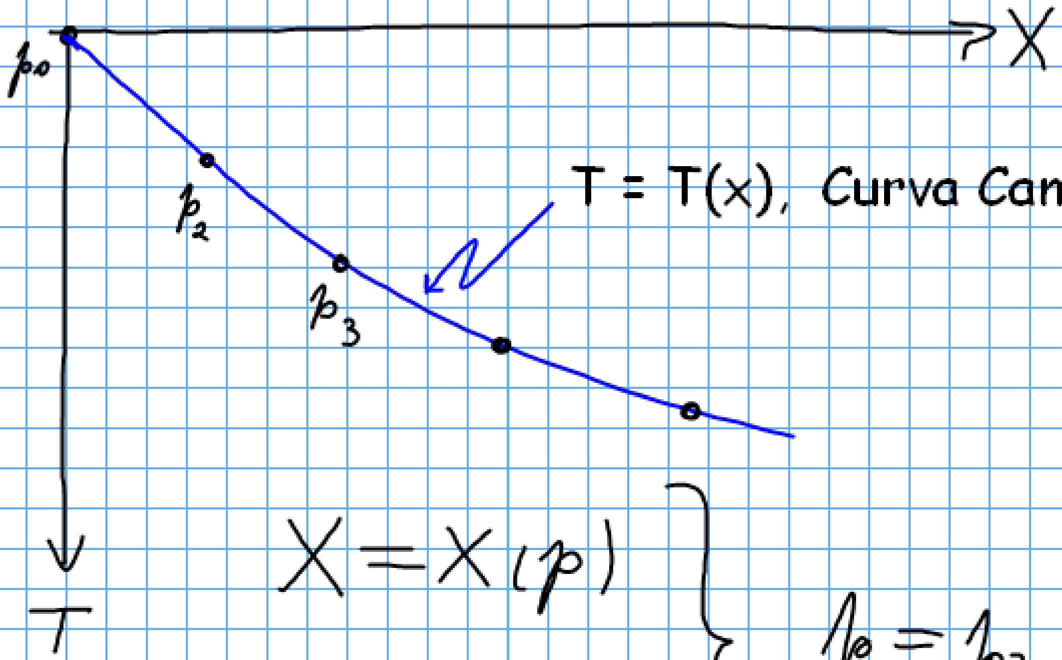
$$X(p) = \frac{2}{kp}\sqrt{1-(pV_0)^2}$$

$$T(p) = \frac{2}{k}\ln\left[\frac{1+\sqrt{1-(pV_0)^2}}{pV_0}\right]$$

En este caso se puede obtener $T=T(x)$ explícitamente:

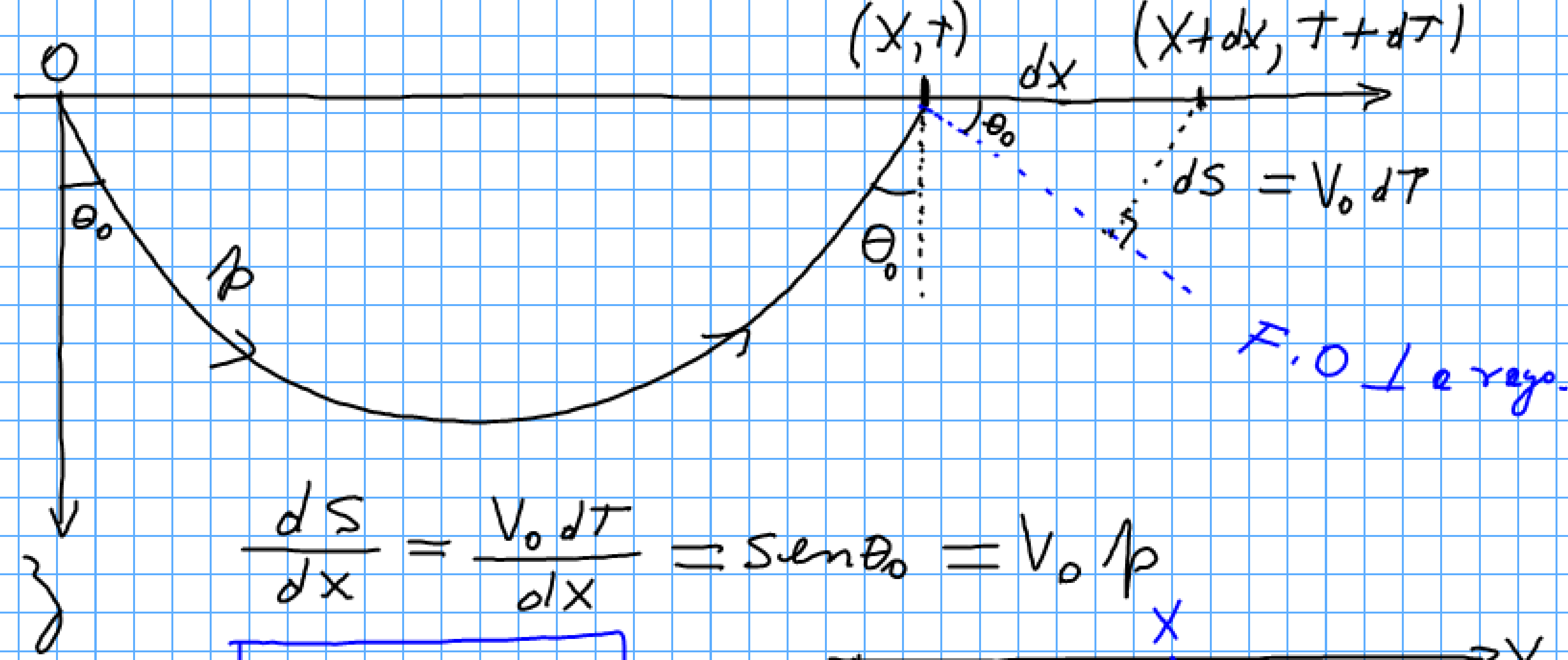
$$T(x) = \frac{2}{k}\,\mathrm{senh}^{-1}\left(\frac{kx}{2V_0}\right)$$

Curva Camino - tiempo. En general $\longrightarrow$



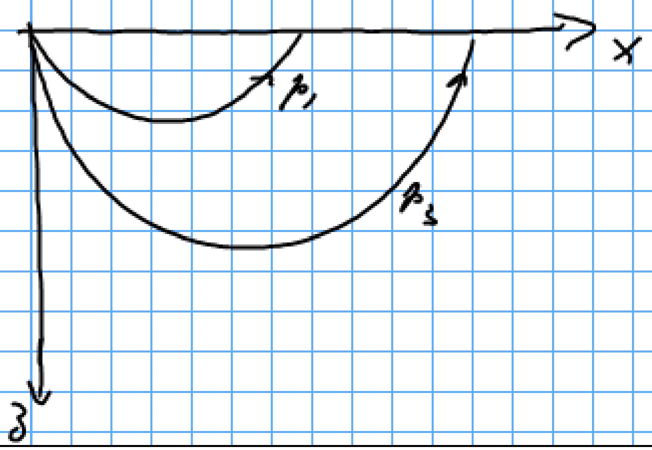
T = T(x), Curva Camino - Tiempo

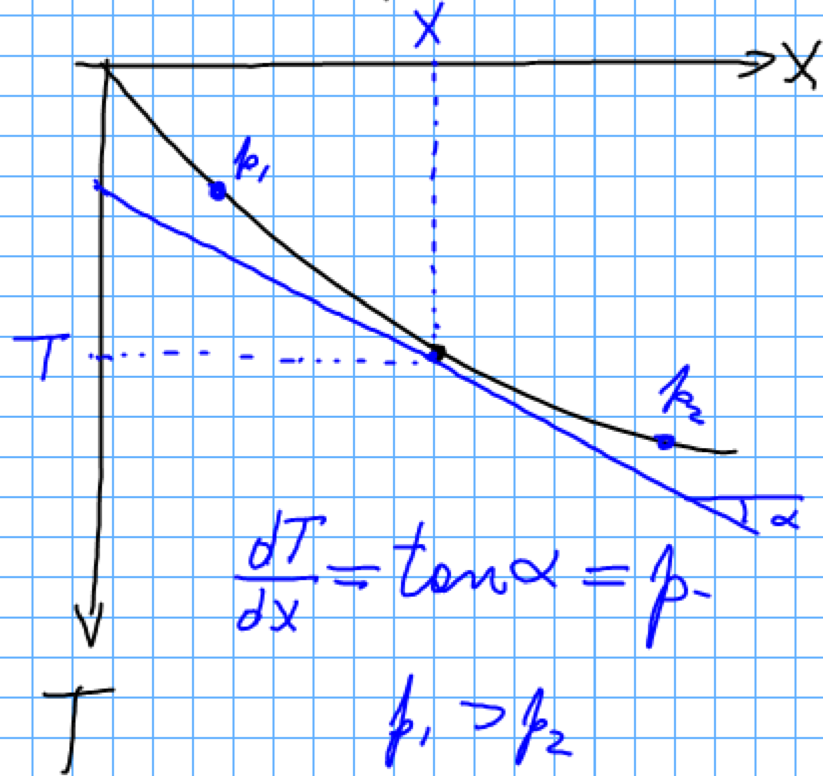
$$\left.\begin{array}{l} X = X(p) \\ T = T(p) \end{array}\right\} \quad p = p_0, p_1, p_2 \ldots$$



$$\frac{dS}{dX} = \frac{V_0\, dT}{dX} = \operatorname{sen}\theta_0 = V_0\, p$$

$$\boxed{\frac{dT}{dX} = p.}$$

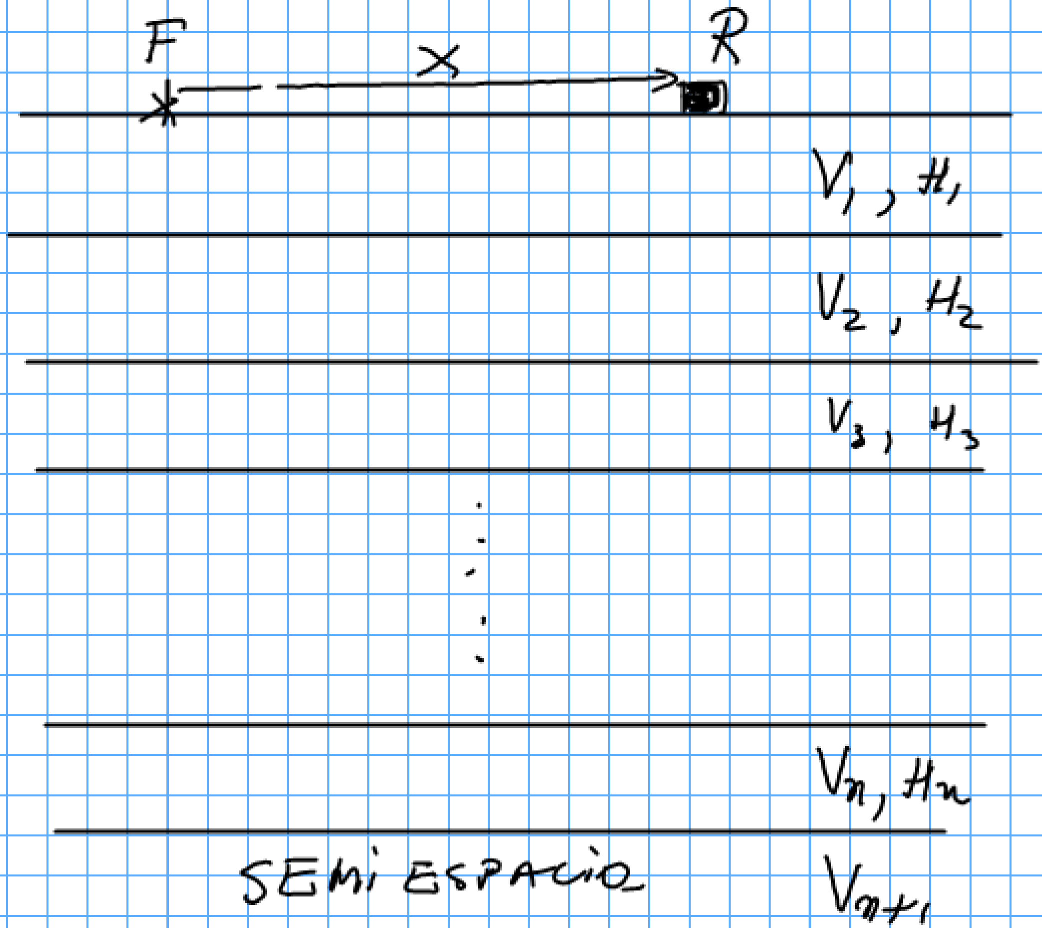


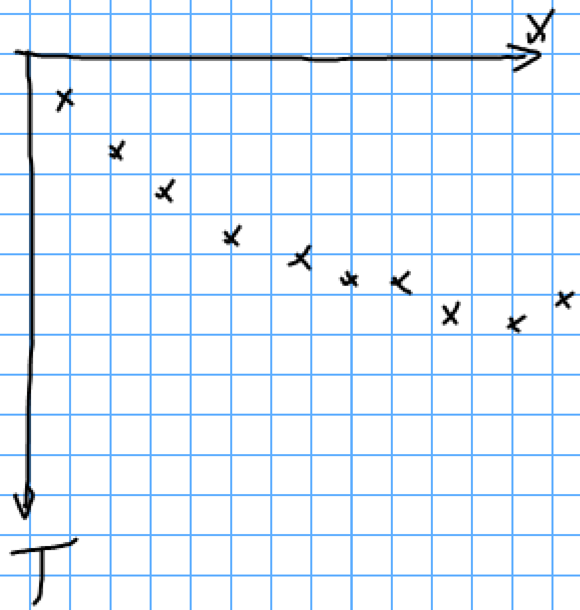


$$\frac{dT}{dX} = \tan\alpha = p.$$

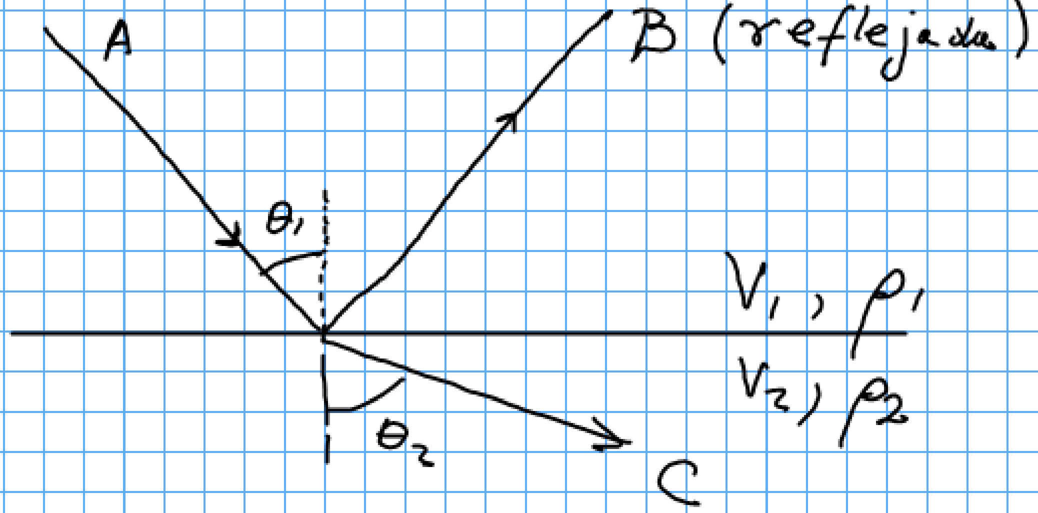
$$p_1 > p_2$$

Modelo de capas planas homogéneas





1 Interfaz



$$R = \frac{B}{A}$$ = Coeficiente de reflexión

$$T = \frac{C}{A}$$ = Coeficiente de Transmisión

En el caso acústico $\beta_1 = \beta_2 = 0$, $\alpha_1 = V_1$, $\alpha_2 = V_2$
(APROXIMACIÓN ADECUADA), se tiene:

$$R = \frac{\rho_2 q_1 - \rho_1 q_2}{\rho_2 q_1 + \rho_1 q_2} \qquad q_1 = \sqrt{\frac{1}{V_1^2} - p^2}$$

$$q_2 = \sqrt{\frac{1}{V_2^2} - p^2}$$

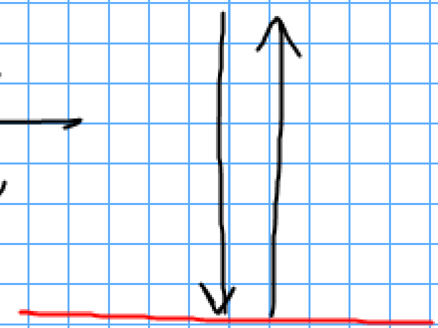
$$p = \frac{\operatorname{sen}\theta_1}{V_1} = \frac{\operatorname{sen}\theta_2}{V_2} = \text{cte.}$$

$$R = \frac{\rho_2 \sqrt{\frac{1}{V_1^2} - p^2} - \rho_1 \sqrt{\frac{1}{V_2^2} - p^2}}{\% \quad + \quad \%} = R(p)$$

$$T = 1 + R \quad !!$$

(Coeficientes para amplitudes, no energías) para $p=0$ $(\theta_1=\theta_2=0)$ incidencia normal.

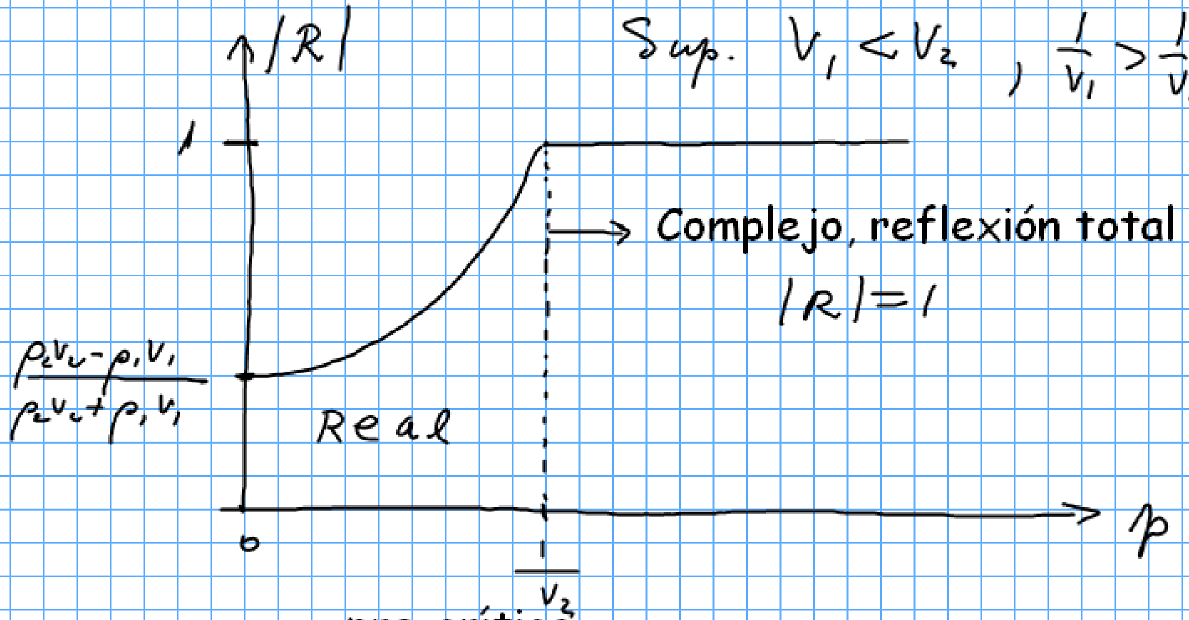
$$R(p=0) = \frac{\frac{\rho_2}{V_1} - \frac{\rho_1}{V_2}}{\% + \%} = \frac{\rho_2 V_2 - \rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1}$$

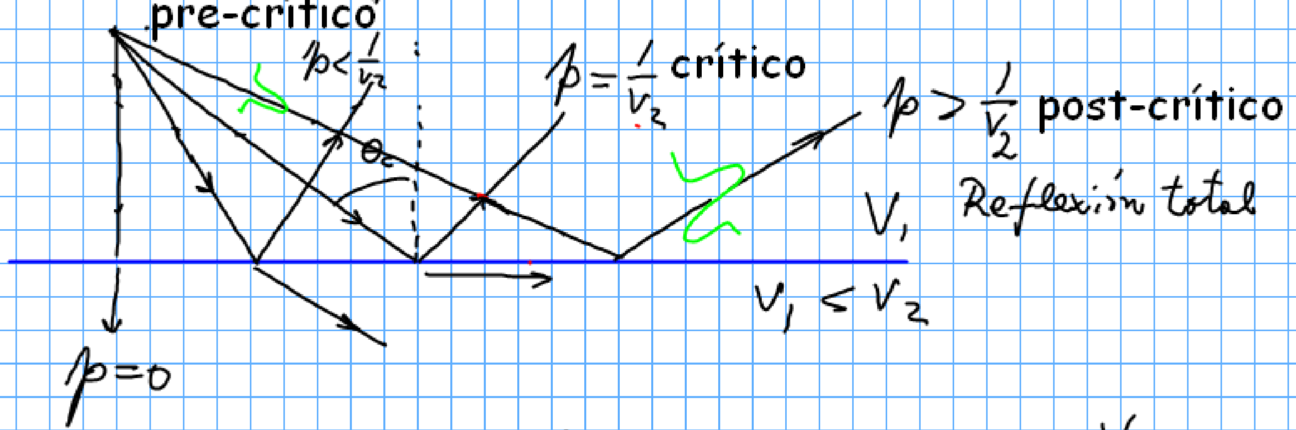


$$= \frac{I_2 - I_1}{I_2 + I_1}$$

$I = \rho V =$ Impedancia Acústica.

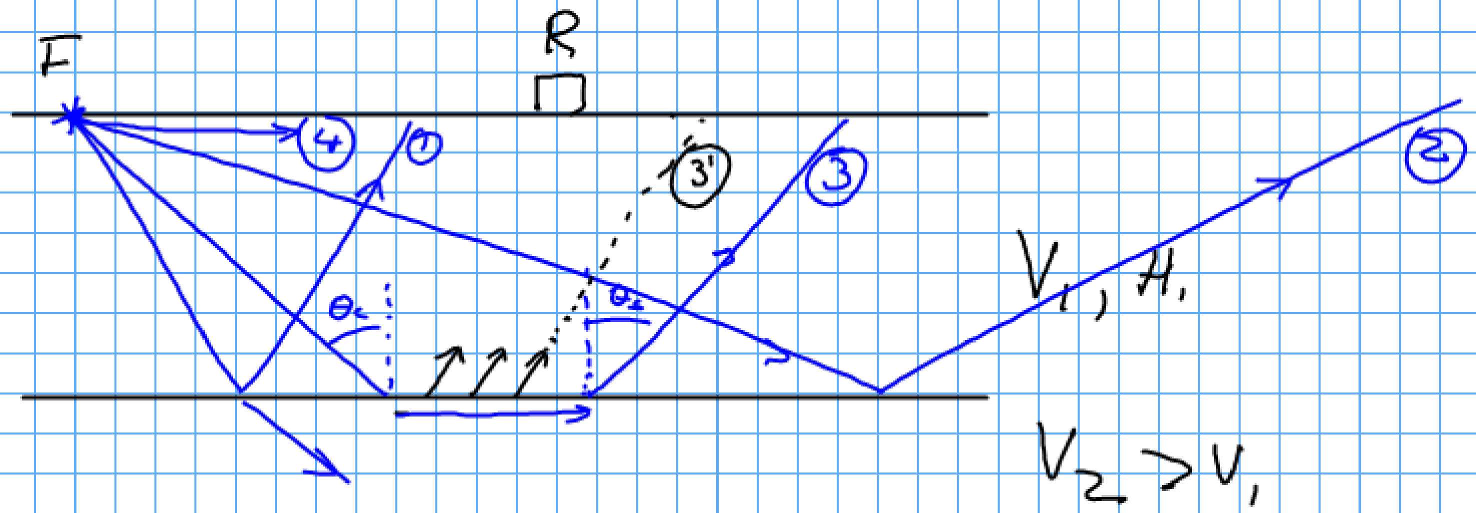
Sup. $V_1 < V_2$, $\frac{1}{V_1} > \frac{1}{V_2}$





$$\frac{\operatorname{sen}\theta_c}{V_1} = \frac{\operatorname{sen} 90^\circ}{V_2} = \frac{1}{V_2} \longrightarrow \operatorname{sen}\theta_c = \frac{V_1}{V_2}$$

1 Capa sobre semiespacio

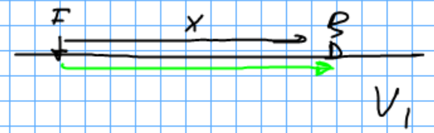


1, 2 Reflejadas ----> T_R

3 Refractada ----> T_H

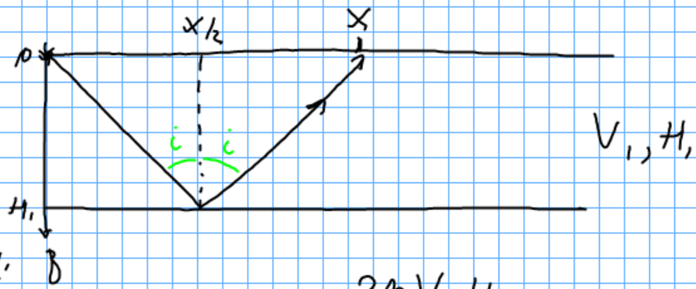
4 Directa -----> T_D

Directa



$$\boxed{T_D = \frac{X}{V_1}}$$

Reflejada



$$X = 2\int_0^{H_1} \frac{pV_1}{\sqrt{1-(pV_1)^2}}\,dz = \frac{2pV_1H_1}{\sqrt{1-(V_1p)^2}} = X(p)$$

$$T = 2\int_0^{H_1} \frac{dz}{V_1\sqrt{1-(pV_1)^2}} = \frac{2H_1}{V_1\sqrt{1-(V_1p)^2}} = T(p)$$

De X(p) se obtiene:

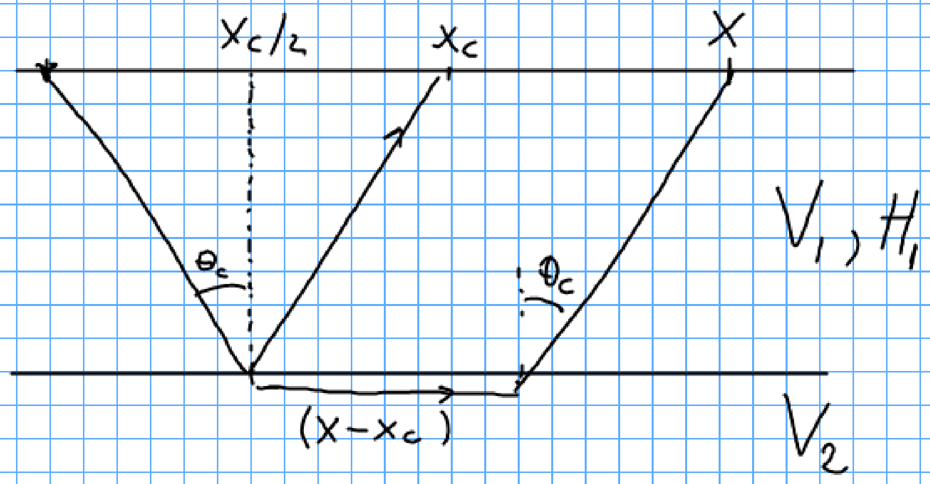
$$(pV_1)^2 = \frac{x^2}{x^2 + (2H_1)^2}$$

Reemplazando en T(p) :

$$\boxed{T^2 = T_R^2 = T_0^2 + \frac{x^2}{V_1^2} \quad \text{hipérbola}}$$

$$T_0 = \frac{2H_1}{V_1}$$ = Tiempo de reflexión normal

Refractada $(V_2 > V_1)$



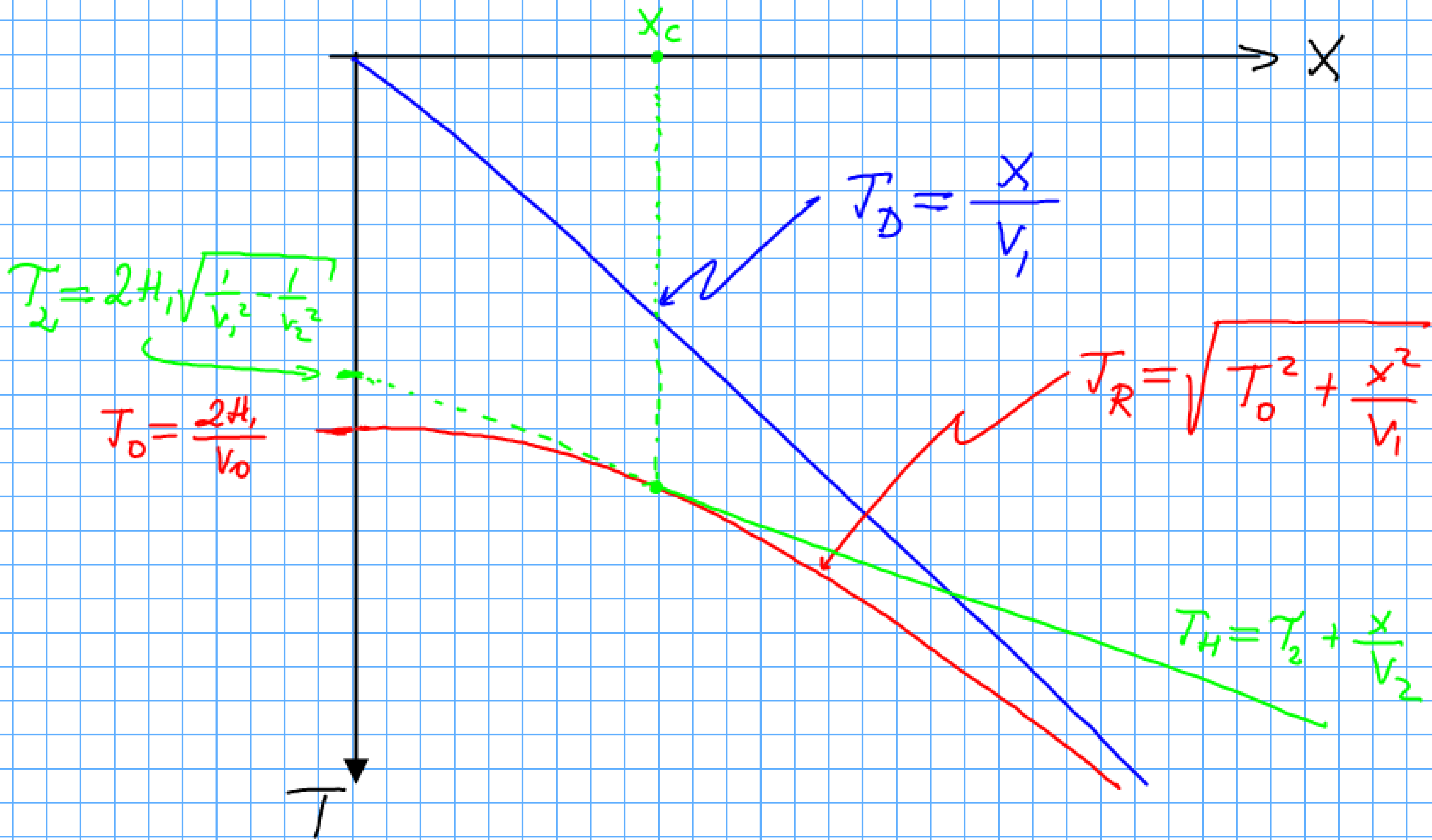
$$T_H(x) = T_R(x_c) + \frac{X - X_c}{V_2} \quad , \quad \operatorname{Sen}\theta_c = \frac{V_1}{V_2}$$

$$\frac{\frac{X_c}{2}}{H_1} = \tan\theta_c = \frac{\operatorname{sen}\theta_c}{\cos\theta_c} = \frac{\frac{V_1}{V_2}}{\sqrt{1 - \left(\frac{V_1}{V_2}\right)^2}}$$

$$\therefore \quad X_C = \frac{2 H_1 V_1}{\sqrt{V_2^2 - V_1^2}}$$

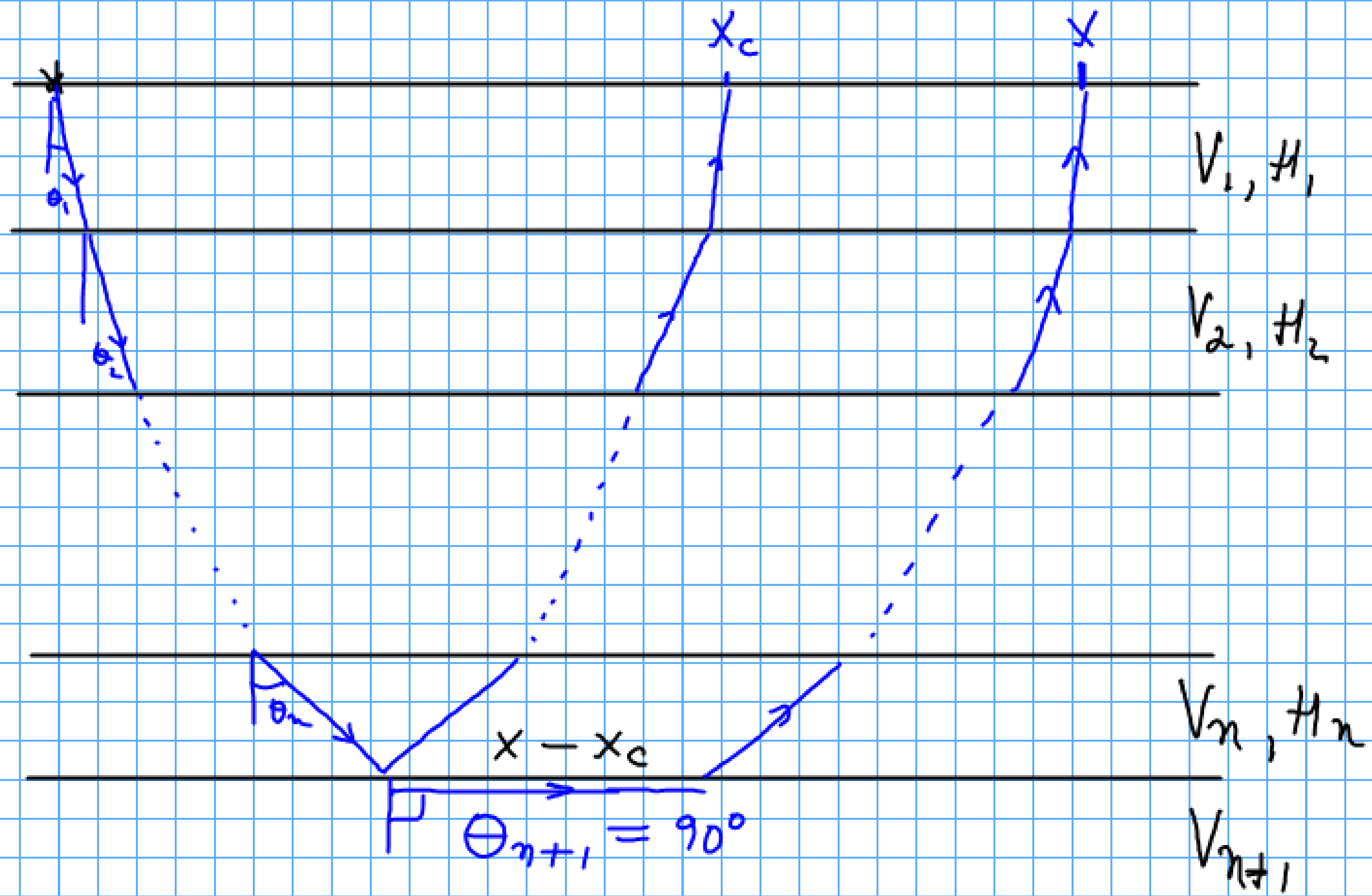
$$T_R(x_c) = \sqrt{T_0^2 + \frac{x_c^2}{V_1^2}} = \frac{2H_1 V_2}{V_1\sqrt{V_2^2 - V_1^2}}$$

$$\therefore \quad \boxed{T_H(x) = 2H_1\sqrt{\frac{1}{V_1^2} - \frac{1}{V_2^2}} + \frac{x}{V_2}, \quad x \geq x_c}$$



X_c
X
T
$T_D = \frac{X}{V_1}$
$T_2 = 2H_1\sqrt{\frac{1}{V_1^2} - \frac{1}{V_2^2}}$
$T_0 = \frac{2H_1}{V_0}$
$T_R = \sqrt{T_0^2 + \frac{x^2}{V_1}}$
$T_H = T_2 + \frac{X}{V_2}$

Onda refractada (H) al fondo de capa n



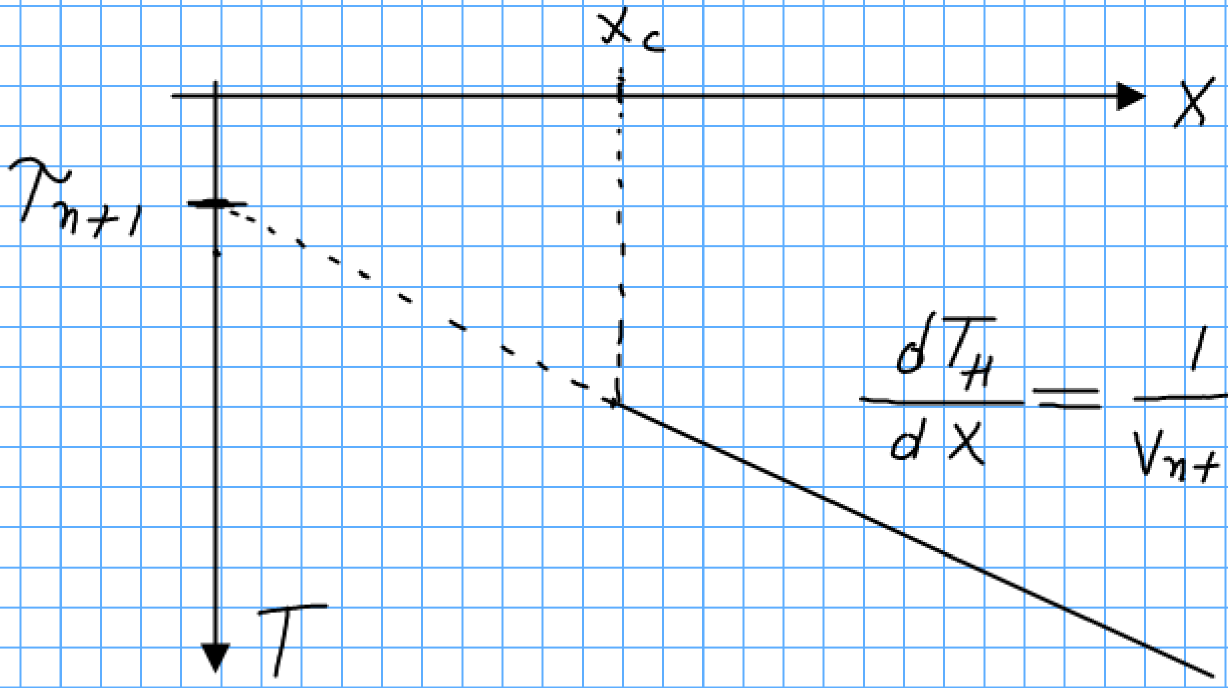
$$\frac{\operatorname{sen}\theta_1}{V_1} = \frac{\operatorname{sen}\theta_2}{V_2} = \cdots = \frac{\operatorname{sen}\theta_n}{V_n} = \frac{\operatorname{sen}\theta_{n+1}}{V_{n+1}} = \frac{1}{V_{n+1}} = p$$

$$\operatorname{sen}\theta_n = \frac{V_n}{V_{n+1}}, \quad \cdots \quad \operatorname{sen}\theta_i = \frac{V_i}{V_{n+1}} = pV_i$$

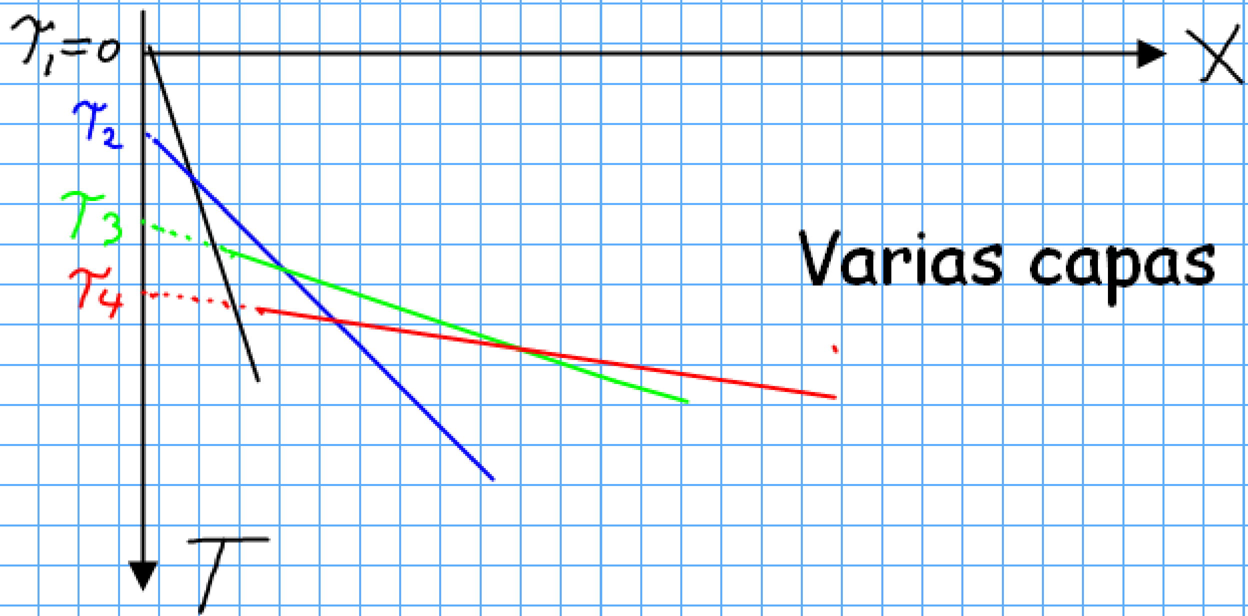
$$T_H(x) = T_R(x_c) + \frac{x - x_c}{V_{n+1}}$$

Después de desarrollos y álgebra (Tarea), se tiene:

$$T_H(x) = \underbrace{\sum_{i=1}^{n} 2H_i \sqrt{\frac{1}{V_i^2} - \frac{1}{V_{n+1}^2}}}_{T_{n+1}} + \frac{x}{V_{n+1}}$$

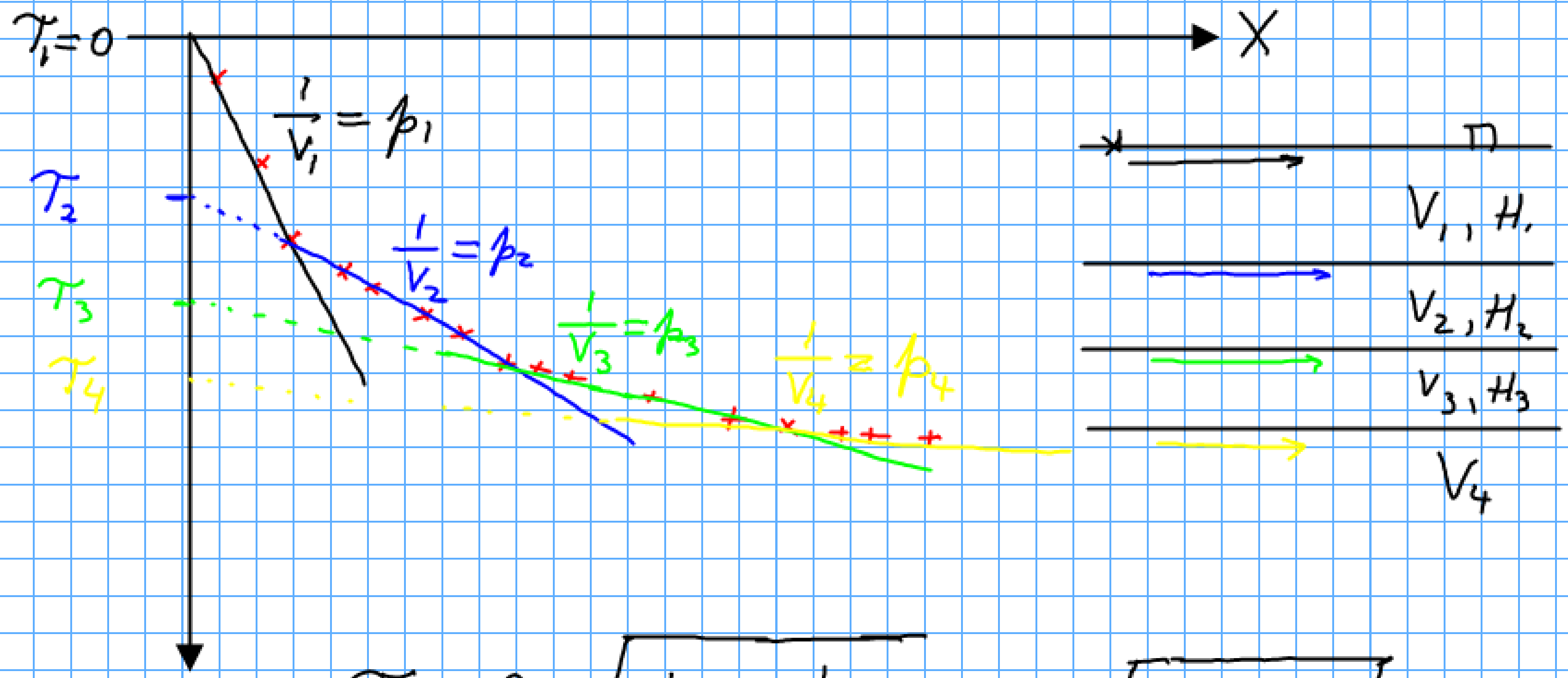


$$\frac{dT_H}{dX} = \frac{1}{V_{n+1}} = p$$



Varias capas

Interpretación de primeras llegadas en base a refracciones



$$\tau_2 = 2H_1\sqrt{\frac{1}{V_1^2} - \frac{1}{V_2^2}} = 2H_1\sqrt{p_1^2 - p_2^2}$$

$$\longrightarrow \quad H_1 = \frac{\tau_2/2}{\sqrt{p_1^2 - p_2^2}}$$

$$T_3 = 2H_1\sqrt{p_1^2 - p_3^2} + 2H_2\sqrt{p_2^2 - p_3^2}$$

$$\therefore \quad H_2 = \frac{T_3/2 - H_1\sqrt{p_1^2 - p_3^2}}{\sqrt{p_2^2 - p_3^2}}$$

$$\vdots$$

$$H_i = \frac{\tau_{i+1}/2 - \sum_{k=1}^{i-1} H_k\sqrt{p_k^2 - p_{i+1}^2}}{\sqrt{p_i^2 - p_{i+1}^2}}$$

Recursión TAU - SUM