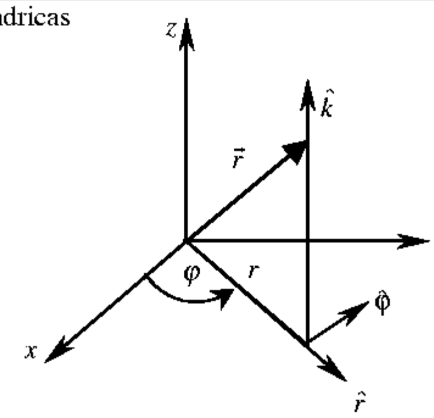
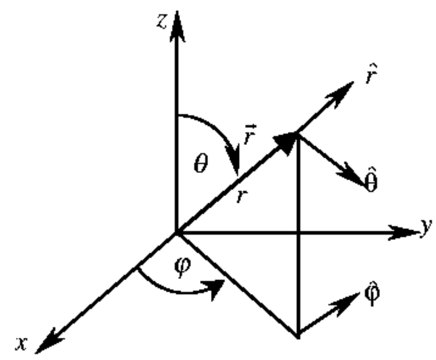
Formulario Matemático de Electromagnetismo

C.Cilíndricas	C. Esféricas
	

$$\vec{r} = r\hat{r} + z\hat{k}$$
$$x = r\cos\varphi$$
$$y = r\,\mathrm{sen}\,\varphi$$
$$z = z$$

$$\vec{r} = r\hat{r}$$
$$x = r\,\mathrm{sen}\,\theta\cos\varphi$$
$$y = r\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi$$
$$z = r\cos\theta$$

1. Gradientes

Cartesianas	Cilíndricas	Esféricas
$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$	$\nabla\phi = \frac{\partial\phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\varphi}\hat{\varphi} + \frac{\partial\phi}{\partial z}\hat{k}$	$\nabla\phi = \frac{\partial\phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial\phi}{\partial\varphi}\hat{\varphi}$

2. Divergencias

Cartesianas

$$\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Cilíndricas

$$\nabla\cdot\vec{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$$

Esféricas

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial(\mathrm{sen}\,\theta A_\theta)}{\partial\theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial A_\varphi}{\partial\varphi}$$

3. Rotores

Cartesianas

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i} + \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)\hat{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}$$

Cilíndricas

$$\nabla\times\vec{A} = \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\varphi} & \hat{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ A_r & rA_\varphi & A_z \end{vmatrix} = \frac{1}{r}\left\{\left(\frac{\partial A_z}{\partial\varphi} - \frac{\partial(rA_\varphi)}{\partial z}\right)\hat{r} + r\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\varphi} + \left(\frac{\partial(rA_\varphi)}{\partial r} - \frac{\partial A_r}{\partial\varphi}\right)\hat{k}\right\}$$

Esféricas

$$\nabla\times\vec{A} = \frac{1}{r^2\,\mathrm{sen}\,\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\,\mathrm{sen}\,\theta\hat{\varphi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ A_r & rA_\theta & r\,\mathrm{sen}\,\theta A_\varphi \end{vmatrix}$$

$$= \frac{1}{r^2\,\mathrm{sen}\,\theta}\left\{\left(\frac{\partial(r\,\mathrm{sen}\,\theta A_\varphi)}{\partial\theta} - \frac{\partial(rA_\theta)}{\partial\varphi}\right)\hat{r} + \left(\frac{\partial A_r}{\partial\varphi} - \frac{\partial(r\,\mathrm{sen}\,\theta A_\varphi)}{\partial r}\right)\hat{\theta} + \left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)r\,\mathrm{sen}\,\theta\hat{\varphi}\right\}$$

4. Laplacianos

Cartesianas	Cilindricas
$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}$	$\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2}+\frac{\partial^2\phi}{\partial z^2}$
Esféricas $\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right)+\frac{1}{r^2 \operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial\phi}{\partial\theta}\right)+\frac{1}{r^2\operatorname{sen}^2\theta}\frac{\partial^2\phi}{\partial\varphi^2}$	

5. Elementos diferenciales

De línea		
Cartesianas $d\vec{l} = dx\hat{i}+dy\hat{j}+dz\hat{k}$	Cilíndricas $d\vec{l} = dr\hat{r}+rd\varphi\varphi+dz\hat{k}$	Esféricas $d\vec{l} = dr\hat{r}+rd\theta\hat{\theta}+r\operatorname{sen}\theta d\varphi\hat{\varphi}$
De superficie		
Cartesianas $d\vec{s} = dydz\hat{i}+dxdz\hat{j}+dxdy\hat{k}$	Cilíndricas $d\vec{s} = rd\varphi dz\hat{r}+drdz\hat{\varphi}+rdrd\varphi\hat{k}$	Esféricas $d\vec{s} = r^2\operatorname{sen}\theta d\theta d\varphi\hat{r}+r\operatorname{sen}\theta drd\varphi\hat{\theta}+rd\theta dr\,\hat{\varphi}$
De volumen		
Cartesianas $dv = dxdydz$	Cilíndricas $dv = rdrd\varphi dz$	Esféricas $dv = r^2\operatorname{sen}\theta drd\varphi d\theta$

donde:

en cartesianas $\vec{A} = A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$

en cilíndricas $\vec{A} = A_r\hat{r}+A_\varphi\hat{\varphi}+A_z\hat{k}$

en esféricas $\vec{A} = A_r\hat{r}+A_\varphi\hat{\varphi}+A_\theta\hat{\theta}$

6. Identidades Vectoriales

$$\nabla\times(\nabla\phi)=0$$
$$\nabla\cdot(\nabla\times\vec{A})=0$$
$$\nabla\times(\nabla\times\vec{A})=\nabla(\nabla\cdot\vec{A})-\nabla^2\vec{A}$$
$$\nabla(\phi\psi)=\phi\nabla\psi+\psi\nabla\phi$$
$$\nabla\cdot(\phi\vec{A})=\phi\nabla\cdot\vec{A}+\vec{A}\cdot\nabla\phi$$
$$\nabla\times(f(r)r)=0$$
$$\nabla\left(\frac{1}{r}\right)=-\frac{\vec{r}}{r^3}\qquad(\text{con}\,|\vec{r}|=r)$$
$$\nabla\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\nabla\times\vec{A})-\vec{A}\cdot(\nabla\times\vec{B})$$
$$\nabla\cdot\vec{r}=3\qquad\nabla\times\vec{r}=0$$
$$\nabla(\vec{A}\cdot\vec{r})=\vec{A}$$
$$\nabla\times(\phi\vec{A})=\nabla\phi\times\vec{A}+\phi(\nabla\times\vec{A})$$
$$\nabla(r^n)=nr^{n-2}\vec{r}$$
$$\nabla^2\left(\frac{1}{r}\right)=\delta(\vec{r})$$
$$\nabla^2\left(\frac{1}{r}\right)=0\qquad(\text{para}\qquad r\neq 0)$$
$$\nabla\times(\vec{A}\times\vec{B})=(\vec{B}\cdot\nabla)\vec{A}-(\vec{A}\cdot\nabla)\vec{B}+\vec{A}\nabla\cdot\vec{B}-B\nabla\cdot\vec{A}$$
$$\nabla\times(\phi\vec{A})=\phi\nabla\times\vec{A}-\vec{A}\times\nabla\phi$$