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An algebraic-habits-of-mind perspective on elementary school

Bar graphs, among the earliest graphs that children make, require attention to two dimensions: which bar (horizontal position) and the bar's height.

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Although it is necessary to infuse courses and curricula with modern content, what is even more important is to give students the tools they will need in order to use, understand, and even make mathematics that does not yet exist. A curriculum organized around habits of mind tries to close the gap between what the users and makers of mathematics do and what they say (Cuoco, Goldenberg, and Mark 1996, p. 376).

The three articles in this cross-journal "contemporary curriculum issues" series are written by Al Cuoco, E. Paul Goldenberg, June Mark, and Sarah Sword—a team of curriculum developers at the Educational Development Center who pioneered work using mathematical habits of mind that are central to the work of mathematicians for organizing school mathematics curricula. Regardless of the level at which you teach, each article has an important message that is relevant across the grades.

This month, the article in *Teaching Children Mathematics* considers the ideas, logic, techniques, and habits of mind that algebra entails. When and to what extent can they be learned with intellectual integrity in the elementary school grades prior to a formal course on algebra?

In the *Mathematics in the Middle School* article, the authors argue that developing mathematical habits of mind in the middle grades is essential for making the crucial transition from arithmetic to algebra.

The authors of the *Mathematics Teacher* article reflect on their work in using the habits-of-mind approach for organizing high school curricula. They indicate that the approach offers a vehicle for paring down the collection of methods and techniques one needs in high school, leaving a small set of general-purpose tools that tie together many seemingly different mathematical terrains.

As we seek improvement in students' mathematical learning in the United States, a key component is to build coherence in the development of mathematical ideas across the grades. Knowing the mathematical experiences, understandings, skills, and habits of mind that students bring to your grade level and what the expectations are for grades following yours can help you build a smoother transition on each end for your students.

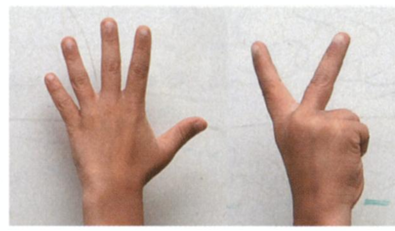
Common wisdom tells teachers to introduce arithmetic first, algebra later. Reality is not so simple. Some algebraic ideas—for instance, those about the properties of binary operations apart from the numbers these operations may combine—develop naturally before children learn arithmetic. In fact, they must develop before arithmetic can make sense.

Using children's natural algebraic ideas to develop mathematical habits of mind can lead to deeper understanding in both algebra and arithmetic (Cuoco, Goldenberg, and Mark 1996; Cuoco, Goldenberg, and Mark 2010; Goldenberg 1996; Goldenberg and Shteingold 2003; Goldenberg and Shteingold 2007; Mark et al. 2010). If children are to become competent at mathematics, including arithmetic, those habits of mind must take precedence over rules, formulas, and procedures that do not derive from logic that the child can grasp. The fact that algebraic ideas, logic, and techniques can be organized around the development of mind makes clear that we are truly talking about habits of mind rather than features of mathematics or idiosyncrasies of mathematicians. This article describes two of these natural habits of mind.

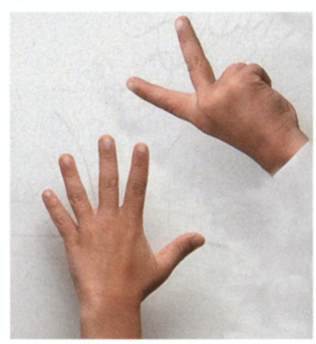
FIGURE 1

Before children learn conservation of number, they may not associate a fixed number with an image.

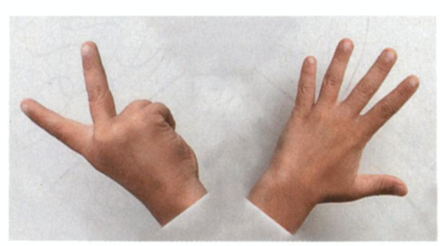
(a) Later, new conservers may not yet know how many fingers are showing without counting them.



(b) But they will be sure that the number stays the same if the hands move this way.



(c) And even if the hands move as in this image, they will be sure that the number stays the same.



A property of addition before addition

For young children, what will later be formalized as the commutative and associative laws of addition begins as an intuitive sense of stability, or invariance, of quantity under rearrangement. Piaget (1952) called it *conservation of number*; Wirtz and others (1962) and Sawyer (2003) called it the *any-order, any-grouping* property. Before conservation, arrangement trumps number, but **figure 1a** may not have a fixed number associated with it. Later, the new conserver may not yet know how many fingers **figure 1a** shows without counting but will be sure that the number, whatever it is, stays put if the hands are moved as in **figure 1b** or even as in **figure 1c**. That algebraic idea, a property of aggregation, must exist before the arithmetic fact—knowing what number 2 + 5 is—can make sense.

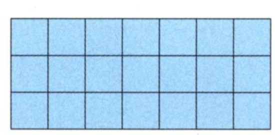
In a similar way, if an instructor hides a group of coins and asks, How much money is there? children for whom the question makes any sense will be absolutely certain that an answer exists and that only one answer is correct. They may be unsure about counting methods and may think that some methods might give incorrect answers, but conservers will know that just one correct answer exists. In fact, any child who really believes that the hidden amount can vary is not cognitively ready for the question of what the amount is. There is no "the amount" if it can vary. Most six-year-olds do not yet conserve number; by age seven, nearly all children do.

Having confidence that all three images in **figure 1** represent the same quantity is not the same as knowing the commutative property, which is not about the arrangement of physical objects in space but about the behavior of a particular element (here, the +, or the plus sign) in a formal syntactic system of written symbols. In some contexts, children can make perfect sense of written symbols—even significant parts of algebraic notation—but most young children cannot make sense of formal operations on a string of symbols. So, at this early stage, commutativity remains largely an intuitively obvious idea about the physics of mathematics: the nature of aggregation, not the nature of symbols. Even so, educators can support a young child's logic better by recognizing that it already relies on the underlying ideas that formal mathematics will later codify. Children see that the principle applies regardless of the numbers. The principle captures the essential algebraic aspect of the structure of addition that commutativity is about.

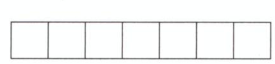
FIGURE 2

Such expressions as 7 + 7 + 7 and 3 × 7 are both language and are best introduced as mathematical descriptions that communicate partly without analyzing the language formally.

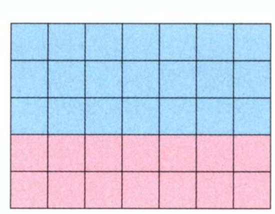
(a) Images require visual rather than linguistic analysis. This array shows something *threeish* and something *sevenish*. It is twenty-one because of its twenty-one squares, but it is also a picture of three sevens: a multiplication fact.



(b) To connect three sevens with twenty-one, we must agree that what makes this figure *seven* is its seven squares.



(c) Three sevens and two sevens makes five sevens.



Logical precursors

Pick a number. Multiply it by five; also multiply your original number by two. Now add the results. You get the same answer you would get if you multiplied your original number by seven. A general statement of that fact, the distributive property, is possibly the most central idea in elementary school arithmetic, key to understanding the algorithms, at the core of fluent mental calculations (e.g., 102 × 27 can be computed in two parts, as 100 × 27 + 2 × 27), and the logical basis for many rules of algebra that might otherwise seem arbitrary. This property relates multiplication and addition, but children know it long before they ever meet multiplication. The property is in the language (and logic) that youngsters use when they say that five (fingers, pennies, or 27s) plus two (fingers, pennies, or 27s) makes seven (fingers, pennies, or 27s). The following dialogues with six-year-olds late in their kindergarten year give a sense of what their logic does and does not handle. What distinguishes the questions the children get right from those they get wrong? What logic might explain the particular wrong answers they get? (**T** indicates the teacher's comments; **S1** and **S2** are both female students.)

T: What's a really big number?
S1: A million!
T: Suppose I asked, "How much is a thousand plus a thousand?" What would you say?
S1: [*with a big smile*] I have no idea!
T: And suppose I asked, "How much is two thousand plus three thousand?"
S1: [*thinking, then with confidence*] Five thousand!
T: Suppose I asked, "How much is a hundred plus a hundred?" What would you say?
S2: A hundred.
T: What about, "How much is two hundred plus three hundred?"

S2: Five hundred.

T: [*playfully*] And what if I asked, "How much is a thousand plus a thousand?"

S2: A million!

As soon as children are comfortable enough with the idea, the language, and the knowledge—perhaps late in kindergarten or early in first grade—to answer the question, How much is three sheep plus two sheep? they will happily apply the idea, the language, and the knowledge to give the correct answer to the spoken question, How much is three hundred plus two hundred? or even to the question, How much is three-eighths plus two-eighths? However, what they have in mind may well be quite different from what adults have in mind when we give the same answer.

When teachers ask a slightly different question, How much is a hundred plus a hundred? (with no audible preceding small numbers such as two or three), young six-year-olds may repeat the words *a hundred* or say something such as *a million*.

If, instead, a teacher asks, How much is an eighth plus an eighth? little ones may give just a puzzled stare and no answer at all. If their arithmetic is strong enough, they might possibly count and answer *sixteen* (or, sometimes, *nine*).

Why such errors are made and why *hundred* and *eighth* lead to different errors are beyond the scope of this article. The point is that when no audible small numbers are given, little children tend to give wrong answers. But when the numbers are not too large, even some kindergartners tend to answer correctly; more first graders do; and we can absolutely count on it in second grade. Whatever an *eighth* or a *hundred* is, children are sure that three of them plus two of them is five of them! Although this does not constitute knowing the distributive property, it does tell us that the children already grasp the underlying idea that the distributive property will later formally encode.

If we use *sevens* (a fully understood fixed quantity) in place of *hundreds* (which may still be a nonspecific *zillions* for young children), youngsters still know that three of them plus two of them yields five of them. Once a child has a meaning for three sevens and that meaning is a specific number (even if the child does not yet remember which number), the child's long-standing logic, intuition, or linguistic knowledge that three sevens plus two sevens is five sevens becomes arithmetically usable.

The meaning might be given as an image (see **fig. 2a**), a sum ($7 + 7 + 7$), a product (3×7), or in other ways. Each way shows something *threeish* and something *sevenish*. Because $7 + 7 + 7$ and 3×7 are both language, such expressions are best introduced as (mathematical) descriptions of a situation—for example, the array image—that communicate partly without analyzing the language formally. The image, of course, requires some analysis, too—visual rather than linguistic—to see the three sevens. To connect three sevens with twenty-one, we must agree that what makes **figure 2b** *seven* is its seven squares. **Figure 2a** is twenty-one because of its twenty-one squares, but it is also a picture of three sevens: a multiplication fact. Then, **figure 2c** shows that three sevens and two sevens makes five sevens.

The spoken form is familiar: "Three sevens plus two sevens makes five sevens." The pictures support the semantics of the situation, helping to establish the role of sevens and preserve its numerical meaning rather than letting it degenerate into a nonnumeric object, like sheep. In contrast, the classical written form—$(3 \times 7) + (2 \times 7) = 5 \times 7$—is quite another story.

Spoken versus written symbols

A child's knowledge that the finger collections in **figure 1** can be described by the same number does not guarantee that he or she will know that the print statements $5 + 2$ and $2 + 5$ refer to the same number. The written language of mathematics presents challenges that can be finessed by spoken language and by appropriate visual presentations. Perhaps the most glaring example is the canonically incorrect fourth-grade response to $3/8 + 2/8 = ?$ Although $5/16$ is a common answer from fourth graders (and beyond), no first grader would ever respond, "Five-sixteenths." It is uninformative—in fact, misleading—to explain such errors simply by claiming that these expressions are too abstract or that children cannot handle symbols. Spoken words are symbols, too, and such words as *the*—which young children use flawlessly—are about as abstract as one can get. It is worth understanding the difference between **figure 1** and 5 + 2 = 2 + 5 to see why the challenge of print for children may not be a mathematical challenge.

Humans have evolved to be quite flexible about visual order and orientation, but in the life of any individual human, it takes some learning. Infants who have come to recognize a bottle when it is handed to them in the proper orientation (see **fig. 3a**) do not, at first, reach for it when it is handed to them in some unfamiliar orientation, for example, with the nipple visible but facing away (see **fig. 3b**). Very soon they *do* learn to recognize objects regardless of their orientation. Considering the visual processing required, this is quite an impressive accomplishment. Even if the bottle is presented in the same orientation but at different distances, very different images are projected onto the retina. The distortion of parts relative to each other can be extreme, and yet babies recognize all these projections—most of which they have never seen before—as the same object.

Although this complex neural computation needs data (learning) to tune it up, the ability is hard wired, an evolutionary gift essential for survival. Otherwise, we would have been meals for tigers that we did not recognize because they did not happen to be facing exactly the same way as when we first saw them. Our ancestors had to interpret different retinal images as being the same object as long as those images could be made *the same* under rotation, reflection, dilation, or certain projective transformations. As a result, our brains are adept at them.

FIGURE 3

Infants recognize a bottle handed to them in

(a) a familiar orientation

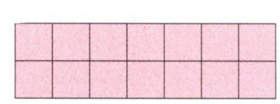
(b) but not when the orientation is unfamiliar.

But those ancestors did not read. The letters *d*, *b*, *q*, and *p* are all the same shape and differ only by rotation or reflection. To read, children must learn to see them as different objects, not as the same object in different orientations. So, young children's letter reversals are part of evolution's gift. To decode print, children must unlearn a principle that applies nearly everywhere else. They must treat print as an exception to the usual rules of seeing.

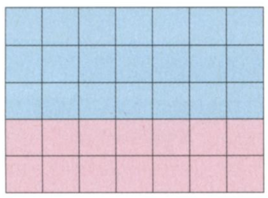
Moreover, *was* and *saw*—each just three print squiggles arranged in a different order—must *not* be recognized as the same. Alas, then come 2 + 5 and 5 + 2, two perfectly good examples of print squiggles that *are* to be treated as the same. (As always, the truth is not so simple. On a number line, numbers represent addresses—the names of specific points or locations along the line—and also distances between addresses. The child who enacts 2 + 5, perhaps by jumping along a large number line on the floor, would enact 5 + 2 differently.) It is therefore not surprising that the notation can cause confusion in some contexts, but this is an issue of notation, not of concept. Such written descriptions as $(3 \times 7) + (2 \times 7) = (3 + 2) \times 7$ are typically opaque, unless they arise as abbreviations of language that the children themselves use to describe such displays as **figure 2c**.

The trouble is not with the underlying mathematical idea but with the notation through which it is communicated. In fact, the way instructors of kindergartners and early first graders teach writing can help here, too. Children tell stories. The teacher encodes their language in writing. For example, children say that

combines with



to make



As they speak, the teacher writes

$$(3 \times 7) + (2 \times 7) = (5 \times 7).$$

Getting arithmetically good enough to use this valuable property takes time and practice. But the underlying idea is part of the child's cognitive structure as soon as the child can meaningfully make such statements as, "Two sheep plus three sheep is five sheep." Again, the underlying idea must be there before any practice of it can make sense.

Possibly because of print's special status, the logic that children apply when information is presented in spoken symbols may not be applied when the same information is presented in print. The canonical error with fractions is a perfect example. The spoken question, "How much is three-eighths plus two-eighths?" focuses attention on three plus two and tends to evoke the correct reasoning and get the correct answer. By contrast, the written question does not focus attention on the top numbers only:

$$\frac{3}{8} + \frac{2}{8} = ?$$

Children for whom the meaning is not already established tend to interpret the plus sign as *add everything in sight.* Mathematical reading and writing are different from prose reading and writing. Prose flows strictly left to right, in one dimension. Bar and coordinate graphs, histograms, charts, tables, and so on are two-dimensional records. One must attend to horizontal and vertical positions to interpret them. Even such symbolic expressions as

$$\frac{3}{8} + \frac{2}{8}$$

require attention to vertical position, as does 3^2, which is not the same as 32. Mathematical writing that is only horizontal cannot be read

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In the photograph, the distance from the tip of the nipple to the top of the bottle is the same as the length of the bottle. Measure to see for yourself.

strictly left to right. Both

$$2 \times (3 + 5) \text{ and } 7 + \_\_\_ = 5 + 4$$

require attention to the right side of the equation before attention to the left. In fact, $7 + 6 \div 2$ requires both left-to-right and right-to-left analysis: $6 \div 2$ must be evaluated left to right (because $2 \div 6$ is different), and yet the convention about order of operations dictates that the $6 \div 2$ part be evaluated before the addition that is specified by the $7 +$ part.

Algebra as a language

Algebraic notation is used in two distinct ways: for describing what we know and for deriving what we do not know. In the former, algebra is a language for describing the structure of a computation, a numerical pattern we have observed, a relationship among varying quantities, and so on. Young children are phenomenal language learners.

Exercises such as the one in **table 1** (but without the leftmost column) are familiar enough in many curricula. Children look for a pattern in the inputs and outputs, determine a rule, and complete the table. The Think Math! curriculum (EDC 2008) often adds a pattern indicator (the leftmost column) to problems of this kind.

Michelle, a second grader in a classroom using Think Math! completed **table 1** before her teacher had finished distributing copies to all the children. When the teacher asked how she had done it so fast, Michelle answered, "Well, I saw it was take away eight, because I looked at the twenty-eight and the twenty, and then I saw that ten and two was take away eight again, and then I saw eight and zero." Pointing to the leftmost column and grinning as if the teacher had left a clue by mistake, Michelle exclaimed, "Besides, it *says* it right here!"

How *did* she know? Nobody had ever discussed *variables* or *letters standing for numbers* or had even mentioned that first column. If Michelle had seen *only* **table 2**, with no examples to infer from, she most likely would *not* have felt that the symbols say anything. But having discovered the pattern, she thought that the symbols looked close enough to mean the same thing, so she *then* assigned them that meaning.

In other words, she did what little children excel at: She learned language (in this case, $n - 8$) from context. If algebraic language is part of the environment, used where context gives it meaning, children can apply their natural—and extraordinary—language-learning prowess to it and learn to use it descriptively.

TABLE 1

A pattern indicator gains meaning from context when it accompanies a find-a-rule exercise.

n	10	8	28	18	17			58	57
$n - 8$	2	0	20			3	4		

TABLE 2

A pattern indicator without a pattern from which to infer its meaning would be simply more to learn.

n	18	17			58	57
$n - 8$			3	4		

Just as children learning their native language understand, at first, more than they can say, Michelle could not immediately produce such descriptive language, but she and others try these interesting ways of writing what they know and, over time, become good at it. For instance, fourth graders learn this trick:

> Think of a number; add three; double that; subtract four; cut it in half; subtract your original number; your result is one.

They love the trick and want to show their parents and friends. They also want to know how it works. To explain, we can add pictures (see **fig. 4**). The act of doubling, which most fourth graders find quite natural and obvious, is, again, the distributive property in action. Although the expression $2(b + 3)$ does not make obvious what the result is, children readily learn to describe the third picture (see **fig. 4c**) as *two bags plus six* and abbreviate that description as $2b + 6$. They do not have to talk about *variables* or *letters standing for numbers*; they simply describe what they know and then write it as simply as they can. See a detailed description of this algebraic thinking with children on the Think Math! Web site (EDC 2009), and see Sawyer (1964) for the original source of the idea. Furthermore, Mark and her colleagues describe yet another way in which Think Math! gives students this algebra-as-description-of-what-you-know experience (Mark et al. 2010).

Why not teach algebra in grade 4?

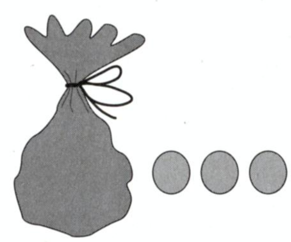
The other use of algebra—deriving what we do not know—is a formal syntactic operation on a set of symbols. Children are generally unable to divorce symbols from meanings before roughly age twelve; so, algebra as a course is not taught before fourth grade. This is not because fourth graders cannot handle symbols or abstract ideas—words are symbols; pictures are symbols; little children can be symbolic and abstract from babyhood—but because the use of the symbols differs. Formal operations on strings of algebraic symbols—rearranging them, apart from their semantics, to create other strings of symbols that solve a problem—are, well, formal operations, and children are not, by and large, formally opera-

FIGURE 4

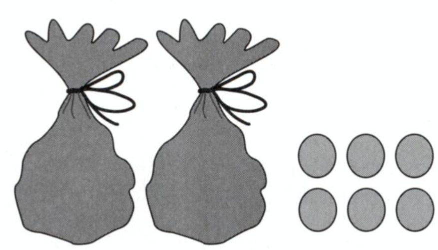
Children simply describe what they know and write it as simply as they can.

(a) For the "Think of a number" activity, picture a bag with a certain number of grapes in it.

(b) For *add three*, picture this:



(c) *Double that number* is shown as the following:



tional before age eleven and not reliably so before about age thirteen, thus the common need to wait until that age for algebra.

However, only the part of algebra that requires deduction by formal rules must wait that long. The part of algebra that is expressive of what we already know—that is, essentially, shorthand for semantic content clearly tied to a context we already understand—*that* part can be learned earlier. It is just language to express oneself, and children are excellent language learners. They do not learn language from explanations or formal lessons; they learn it from use in context. And, if it is learned all along, as it becomes developmentally possible, then, when the child is in late middle school, the transition to the new use of that language for deductive purposes could, presumably, be much easier, much more accessible for all children, much less of a brick wall of a million seemingly new things to learn all at once.

What about elementary school?

Taking advantage of children's natural algebraic ideas and honing them is a focus on habits of mind rather than on rules that can otherwise seem arbitrary. The precursors of commutative and distributive properties described earlier must be refined, honed, extended, practiced, codified, and generalized, but they are already there as natural logic, the child's natural habits of mind and the building blocks of higher mathematics. If children are to become competent at mathematics, including arithmetic, those habits of mind must take precedence over rules, formulas, and procedures that do not derive from logic that the child can grasp. In fact, children can grasp a lot more if the foundations for their learning are grounded in their logic, which gives students all the tools to understand, not just memorize, the algorithms for arithmetic with whole numbers and fractions. The dramatically disappointing result of learning rules apart from understanding is the tendency to easily get mixed up and use procedures that do not work (Carpenter et al. 1997).

Organizing the arithmetic part of the elementary school mathematics curriculum around mathematical habits of mind would not shift the curriculum dramatically in content, except to give more attention to mental arithmetic than is usual. Paper-and-pencil methods are engineered to make the work easy and to reduce the cognitive load of calculation, the amount of thinking one needs to do. Judiciously chosen mental arithmetic tasks both exercise and depend on mathematical ways of thinking that the paper-and-pencil algorithms deliberately try to avoid—mathematical ways of thinking that are the backbone of the successful preparation for algebra that we want for our students. What *would* shift if we were to emphasize habits of mind is the order in which students acquire content. Instead of being the preparatory step for computing, algorithms would become the culmination of understanding how the computation works, another case of describing what we already know and abbreviating that description. Taking full advantage of the natural logic and algebraic ideas of young learners, and helping them refine and communicate those ideas in mathematical language, would produce students who are better at arithmetic as well as better prepared for and familiar with algebra.

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